

## Reading

Optional:

- Angel 6.1-6.5, 6.7-6.8, 9.1-9.10
- OpenGL red book, chapter 5.


## Introduction

So far, we've talked exclusively about geometry.
-What is the shape of an object?

- How do I place it in a virtual 3D space?
- How do I know which pixels it covers?
- How do I know which of the pixels I should actually draw?

Once we've answered all those, we have to ask one more important question:

- To what value do I set each pixel?


## shading

model

Other names:

- Lighting model
- Light reflection model
- Local illumination model
- Reflectance model
- BRDF


## Our problem

Modeling the flow of light in a scene is very complex: photons pour out of light sources and bounce around and around before reaching a camera.

Here we focus on local illumination happens for a single bounce:
light source $\rightarrow$ surface $\rightarrow$ viewer
No interreflections, no shadows.

We're going to explore two models: the Phong Blinn-Phong illumination models

They have the following characteristics:

- physically plausible (albeit not strictly correct)
- very fast
- widely used


## Setup...



Given:
-

- The normal $\mathbf{N} \mathbf{P}$
- The lighting direction, $\mathbf{L}$ P
- The viewing direction, $\mathbf{V} \mathbf{P}$
- The shading coefficients at $\mathbf{P}$

Compute the color, , of pixel .
Assume that the direction vectors are normalized:

$$
\|\mathbf{N}\|=\|\mathbf{L}\|=\|\mathbf{V}\|=1
$$

## "Iteration zero"

The simplest thing you can do is...
Assign each polygon a single color:

$$
I=k_{e}
$$

where

- $\quad$ is the resulting intensity
- $k_{e}$ emissivity with the object

This has some special-purpose uses, but not really good for drawing a scene.
[Note: is omitted in Angel.]

## "Iteration one"

Let's make the color at least dependent on the overall quantity of light available in the scene:

$$
I=k_{e}+k_{a} I_{L a}
$$

- ${ }_{a}$ is the ambient reflection coefficient.
- really the reflectance of ambient light
- "ambient" light is assumed to be equal in all directions
- is the ambient light intensity.

Physically, what is "ambient" light?
[Note: Angel uses $L_{a} \quad I_{L a}$

## Wavelength dependence

Really, $k_{e} k_{a} \quad I_{\text {L }}$ wavelengths $\lambda$.

Ideally, we would do the calculation on these functions. For the ambient shading equation, we would start with:

$$
I(\lambda)=k_{a}(\lambda) I_{L a}(\lambda)
$$

then we would find good RGB values to represent the spectrum $I(\lambda)$.

Traditionally, though, $k_{a}$ and $I_{L a}$ are represented as RGB triples, and the computation is performed on each color channel separately:

$$
\begin{aligned}
I^{R} & =k_{a}^{R} I_{L a}^{R} \\
I^{G} & =k_{a}^{G} I_{L a}^{G} \\
I^{B} & =k_{a}^{B} I_{L a}^{B}
\end{aligned}
$$

## Diffuse reflection

Let's examine the ambient shading model:

- objects have different colors
- we can control the overall light intensity
- what happens when we turn off the lights?
- what happens as the light intensity increases?
- what happens if we change the color of the lights?

So far, objects are uniformly lit.

- not the way things really appear
- in reality, light sources are localized in position or direction

Diffuse Lambertian
intensity to vary with the direction of the light.

## Diffuse reflectors

Diffuse reflection occurs from dull, matte surfaces, like latex paint, or chalk.

These diffuse or Lambertian reflectors reradiate light equally in all directions.

Picture a rough surface with lots of tiny microfacets.


## Diffuse reflectors

...or picture a surface with little pigment particles embedded beneath the surface (neglect reflection at the surface for the moment):


The microfacets and pigments distribute light rays in all directions.

Embedded pigments are responsible for the coloration of diffusely reflected light in plastics and paints.

Note: the figures above are intuitive, but not strictly (physically) correct.

## Diffuse reflectors, cont.

The reflected intensity from a diffuse surface does not depend on the direction of the viewer. The incoming light, though, does depend on the direction of the light source:


## "Iteration two"

The incoming energy is proportional to $\qquad$ giving the diffuse reflection equations:

$$
\begin{aligned}
I & =k_{e}+k_{a} I_{L a}+k_{d} I_{L} B \\
& =k_{e}+k_{a} I_{L a}+k_{d} I_{L} B(\quad)
\end{aligned}
$$

where:

- d
- $I_{L}$ is the (color) intensity of the light source
- $\mathbf{N}$ is the normal to the surface (unit vector)
- L is the direction to the light source (unit vector)
- B
surface:

$$
B= \begin{cases}1 & \text { if } \mathbf{N} \cdot \mathbf{L}>\mathbf{0} \\ 0 & \text { if } \mathbf{N} \cdot \mathbf{L} \leq \mathbf{0}\end{cases}
$$

[Note: Angel uses $L_{d}$ instead of $I_{L}$ and $f$

## Specular reflection "derivation"


$N$

$$
I=\left\{\begin{array}{cc}
I_{L} & \text { if } \mathbf{V}=\mathbf{R} \\
0 & \text { otherwise }
\end{array}\right.
$$

For a near-perfect reflector, you might expect the highlight to fall off quickly with increasing angle $\phi$.

## Also known as:

- "rough specular" reflection
- "directional diffuse" reflection
- "glossy" reflection


## Specular reflection

## Specular reflection

you see on some objects.

It is particularly important for smooth, shiny such as:

- metal
- polished stone
- plastics
- apples
- skin

Properties:

- Specular reflection depends on the viewing direction $\mathbf{V}$.
- For non-metals, the color is determined solely by the color of the light.
- For metals, the color may be altered (e.g., brass)


## Phong specular reflection




One way to get this effect is to take ( $\mathbf{R} \cdot \mathbf{V}$ power $n_{s}$

As $n_{s}$

- the dropoff becomes \{more,less\} gradual
- gives a \{larger,smaller\} highlight
- simulates a \{more,less\} mirror-like surface

Phong specular reflection is proportional to:

$$
I_{\text {specular }}=B(\mathbf{R} \cdot \mathbf{V})_{+}^{n_{s}}
$$

where $\left(x_{+} \equiv \max (0, \quad)\right.$.

## Blinn-Phong specular reflection

A common alternative for specular reflection is the Blinn-Phong model (sometimes called the modified Phong model.)

We compute the vector halfway between $\mathbf{L}$ and $\mathbf{V}$ as:


Analogous to Phong specular reflection, we can compute the specular contribution in terms of ( $\mathbf{N} \cdot \mathbf{H}$ raised to a power $n_{s}$ :

$$
l_{\text {specular }}=B(\mathbf{N} \cdot \mathbf{H})_{+}^{n_{s}}
$$

where, again, $(x)_{+} \equiv \max (0, x)$.

## "Iteration three"

The next update to the Blinn-Phong shading model is then:

$$
\begin{aligned}
I & =k_{e}+k_{a} I_{L a}+k_{d} I_{L} B(\mathbf{N} \cdot \mathbf{L})+k_{s} l_{L} B(\mathbf{N} \cdot \mathbf{H})_{+}^{n_{s}} \\
& =k_{e}+k_{a} I_{L a}+l_{L} B\left[k_{d}(\mathbf{N} \cdot \mathbf{L})+k_{s}(\mathbf{N} \cdot \mathbf{H})_{+}^{n_{s}}\right]
\end{aligned}
$$

where:

- is the specular reflection coefficient
- is the specular exponent shininess
- $\mathbf{H}$ is the unit halfway vector between $\mathbf{L}$ and $\mathbf{V}$, where $\mathbf{V}$ is the viewing direction.
[Note: Angel uses $\alpha$ instead of $n_{5^{\prime}}$ and maintains a separate $L_{d}$ and $L_{s^{\prime}}$, instead of a single $I_{L}$. This choice reflects the flexibility available in OpenGL.]


## Directional lights

OpenGL supports three different kinds of lights: ambient, directional, and point. Spot lights are also supported as a special form of point light.

We've seen ambient light sources, which are not really geometric.

Directional light sources have a single direction and intensity associated with them.


Using affine notation, what is the homogeneous coordinate for a directional light?

## Point lights

The direction of a point light sources is determined by the vector from the light position to the surface point.


Physics tells us the intensity must drop off inversely with the square of the distance:

$$
f_{\text {atten }}=\frac{1}{r^{2}}
$$

Sometimes, this distance-squared dropoff is considered too "harsh." A common alternative is:

$$
f_{\text {atten }}=\frac{1}{a+b r+c r^{2}}
$$

with user-supplied constants for $a b \quad c$
Using affine notation, what is the homogeneous coordinate for a point light?

## Spotlights

## attenuation

 effect.

The spotlight intensity factor is computed in OpenGL as:

$$
f_{\text {spot }}=(\mathbf{L} \cdot \mathbf{S})_{\beta}^{e}
$$

where

- $\mathbf{L}$ is the direction to the point light.
- $\mathbf{S}$ is the center direction of the spotlight.
- $\beta$ is the cutoff angle for the spotlight
- $e$ is the angular falloff coefficient
- $(x)_{\beta}^{e}=[\max \{\operatorname{acos}(x)-\beta, 0\}]^{e}$


## "Iteration four"

Since light is additive, we can handle multiple lights by taking the sum over every light.

Our equation is now (for point lights):

$$
\begin{aligned}
& I=k_{e}+k_{a} I_{L a}+ \\
& \\
& \qquad \sum_{j} \frac{1}{a_{j}+b_{j} r_{j}+c_{j} r_{j}^{2}} I_{L, j} B_{j}\left[k_{d}\left(\mathbf{N} \cdot \mathbf{L}_{j}\right)+k_{s}\left(\mathbf{N} \cdot \mathbf{H}_{j}\right)_{+}^{n_{s}}\right]
\end{aligned}
$$

This is the Blinn-Phong illumination model.
Which quantities are spatial vectors?

Which are RGB triples?

Which are scalars?

## Choosing the parameters

Experiment with different parameter settings. To get you started, here are a few suggestions:

- Try $n_{s}$
- Try $k_{a}+k_{d} \quad k_{s}<1$
- Use a small $k_{a}(\sim 0.1)$

|  | $n_{s}$ | $k_{d}$ | $k_{s}$ |
| :--- | :--- | :--- | :--- |
| Metal | large | Small, color <br> of metal | Large, color <br> of metal |
| Plastic | medium | Medium, <br> color of <br> plastic | Medium, <br> white |
| Planet | 0 | varying | 0 |

## Materials in OpenGL

The OpenGL code to specify the surface shading properties is fairly straightforward. For example:

GLfloat ke[] = \{ 0.1, 0.15, 0.05, 1.0 \};
GLfloat ka[] = \{ 0.1, 0.15, 0.1, 1.0 \};
GLfloat $k d[]=\{0.3,0.3,0.2,1.0\}$;
GLfloat ks[] = \{ 0.2, 0.2, 0.2, 1.0 \};
GLfloat ns[] = \{ 50.0 \};
glMaterialfv(GL_FRONT, GL_EMISSION, ke);
glMaterialfv(GL_FRONT, GL_AMBIENT, ka);
glMaterialfv(GL_FRONT, GL_DIFFUSE, kd); glmaterialfv(GL_FRONT, GL_SPECULAR, ks); glMaterialfv(GL_FRONT, GL_SHININESS, ns);

Notes:

- The GL_front parameter tells OpenGL that we are specifiying the materials for the front of the surface.
- Only the alpha value of the diffuse color is used for blending. It's usually set to 1 .


## Shading in OpenGL

The OpenGL lighting model allows you to associate different lighting colors according to material properties they will influence.

Thus, our original shading equation:

$$
\begin{aligned}
& I=k_{e}+k_{a} I_{L a}+ \\
& \\
& \qquad \sum_{j} \frac{1}{a_{j}+b_{j} r_{j}+c_{j} r_{j}^{2}} I_{L, j} B_{j}\left[k_{d}\left(\mathbf{N} \cdot \mathbf{L}_{j}\right)_{+}+k_{s}\left(\mathbf{N} \cdot \mathbf{H}_{j}\right)_{+}^{n_{s}}\right]
\end{aligned}
$$

becomes:

$$
\begin{aligned}
& I=k_{e}+k_{\mathrm{a}} I_{L a}+ \\
& \sum_{j} \frac{1}{a_{j}+b_{j} r_{j}+c_{j} r_{j}^{2}}\left[k_{a} I_{L a, j}+B_{j}\left\{k_{d} I_{L d, j}\left(\mathbf{N} \cdot \mathbf{L}_{j}\right)_{+}+k_{s} I_{L s, j}\left(\mathbf{N} \cdot \mathbf{H}_{j}\right)_{+}^{n_{s}}\right\}\right]
\end{aligned}
$$

where you can have a global ambient light with intensity $I_{\text {La }}$
intensity $I_{\text {La,j }}$ associated with each individual light, as well as separate diffuse and specular intensities, $I_{L d, j}$ and $I_{L s, j}$, repectively.

## Shading in OpenGL, cont'd

In OpenGL this equation, for one light source (the $0^{\text {th }}$ ) is specified something like:

GLfloat La[] = \{ 0.2, 0.2, 0.2, 1.0 \};
GLfloat La0[] = \{ 0.1, 0.1, 0.1, 1.0 \};
GLfloat Ld0[] = \{ 1.0, 1.0, 1.0, 1.0$\}$;
GLfloat Ls0[] = \{ 1.0, 1.0, 1.0, 1.0 \};
GLfloat pos0[] = \{ 1.0, 1.0, 1.0, 0.0 \};
GLfloat a 0[]$=\{1.0\}$;
GLfloat b0[] = \{ 0.5 \};
GLfloat c0[] = \{ 0.25 \};
GLfloat S 0[]$=\{-1.0,-1.0,0.0\}$;
GLfloat beta0[] = \{ 45 \};
GLfloat e0[] = \{ 2 \};
glLightModelfv(GL_LIGHT_MODEL_AMBIENT, La); glLightfv(GL_LIGHT0, GL_AMBIENT, La0); glLightfv(GL_LIGHT0, GL_DIFFUSE, Ld0); glLightfv(GL_LIGHT0, GL_SPECULAR, Ls0); glLightfv(GL_LIGHT0, GL_POSITION, pos0); glLightfv(GL_LIGHT0, GL_CONSTANT_ATTENUATION, a0); glLightfv(GL_LIGHT0, GL_LINEAR_ATTENUATION, b0); glLightfv(GL_LIGHT0, GL_QUADRATIC_ATTENUATION, c0); glLightfv(GL_LIGHT0, GL_SPOT_DIRECTION, S0); glLightf(GL_LIGHT0, GL_SPOT_CUTOFF, beta0); glLightf(GL_LIGHT0, GL_SPOT_EXPONENT, e0);

## Shading in OpenGL, cont'd

## Notes:

You can have as many as GL_MAX_LIGHTS lights in a scene. This number is system-dependent.

For directional lights, you specify a light direction, not position, and the attenuation and spotlight terms are ignored.

The directions of directional lights and spotlights are specified in the coordinate systems of the lights the surface points as we've been doing in lecture.

## BRDF

The diffuse+specular parts of the Blinn-Phong illumination model are a mapping from light to viewing directions:

$$
\begin{aligned}
I & =I_{L} B\left[k_{d}(\mathbf{N} \cdot \mathbf{L})+k_{s}\left(\mathbf{N} \cdot \frac{\mathbf{L}+\mathbf{V}}{\|\mathbf{L}+\mathbf{V}\|}\right)_{+}^{n_{s}}\right] \\
& =I_{L} f_{r}(\mathbf{L}, \mathbf{V})
\end{aligned}
$$

The mapping function $f_{r}$ incoming (light) directions $\omega_{\text {in }}$ and outgoing (viewing) directions $\omega_{\text {out }}$ :

$$
f_{r}\left(\omega_{\text {in }}, \omega_{\text {out }}\right) \quad \text { or } \quad f_{r}\left(\omega_{\text {in }} \rightarrow \omega_{\text {out }}\right)
$$

Bi-directional Reflectance
Distribution Function BRDF

Here's a plot with $\omega_{\text {in }}$


BRDF's can be quite sophisticated...


## Gouraud vs. Phong interpolation

Now we know how to compute the color at a point on a surface using the Blinn-Phong lighting model

Does graphics hardware do this calculation at every point? Not by default...

Smooth surfaces are often approximated by polygonal facets, because:

- Graphics hardware generally wants polygons (esp. triangles).
- Sometimes it easier to write ray-surface intersection algorithms for polygonal models. How do we compute the shading for such a surface?


## Faceted shading

Assume each face has a constant normal:


For a distant viewer and a distant light source and constant material properties over the surface, how will the color of each triangle vary?

Result: faceted, not smooth, appearance.

Faceted shading (cont'd)


## Gouraud interpolation

To get a smoother result that is easily performed in hardware, we can do Gouraud interpolation

Here's how it works:

1. Compute normals at the vertices.
2. Shade only the vertices.
3. Interpolate the resulting vertex colors.



$\stackrel{\downarrow}{\text { Interpolate }}$


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## Rasterization with color

Recall that the z-buffer works by interpolating zvalues across a triangle that has been projected into image space, a process called rasterization.

During rasterization, colors can be smeared across a triangle as well:


Facted shading vs. Gouraud interpolation


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## Gouraud interpolation artifacts

Gouraud interpolation has significant limitations.

1. If the polygonal approximation is too coarse, we can miss specular highlights.

2. We will encounter Mach banding discontinuity enhanced by human eye).

This is what graphics hardware does by default.

A substantial improvement is to do...

## Phong interpolation

To get an even smoother result with fewer artifacts, we can perform Phong interpolation

Here's how it works:

1. Compute normals at the vertices.
2. Interpolate normals and normalize.
3. Shade using the interpolated normals.




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Gouraud vs. Phong interpolation


Williams and Siegel 1990]

## Default pipeline: Gouraud interpolation



Programmable pipeline: Phong-interpolated normals!


## Summary

The most important thing to take away from this lecture is the equation for the Blinn-Phong lighting model described in the "Iteration Four" slide.

- What is the physical meaning of each variable?
- How are the terms computed?
- What effect does each term contribute to the image?
- What does varying the parameters do?

You should also understand the differences between faceted, Gouraud, and Phong interpolated

