




# Lecture Outline: Announcements

- **Announcements** 
- Method Calls Review
- Recursion Review
- Recursive Tracing Practice

# Announcements

- Quiz 1 Tuesday (May 5)!
  - Topics: ListNodes, LinkedIntList, Runtime analysis
  - Topics not on Quiz 1: Recursion
  - Practice quizzes and reference sheet for Quiz 1 will be posted soon!
  - Quiz 0 feedback will be released before Quiz 1
- Programming Assignment 1 due in one week (May 6) @ 11:59pm
- Creative Project 1 and Resubmission Cycle 1 grades released today
- Resubmission Period 2 closes this Friday (May 1) @ 11:59pm
  - Available assignments: **C0**, P0, C1
  - ***Last opportunity to resubmit C0***
- Trien will be giving lecture on Friday!

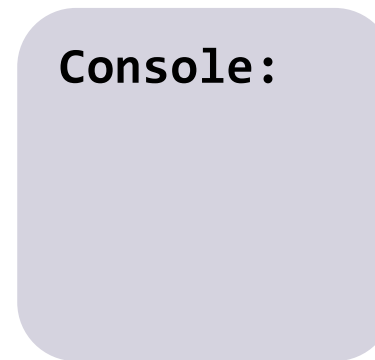
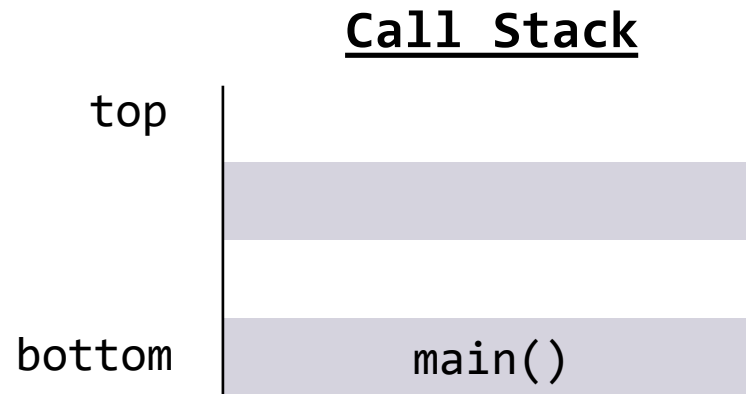
# Lecture Outline: Method Calls Review

- Announcements
- **Method Calls Review** ◀
- Recursion Review
- Recursive Tracing Practice

# Method Calls

- Regardless how you use them, methods work the same way!
  - Pause execution, finish method, return where you left off
- How does Java keep track of the prior method?
  - Sort of like the "history" of the method calls it has paused... (LIFO)
  - Something called the **Call Stack**

# Call Stack: Step 1



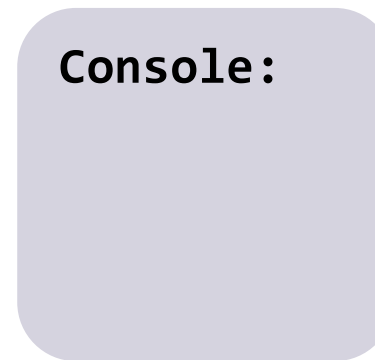
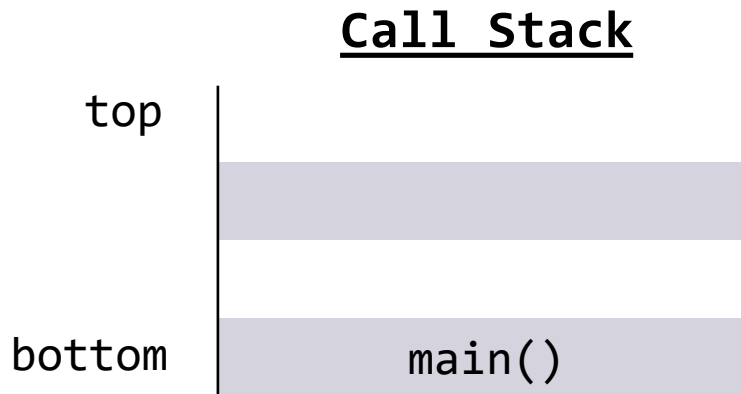
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 2



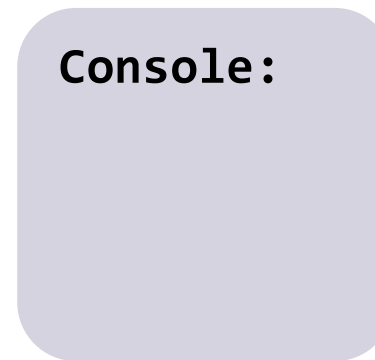
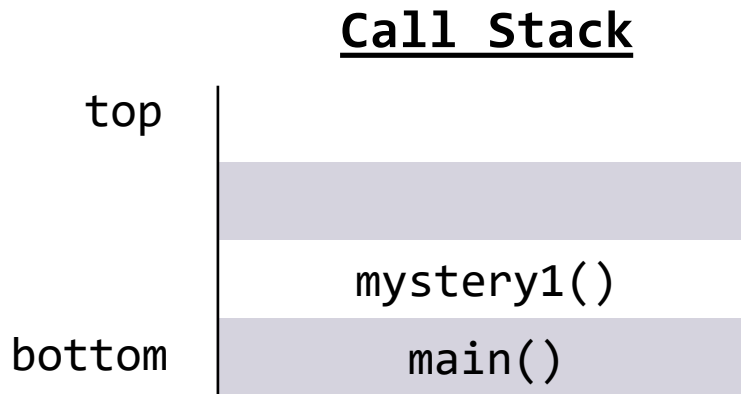
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 3



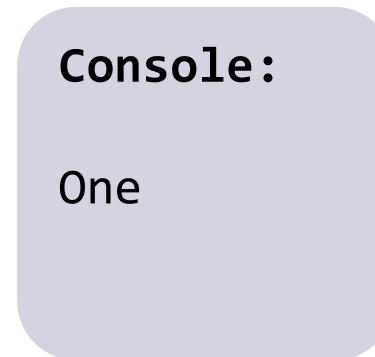
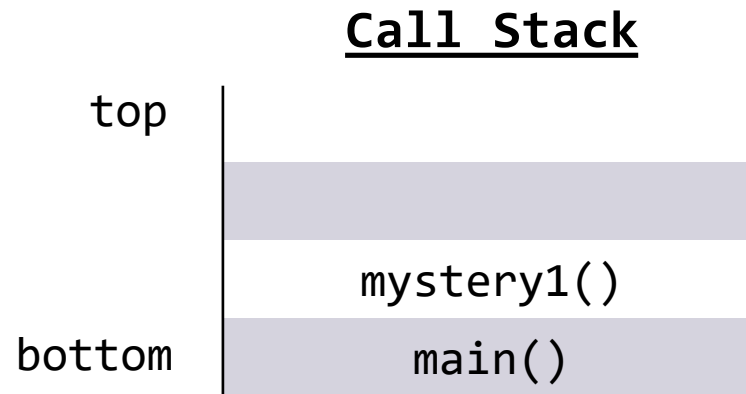
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 4



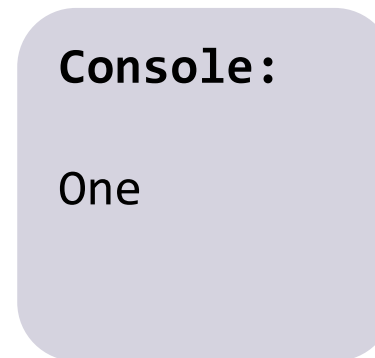
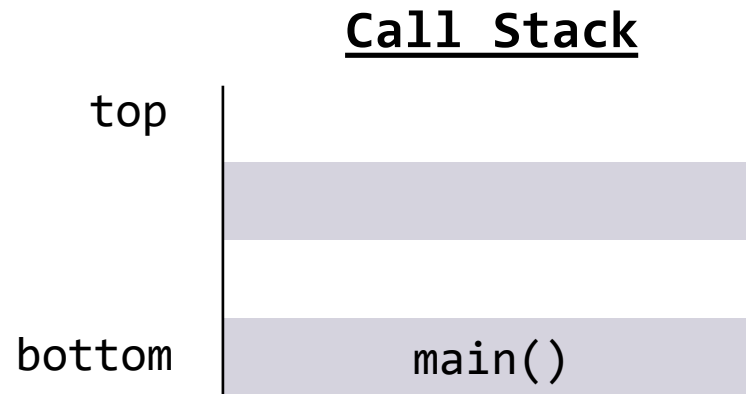
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 5



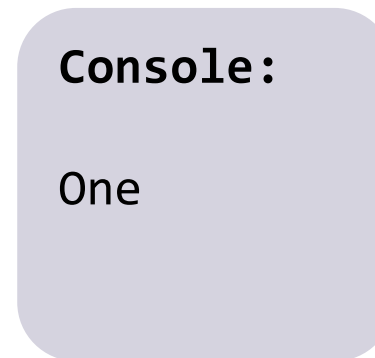
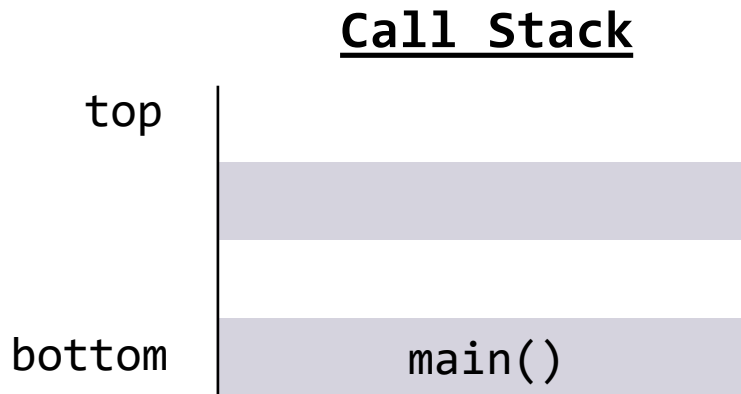
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 6



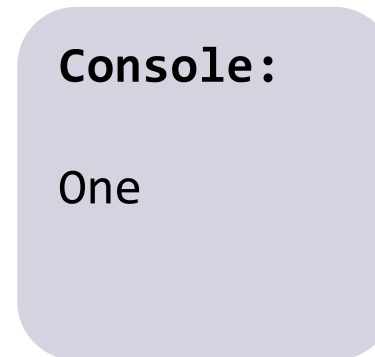
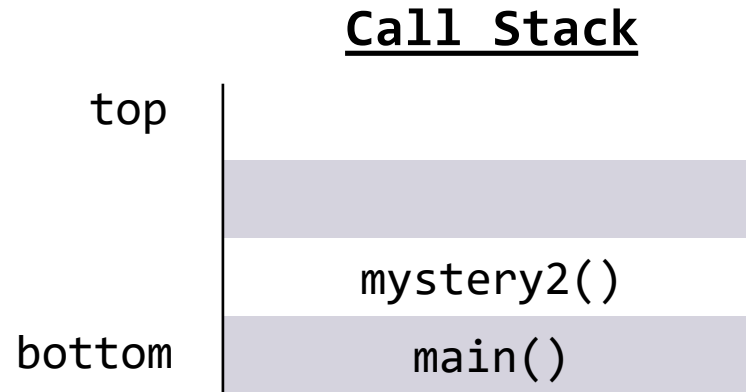
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 7



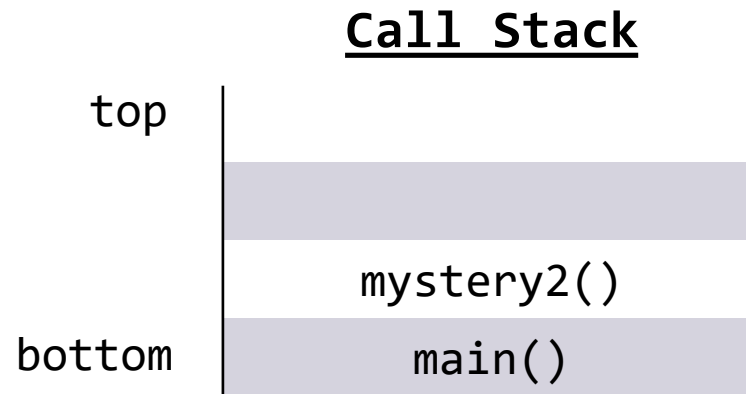
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 8



**Console:**

One  
Two

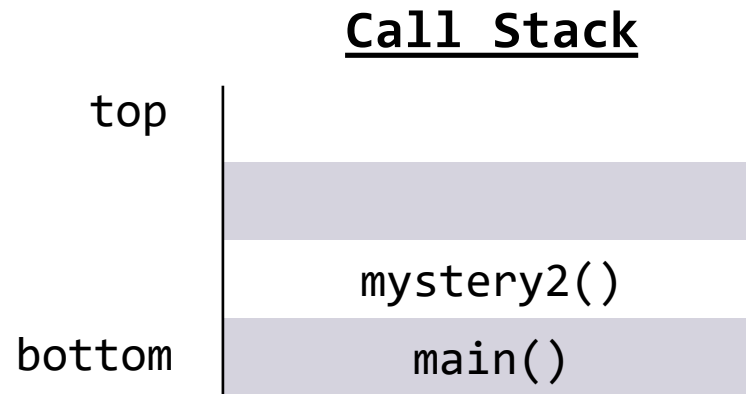
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 9



Console:

One  
Two

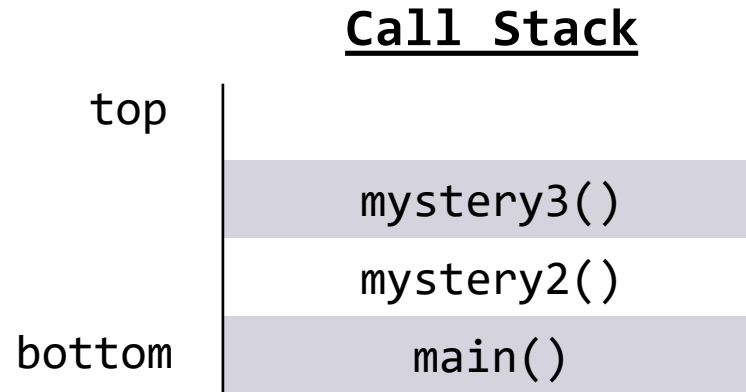
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 10



**Console:**

One  
Two

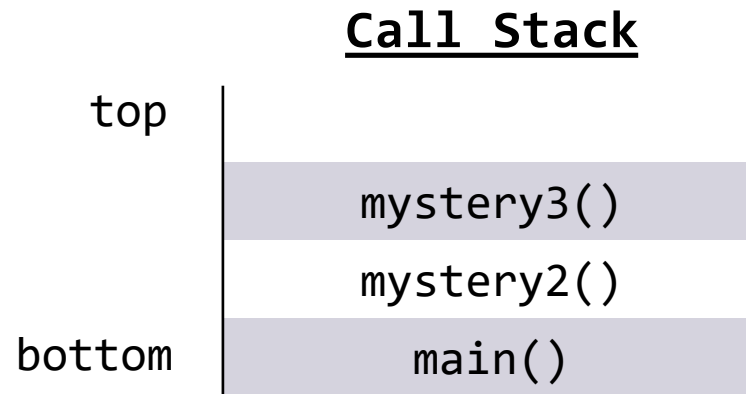
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 11



## Console:

One  
Two  
Three

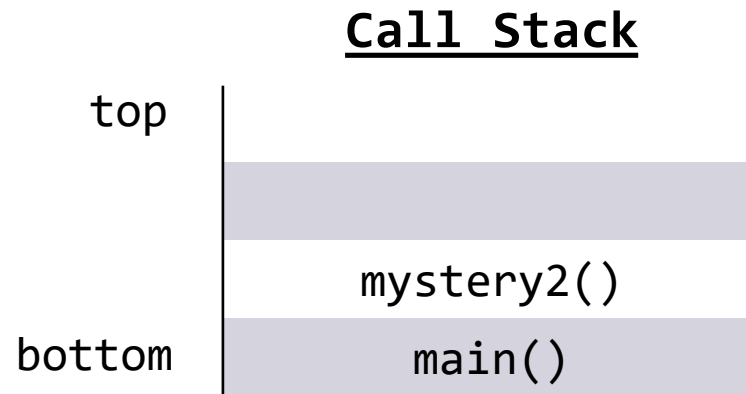
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 12



**Console:**

One  
Two  
Three

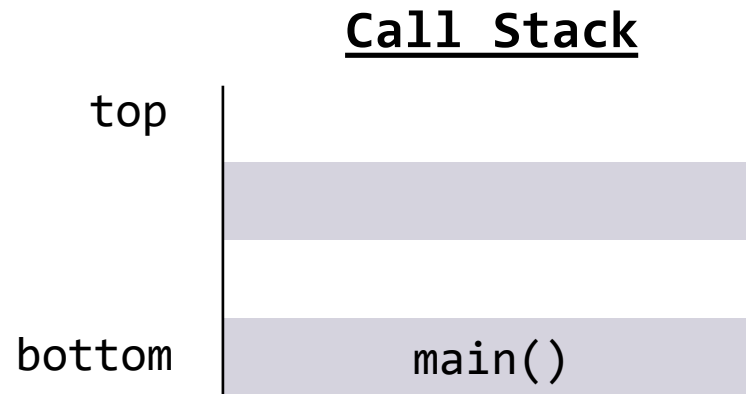
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 13



**Console:**

One  
Two  
Three

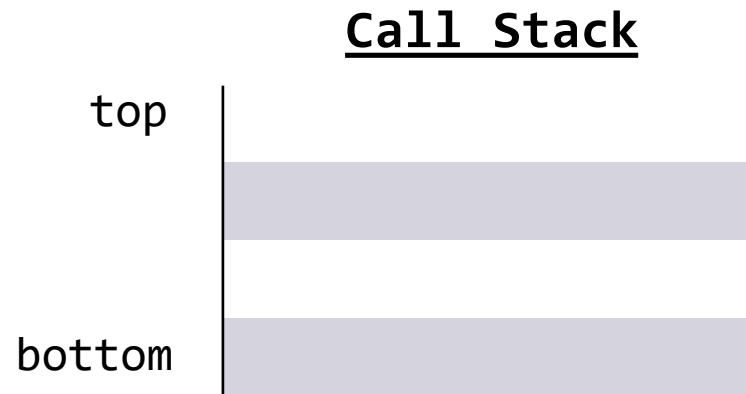
```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Call Stack: Step 14



**Console:**

One  
Two  
Three

```
public static void main(String[] args) {  
    mystery1();  
    mystery2();  
}
```

```
public static void mystery1() {  
    System.out.println("One");  
}
```

```
public static void mystery2() {  
    System.out.println("Two");  
    mystery3();  
}
```

```
public static void mystery3() {  
    System.out.println("Three");  
}
```

# Lecture Outline: Recursion Review

- Announcements
- Method Calls Review
- **Recursion Review** ◀
- Recursive Tracing Practice

# Recursion (1)

*“The repeated application of a recursive procedure or definition”*

- Oxford Languages

- Real-world definition: defining a problem in terms of itself
  - Case in point: above definition
  - Further natural examples:



# Recursion (2)

*“The repeated application of a recursive procedure or definition”*

- Oxford Languages

- Real-world definition: defining a problem in terms of itself
  - Case in point: above definition
  - Further natural examples:



# Recursion (3)

*“The repeated application of a recursive procedure or definition”*

- Oxford Languages

- Real-world definition: defining a problem in terms of itself
  - Case in point: above definition
- Computer science definition: when a method calls itself
  - “Alternative” to iteration (can combine for powerful results)

# Recursion (4)

*“The repeated application of a recursive procedure or definition”*

- Oxford Languages

- Why do we care?
  - It's a fundamental Computer Science topic
  - Some things are easier to describe with recursion than with loops
  - There are some problems that lend themselves more easily to a recursive solution than an iterative (loop-y) solution
    - Though all problems that can be solved with recursion can be solved with loops, and vice versa

# Method Calls

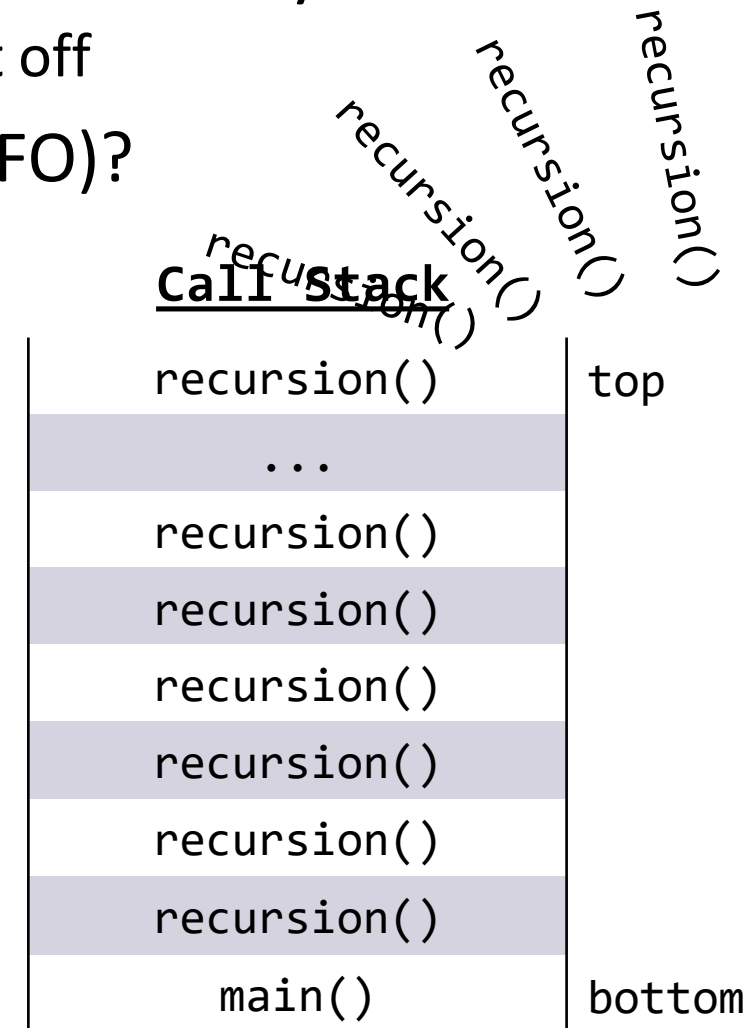


- Regardless how you use them, methods work the same way!
  - Pause execution, finish method, return where you left off
- How does Java keep track of the prior method (LIFO)?
  - Something called the **Call Stack**


🤔 Wouldn't that just lead to an infinite loop?

```
public static void recursion() {  
    System.out.println("Woah");  
    recursion();  
}
```

**StackOverflowException!**



# Method Calls: Avoiding Infinite Recursion

- Regardless how you use them, methods work the same way!
  - Pause execution, finish method, return where you left off
- How does Java keep track of the prior method (LIFO)?
  - Something called the **Call Stack**
-  Wouldn't that just lead to an infinite loop?
  - Yes! We get something called a **StackOverflowException**
- How do we avoid infinite recursion?

# Recursive Methods

- 2 components of every recursive method:
- Recursive case
  - Decompose problem into subproblem
  - Make the actual recursive call
  - Combine results meaningfully
- Base case
  - Simplest version of the problem
  - No subproblems to break into
  - Return known answer



# Recursive Methods: Decomposition

- 2 components of every recursive method:
- Recursive case
  - Decompose problem into subproblem
  - Make the actual recursive call
  - Combine results meaningfully
- Base case
  - Simplest version of the problem
  - No subproblems to break into
  - Return known answer



*If decomposing moves you closer to the base, no infinite recursion!*

# Math Examples

*$n!$  or  $factorial(n)$  = product of all positive integers  $\leq n$*

- Two ways of viewing this idea:
- $n! = n * (n - 1) * (n - 2) * \dots * 2 * 1$ 
  - Iterative approach - loop through all values and multiply together
- $n! = n * (n - 1)!$ 
  - Recursive approach – decompose into subproblem and combine
  - What would our base case / simplest input ( $n$ ) be?

# Math Examples: Hitting the Base Case

$n!$  / *factorial*( $n$ ) = product of all positive integers  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$



$$2! = 2 * 1!$$



$$1! = 1 * 0!$$

$$0! = 1$$

# Math Examples: Popping off callstack 1

$n!$  / *factorial*( $n$ ) = product of all positive integers  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$



$$2! = 2 * 1!$$



$$1! = 1 * 0!$$



$$0! = 1$$

# Math Examples: Popping off callstack 2

$n!$  / *factorial*( $n$ ) = product of all positive integers  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$



$$2! = 2 * 1!$$



$$1! = 1 * 1$$

# Math Examples: Popping off callstack 3

$n!$  / *factorial*( $n$ ) = product of all positive integers  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$



$$2! = 2 * 1!$$



$$1! = 1$$

# Math Examples: Popping off callstack 4

$n!$  / *factorial*( $n$ ) = product of all positive integers  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$



$$2! = 2 * 1$$

# Math Examples: Popping off callstack 5

$n!$  / *factorial*( $n$ ) = *product of all positive integers*  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$



$$2! = 2$$

# Math Examples: Popping off callstack 6

$n!$  / *factorial*( $n$ ) = *product of all positive integers*  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 3 * 2!$$

# Math Examples: Popping off callstack 7

$n!$  / *factorial*( $n$ ) = *product of all positive integers*  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 3!$$



$$3! = 6$$

# Math Examples: Popping off callstack 8

*$n!$  / factorial( $n$ ) = product of all positive integers  $\leq n$*

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 4 * 6$$

# Math Examples: Popping off callstack 9

$n!$  / *factorial*( $n$ ) = *product of all positive integers*  $\leq n$

- Reminder, recursive approach:  $n! = n * (n - 1)!$ 
  - Let's trace through with a simple example



$$4! = 24$$

# Lecture Outline: Recursive Tracing Practice

- Announcements
- Method Calls Review
- Recursion Review
- **Recursive Tracing Practice** ◀

