

CSE 123

Recursion

Questions during Class?

Raise hand or send here

sli.do #cse123



LEC 08

LEC 08: Recursion

BEFORE WE START

Talk to your neighbors:

*What's your favorite
icebreaker question? 🤝*

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Miya Natsuhara

TAs:	Arohan	Neha	Rushil	Johnathan	Nicholas
Sean	Hayden	Srihari	Benoit	Isayah	
Audrey	Chris	Andras	Jessica	Kavya	
Cynthia	Shreya	Kieran	Rohan	Eeshani	
Amy	Packard	Cora	Dixon	Nichole	
Trien	Lawrence	Liza	Helena		

Music: [CSE 123 25wi Lecture Tunes](#)

Lecture Outline

- Announcements 
- Recursion review
- Recursive Tracing Practice

Announcements

- Snow day!
 - Virtual lectures, virtual office hours, lighter IPL staffing today (Wed, Feb 5)
 - As of now, Thursday operations are expected to be "as usual"
- Quiz 1 Tuesday (Feb 11)!
 - Topics: ListNode, LinkedIntList, Runtime analysis
 - Topics not on Quiz: Recursion
 - Practice quiz and reference sheet for Quiz 1 will be posted soon!
 - Quiz 0 feedback will be released before Quiz 1
- Programming Assignment 1 due in one week (Feb 12) @ 11:59pm
- Creative Project 1 and Resubmission Cycle 1 grades released later today
- Resubmission Period 3 closes this Friday (Feb 7) @ 11:59pm
 - Available assignments: **C0**, P0, C1
 - **Last opportunity to resubmit C0**
- Reminder: Grade guarantee calculator

Lecture Outline

- Announcements
- **Recursion review** 
- Recursive Tracing Practice

Recursion (1)

“The repeated application of a recursive procedure or definition”

- Oxford Languages

- Real-world definition: defining a problem in terms of itself
 - Case in point: above definition
 - Further natural examples:



Recursion (2)

“The repeated application of a recursive procedure or definition”

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Recursion (3)

“The repeated application of a recursive procedure or definition”

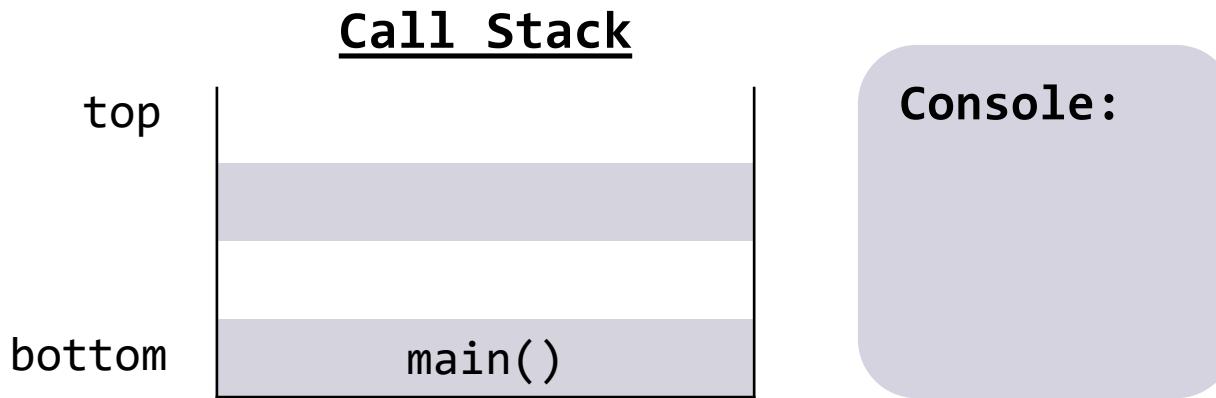
- Oxford Languages

- Real-world definition: defining a problem in terms of itself
 - Case in point: above definition
- Computer science definition: when a method calls itself
 - “Alternative” to iteration (can combine for powerful results)
-  Wouldn't that just lead to an infinite loop?
 - Yes! If you do things wrong...
 - Let's review *how* method calls work

Method Calls

- Regardless how you use them, methods work the same way!
 - Pause execution, finish method, return where you left off
- How does Java keep track of the prior method?
 - Sort of like the "history" of the method calls it has paused... (LIFO)
 - Something called the **Call Stack**

Call Stack



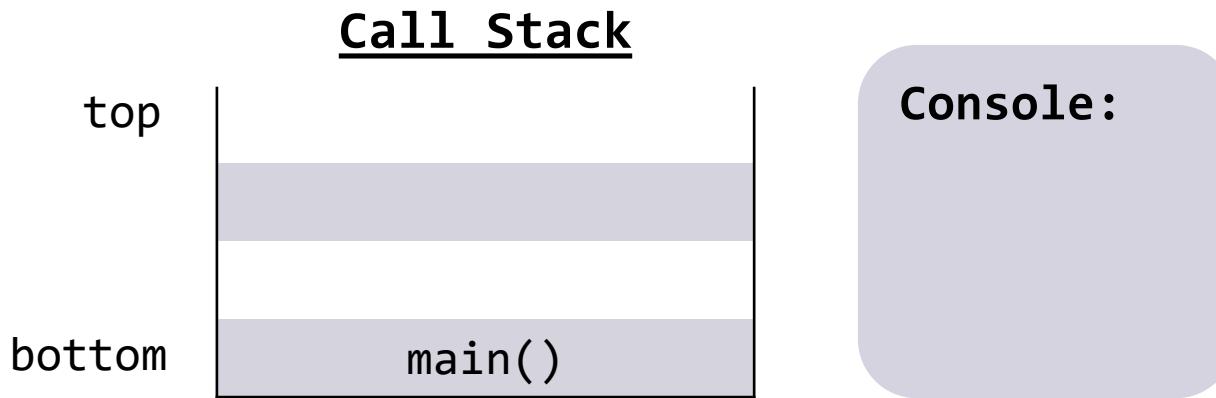
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public static void mystery1() {
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Call Stack



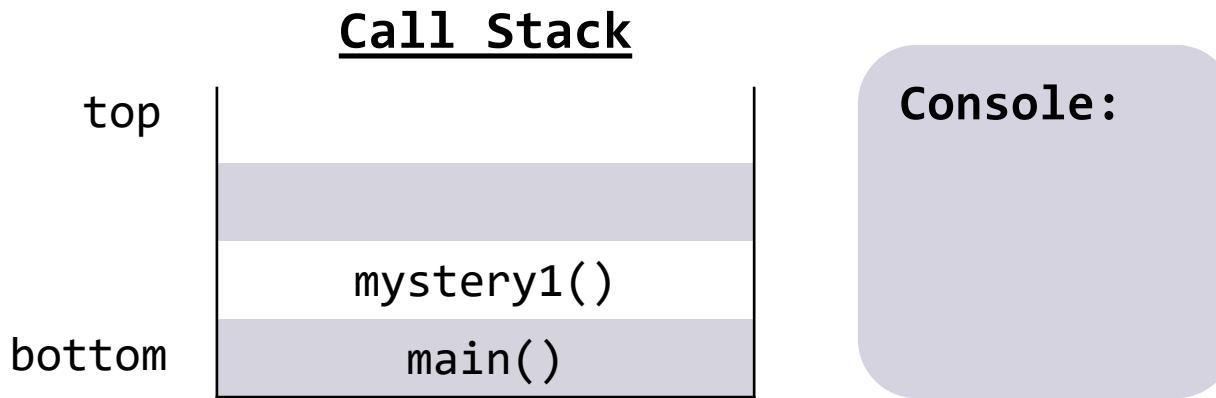
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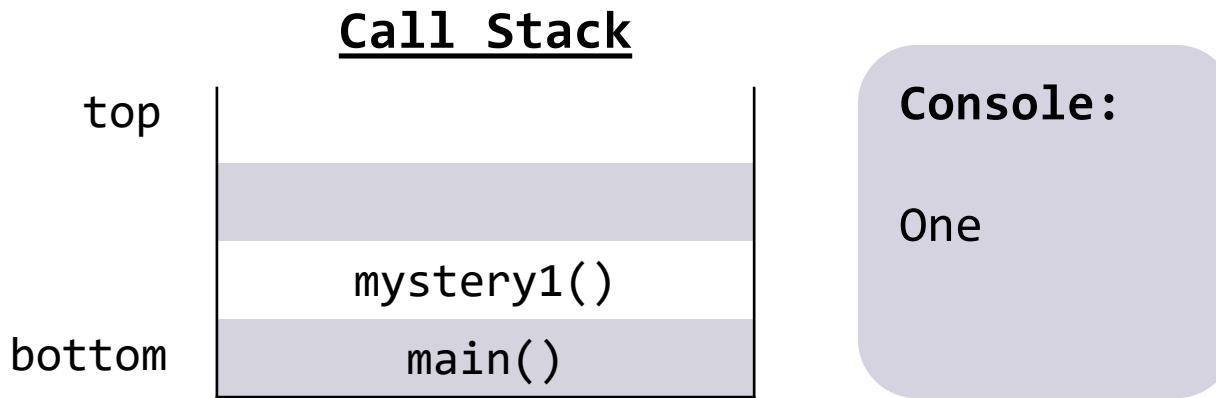
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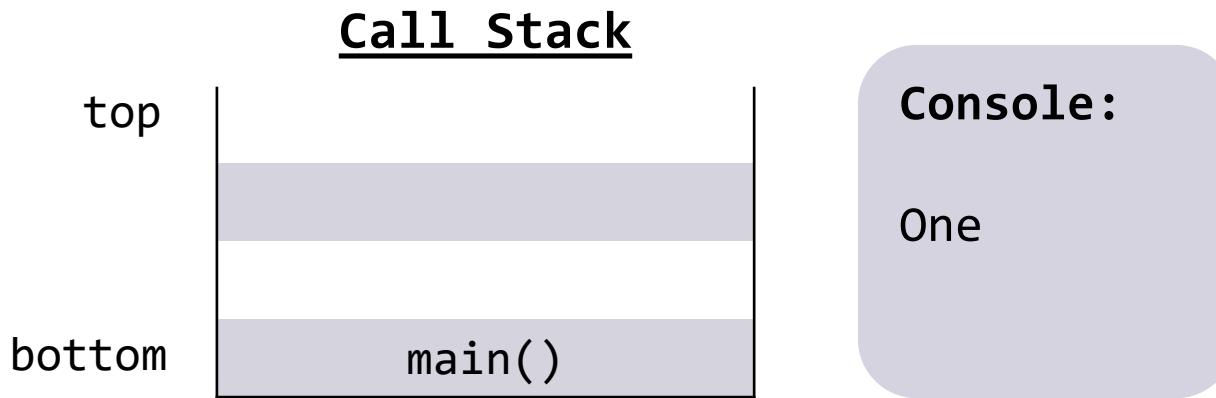
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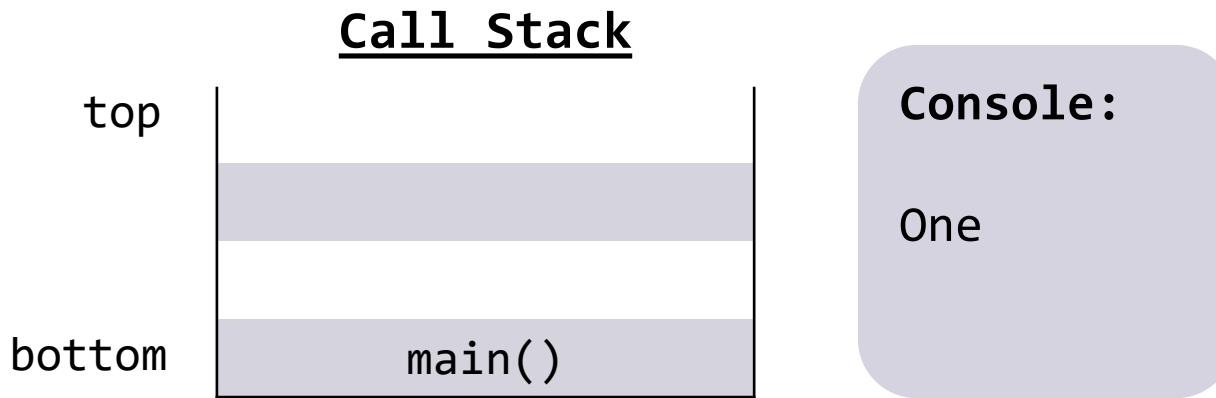
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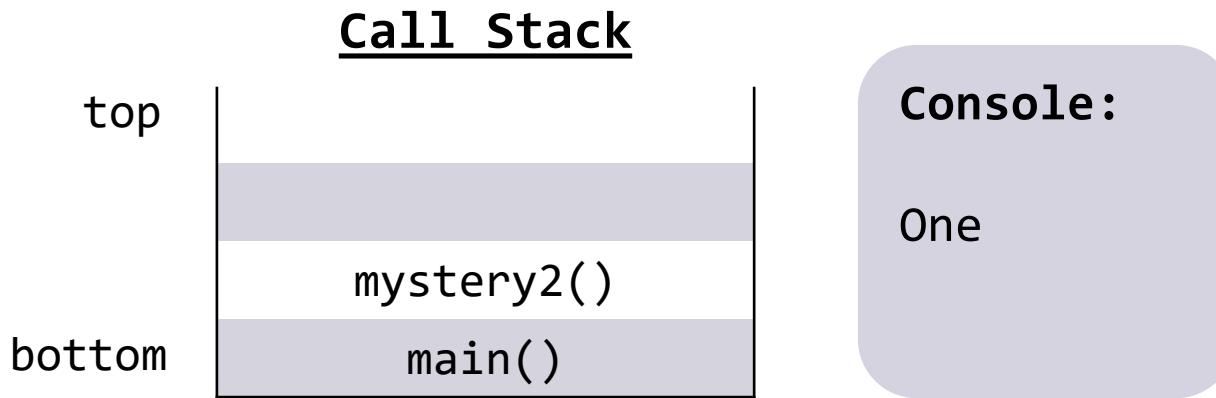
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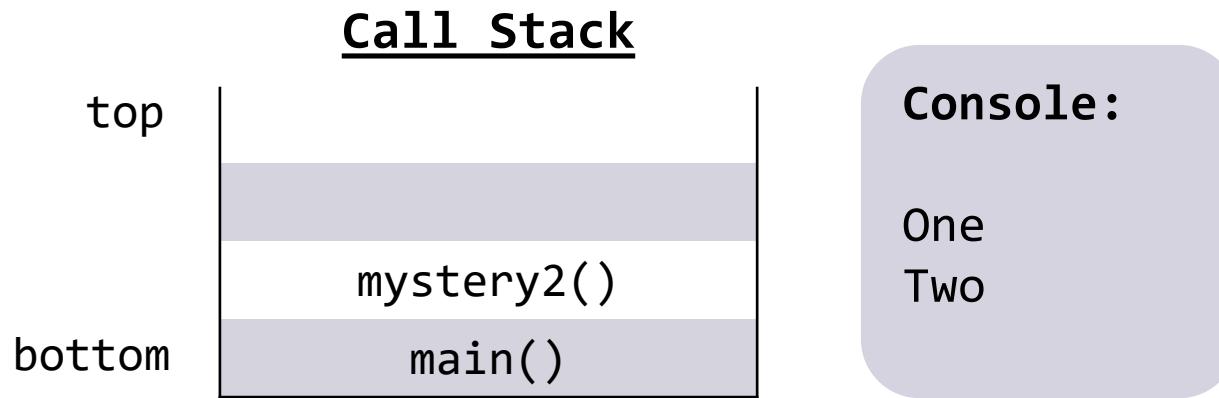
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Call Stack



Console:

One

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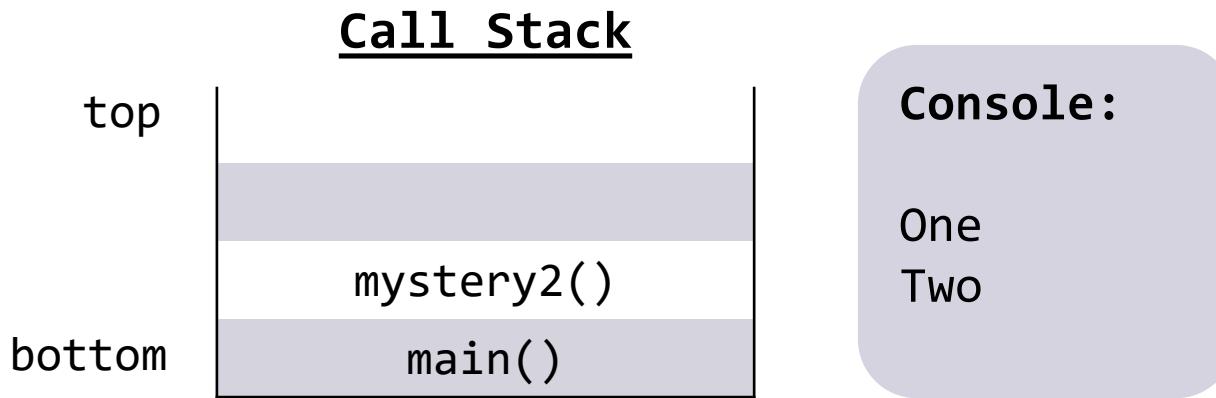
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Call Stack



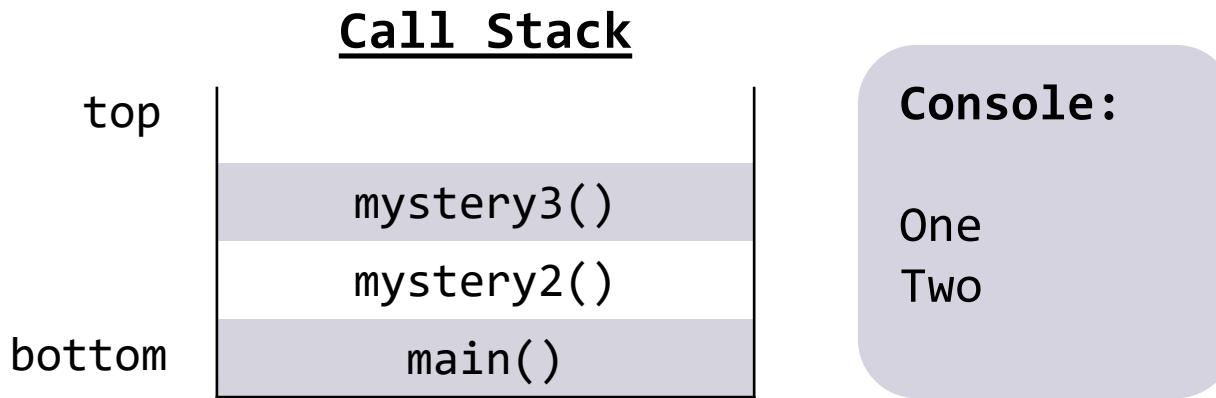
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Call Stack



Console:

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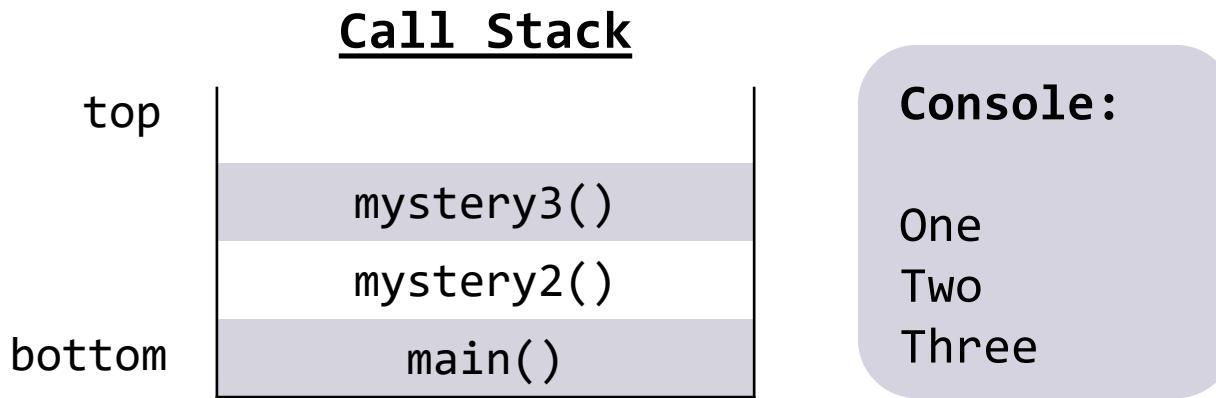
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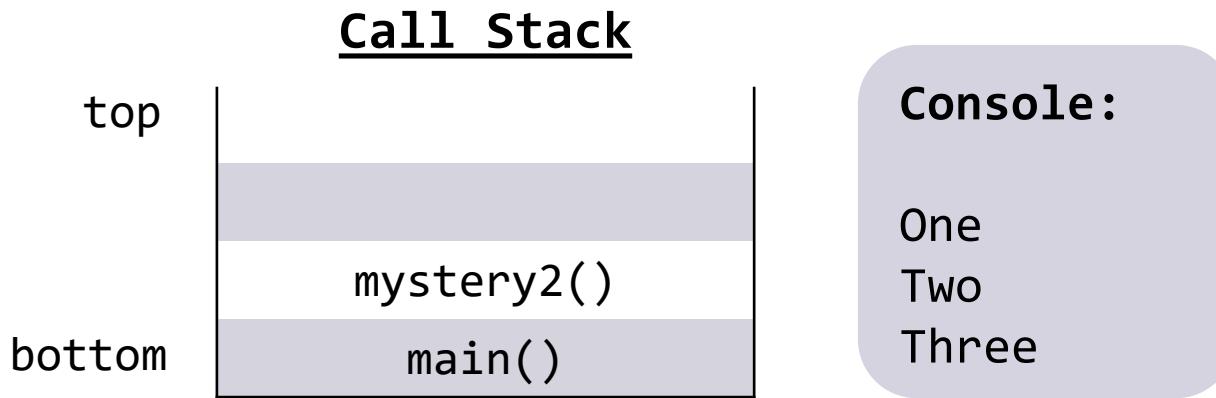
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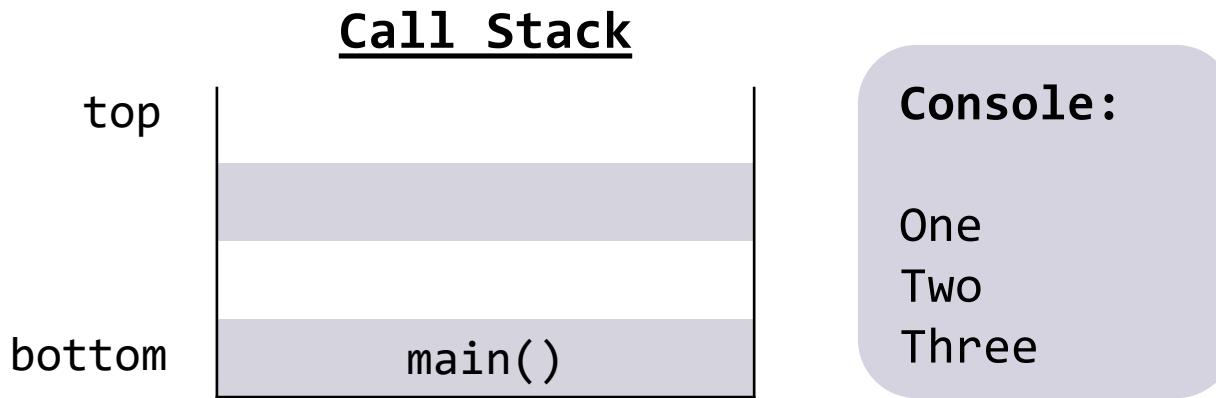
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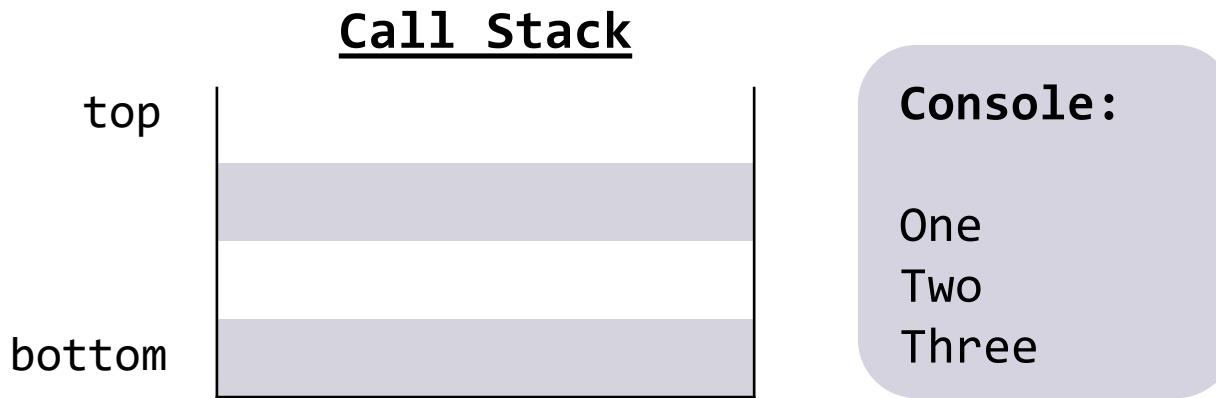
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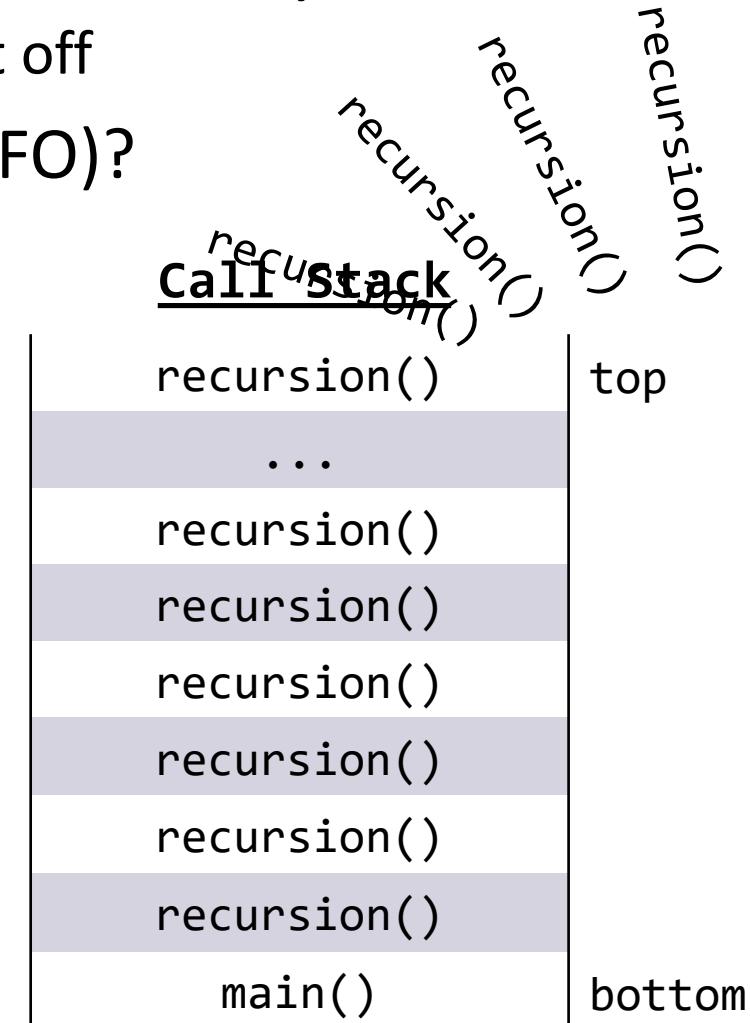
Method Calls

- Regardless how you use them, methods work the same way!
 - Pause execution, finish method, return where you left off
- How does Java keep track of the prior method (LIFO)?
 - Something called the **Call Stack**

⌚ Wouldn't that just lead to an infinite loop?

```
public static void recursion() {  
    System.out.println("Woah");  
    recursion();  
}
```

StackOverflowException!



Method Calls

- Regardless how you use them, methods work the same way!
 - Pause execution, finish method, return where you left off
- How does Java keep track of the prior method (LIFO)?
 - Something called the **Call Stack**



- Wouldn't that just lead to an infinite loop?
 - Yes! We get something called a **StackOverflowException**
- How do we avoid infinite recursion?

Recursion (4)

“The repeated application of a recursive procedure or definition”

- Oxford Languages

- Why do we care?
 - It's a fundamental Computer Science topic
 - Some things are easier to describe with recursion than with loops
 - There are some problems that lend themselves more easily to a recursive solution than an iterative (loop-y) solution
 - Though all problems that can be solved with recursion can be solved with loops, and vice versa

Recursive Methods

- 2 components of every recursive method:
- Recursive case
 - Decompose problem into subproblem
 - Make the actual recursive call
 - Combine results meaningfully
- Base case
 - Simplest version of the problem
 - No subproblems to break into
 - Return known answer



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If decomposing moves you closer to the base, no infinite recursion!

Math Examples

$n!$ or $\text{factorial}(n)$ = product of all positive integers $\leq n$

- Two ways of viewing this idea:
- $n! = n * (n - 1) * (n - 2) * \dots * 2 * 1$
 - Iterative approach - loop through all values and multiply together
- $n! = n * (n - 1)!$
 - Recursive approach – decompose into subproblem and combine
 - What would our base case / simplest input (n) be?

Math Examples

$n! / \text{factorial}(n) = \text{product of all positive integers} \leq n$

- Reminder, recursive approach: $n! = n * (n - 1)!$
 - Let's trace through with a simple example 3!



$$4! = 4 * 3!$$

$$3! = 3 * 2!$$

$$2! = 2 * 1!$$

$$1! = 1 * 0!$$

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$$4! = 4 * 3!$$



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$$2! = 2$$

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$$4! = 4 * 3! \quad 3! = 3 * 2$$

Math Examples

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$$4! = 4 * 3!$$

$$3! = 6$$

Math Examples

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- Reminder, recursive approach: $n! = n * (n - 1)!$
 - Let's trace through with a simple example 3!



$$4! = 4 * 6$$

Math Examples

$n! / \text{factorial}(n) = \text{product of all positive integers} \leq n$

- Reminder, recursive approach: $n! = n * (n - 1)!$
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$$4! = 24$$

