Lecture 11:
Hidden Surfaces

Reading

Required

- Hearn and Baker, 13.1-13.3, 13.6-13.7

Optional

- Foley *et al*, Chapter 15

The Quest for 3D

- Construct a 3D hierarchical geometric model
- Define a virtual camera
- Map points in 3D space to points in an image

- produce a wireframe drawing in 2D from a 3D object

- Of course, there's more work to be done…

Introduction

- Not every part of every 3D object is visible to a particular viewer. We need an algorithm to determine what parts of each object should get drawn.
- Known as “hidden surface elimination” or “visible surface determination”.
- Hidden surface elimination algorithms can be categorized in three major ways:
  - Object space vs. image space
  - Object order vs. image order
  - Sort first vs. sort last
  - Still a very active research area
- Where would we use a hidden surface algorithm?

Object Space Algorithms

- Operate on geometric primitives
  - For each object in the scene, compute the part of it which isn't obscured by any other object, then draw.
  - Must perform tests at high precision
  - Resulting information is resolution-independent

- Complexity
  - Must compare every pair of objects, so $O(n^2)$ for n objects
  - Optimizations can reduce this cost, but…
  - Best for scenes with few polygons or resolution-independent output

- Implementation
  - Difficult to implement!
  - Must carefully control numerical error

Image Space Algorithms

- Operate on pixels
  - For each pixel in the scene, find the object closest to the COP which intersects the projector through that pixel, then draw.
  - Perform tests at device resolution, result works only for that resolution

- Complexity
  - Must do something for every pixel in the scene, so at least $O(R)$.
  - Easiest solution is so test projector against every object, giving $O(nR)$.
  - More reasonable version only does work for pixels belonging to objects: $O(nr)$, r is number of pixels per object
  - Often, with more objects, each is smaller, so we estimate $nr = O(R)$ in practice

- Implementation
  - Usually very simple!

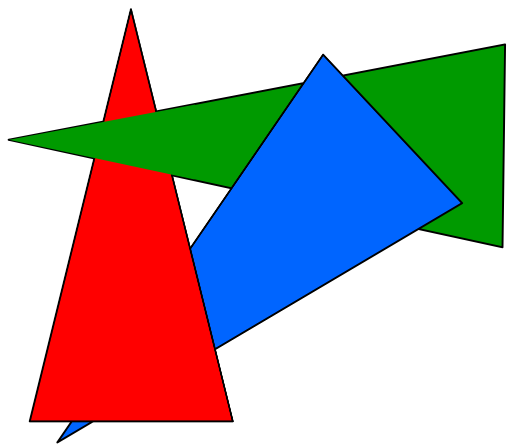
Object Order vs. Image Order

- Object order
  - Consider each object only once - draw its pixels and move on to the next object
  - Might draw the same pixel multiple times
- Image order
  - Consider each pixel only once - draw part of an object and move on to the next pixel
  - Might compute relationships between objects multiple times

Sort First vs. Sort Last

- Sort first
  - Find some depth-based ordering of the objects relative to the camera, then draw from back to front
  - Build an ordered data structure to avoid duplicating work

- Sort last
  - Sort implicitly as more information becomes available

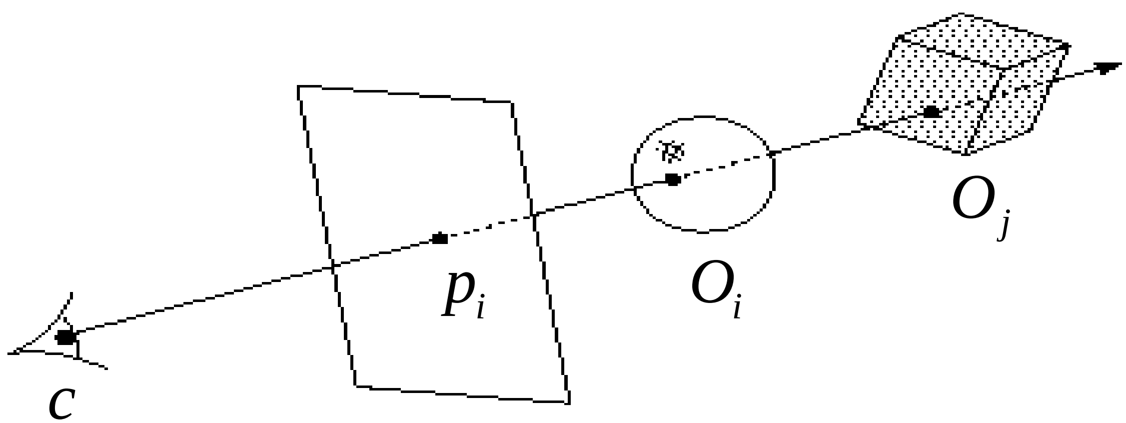


Important Algorithms

- Ray casting
- Z-buffer
- Binary space partitioning
- Back face culling

Ray Casting

- Partition the projection plane into pixels to match screen resolution
- For each pixel p_i, construct ray from COP through PP at that pixel and into scene
- Intersect the ray with every object in the scene, colour the pixel according to the object with the closest intersection

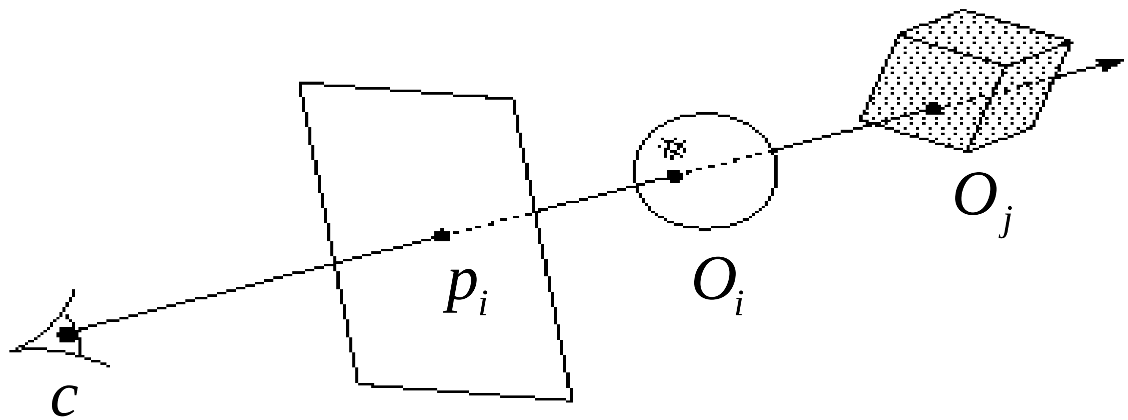


Ray Casting Implementation

- Parameterize the ray:

$$R(t) = (1\text{-}t)c + tp_i$$

- If a ray intersects some object O_i, get parameter t_i such that first intersection with O_i occurs at $R(t_i)$
- Which object owns the pixel?



Aside: Definitions

- An algorithm exhibits *coherence* if it uses knowledge about the continuity of the objects on which it operates
- An *online* algorithm is one that doesn't need all the data to be present when it starts running
  - Example: insertion sort

Ray Casting Analysis

Categorization:

- Easy to implement?
- Hardware implementation?
- Coherence?
- Memory intensive?
- Pre-processing required?
- Online?
- Handles transparency?
- Handles refraction?
- Polygon-based?
- Extra work for moving objects?
- Extra work for moving viewer?
- Efficient shading?
- Handles cycles and self-intersections?

Z-buffer

- Idea: along with a pixel's red, green and blue values, maintain some notion of its *depth*
  - An additional channel in memory, like alpha
  - Called the depth buffer or Z-buffer

```
void draw_mode_setup( void ) {
  …
  GlEnable( GL_DEPTH_TEST );
  …
}
```

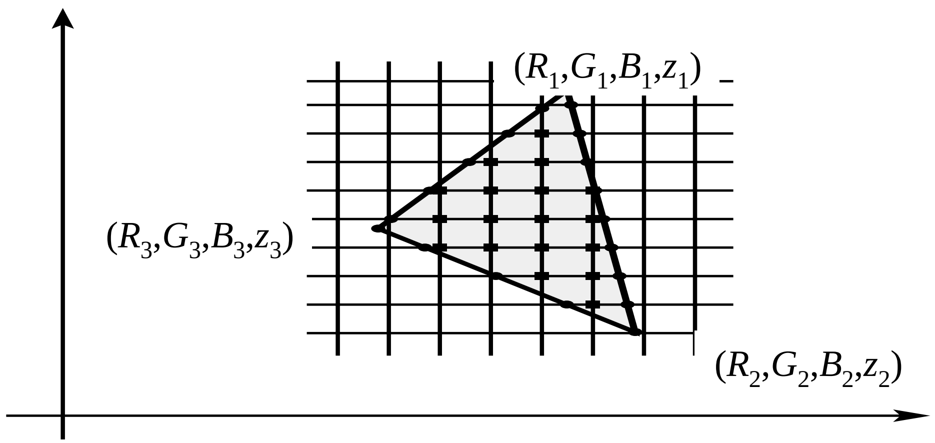
- When the time comes to draw a pixel, compare its depth with the depth of what's already in the framebuffer. Replace only if it's closer
- Very widely used
- History
  - Originally described as "brute-force image space algorithm"
  - Written off as impractical algorithm for huge memories
  - Today, done easily in hardware

Z-buffer Implementation

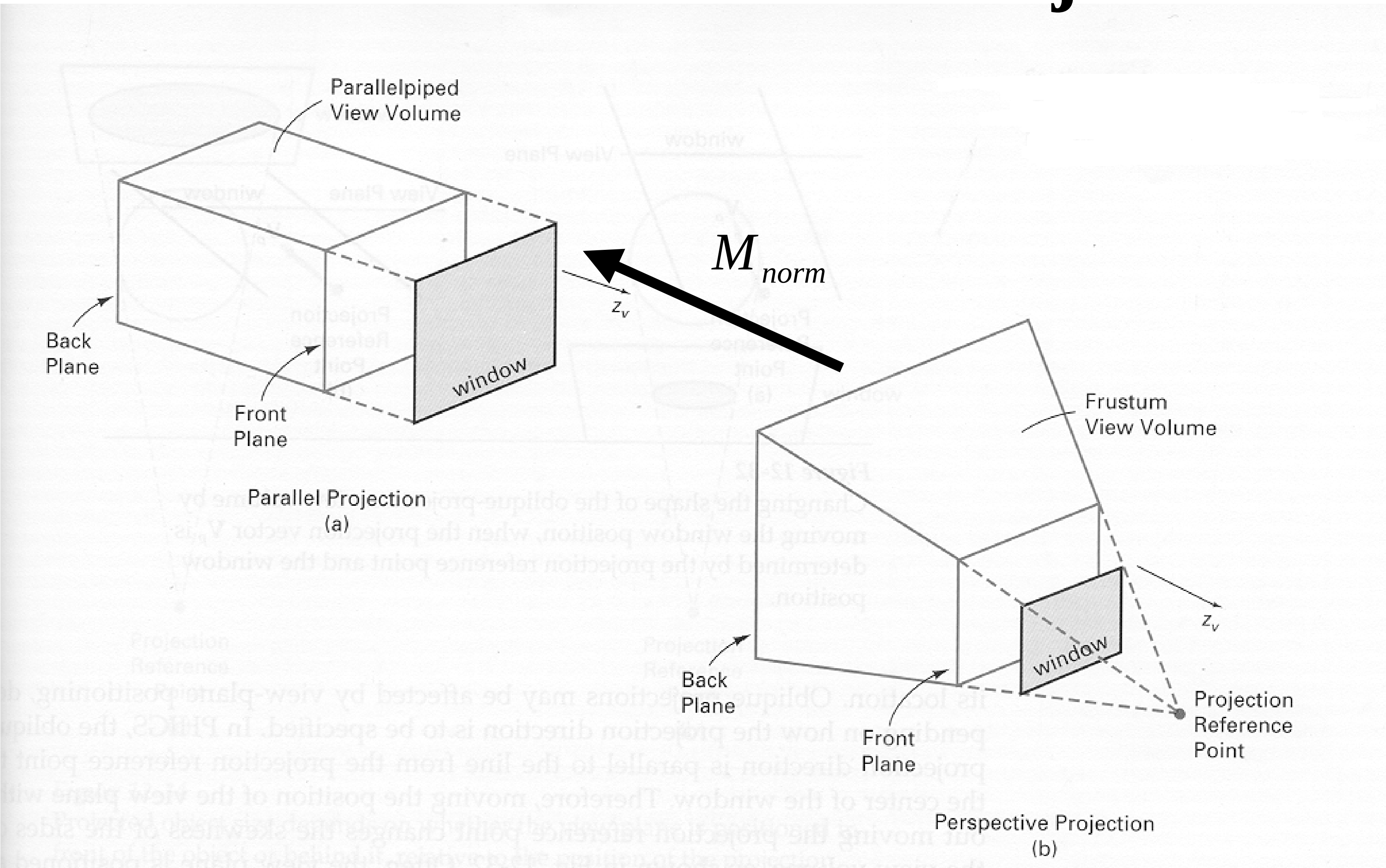
```
for each pixel pi
{
        Z-buffer[ pi ] = FAR
        Fb[ pi ] = BACKGROUND_COLOUR
}

for each polygon P
{
        for each pixel pi in the projection of P
        {
                Compute depth z and shade s of P at pi
                if z < Z-buffer[ pi ]
                {
                        Z-buffer[ pi ] = z
                        Fb[ pi ] = s
                }
        }
}
```

Z-buffer Tricks

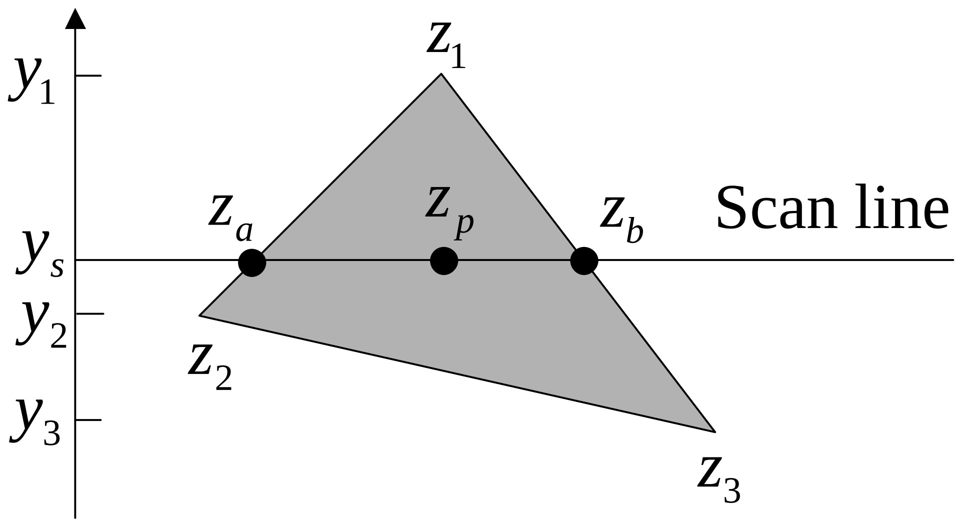
- The shade of a triangle can be computed incrementally from the shades of its vertices
- Can do the same with depth



Depth Preserving Conversion to Parallel Projection



Z value interpolation



$$z_a = z_1 - (z_1 - z_2)\frac{y_1 - y_s}{y_1 - y_2}$$

$$z_b = z_1 - (z_1 - z_3)\frac{y_1 - y_s}{y_1 - y_3}$$

$$z_p = z_b - (z_b - z_a)\frac{x_b - x_p}{x_b - x_a}$$

Z-buffer Analysis

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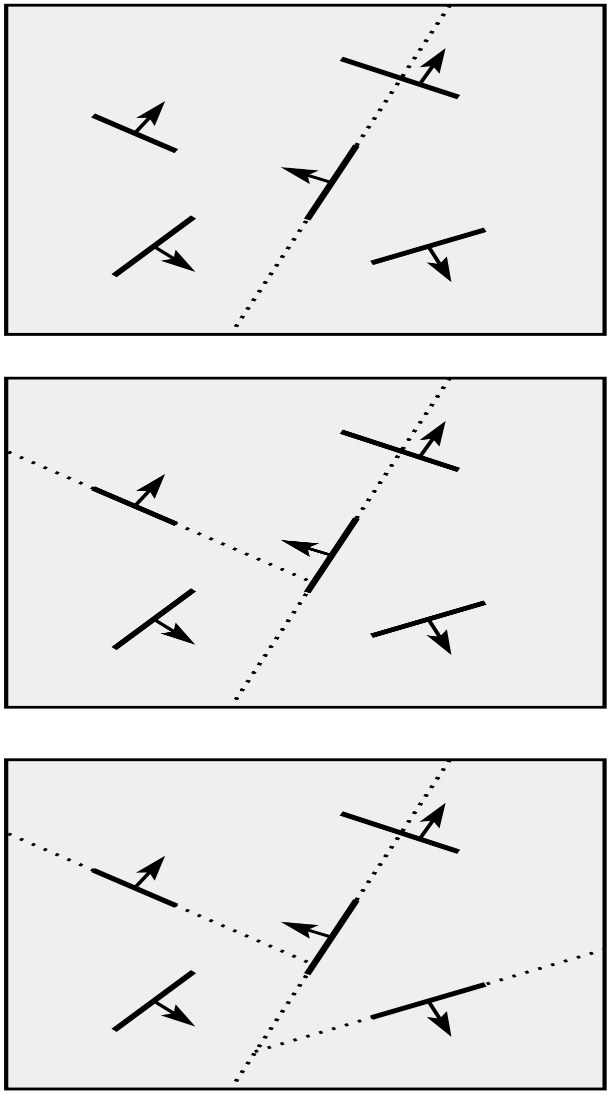
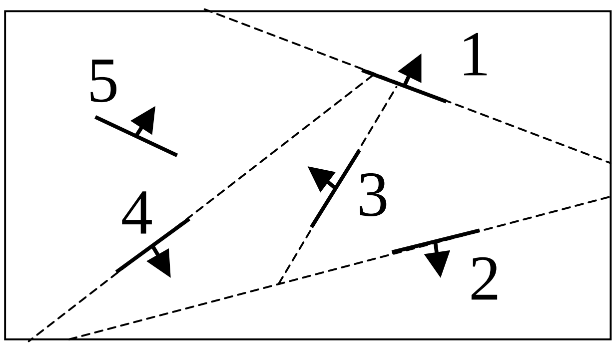
Binary Space Partitioning

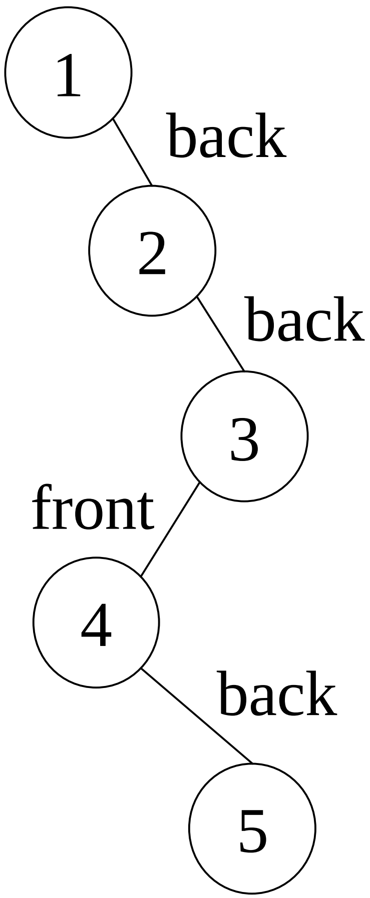
- Goal: build a tree that captures some relative depth information between objects. Use it to draw objects in the right order.
  - Tree doesn't depend on camera position, so we can change viewpoint and redraw quickly
  - Called the binary space partitioning tree, or BSP tree
- Key observation: The polygons in the scene are painted in the correct order if for each polygon *P*,
  - Polygons on the far side of *P* are painted first
  - *P* is painted next
  - Polygons in front of *P* are painted last

Building a BSP Tree (in 2D)



Alternate BSP Tree





BSP Tree Construction

```
BSPtree makeBSP( L: list of polygons )
{
        if L is empty
        {
                return the empty tree
        }

        Choose a polygon P from L to serve as root
        Split all polygons in L according to P
        return new TreeNode(
                P,
                makeBSP( polygons on negative side of P ),
                makeBSP( polygons on positive side of P ))
}
```

- Splitting polygons is expensive! It helps to choose P wisely at each step.
  - Example: choose five candidates, keep the one that splits the fewest polygons

BST Tree Display

```
showBSP( v: Viewer, T: BSPtree )
{
        if T is empty then return

        P := root of T
        if viewer is in front of P
        {
                showBSP( back subtree of T )
                draw P
                showBSP( front subtree of T )
        } else {
                showBSP( front subtree of T )
                draw P
                showBSP( back subtree of T )
        }
}
```

BSP Tree Analysis

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Back Face Culling

- Can be used in conjunction with polygon-based algorithms
- Often, we don't want to draw polygons that face away from the viewer. So test for this and eliminate (cull) back-facing polygons before drawing
- How can we test for this?

Summary

- Classification of hidden surface algorithms
- Understanding ray casting algorithms
- Understanding of Z-buffer
- Familiarity with BSP trees and back face culling