Image processing

Reading

Jain, Kasturi, Schunck, *Machine Vision*. McGraw-Hill, 1995. Sections 4.2-4.4, 4.5(intro), 4.5.5, 4.5.6, 5.1-5.4.

What is an image?

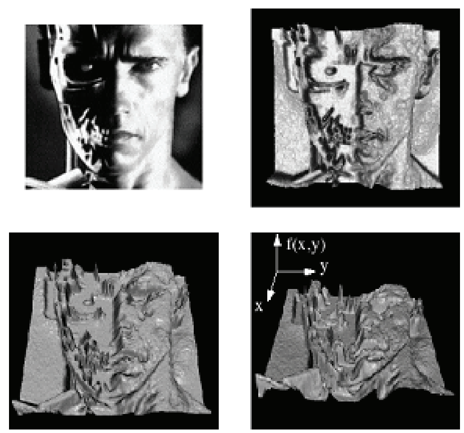
We can think of an **image** as a function, f, from R^2 to R:

- $f(x, y)$ gives the intensity of a channel at position (x, y)
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

Images as functions



What is a digital image?

In computer graphics, we usually operate on **digital** (**discrete**) images:

- **Sample** the space on a regular grid
- **Quantize** each sample (round to nearest integer)

If our samples are Δ apart, we can write this as:

$f[i,j] = \text{Quantize}\{ f(i\,\Delta, j\,\Delta) \}$

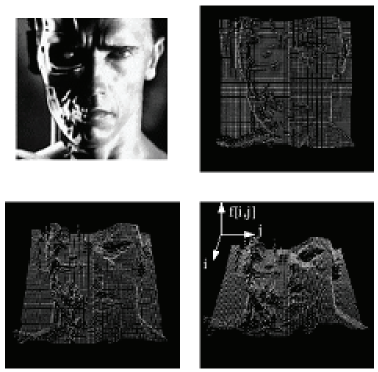


Image processing

An **image processing** operation typically defines a new image g in terms of an existing image f.

The simplest operations are those that transform each pixel in isolation. These pixel-to-pixel operations can be written:

$$g(x,y) = t(f(x,y))$$

Examples: threshold, RGB $\rightarrow$ grayscale

Note: a typical choice for mapping to grayscale is to apply the YIQ television matrix and keep the Y.

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

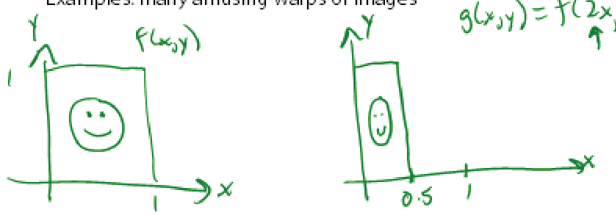
Pixel movement

Some operations preserve intensities, but move pixels around in the image

$$g(x,y) = f(\tilde{x}(x,y), \tilde{y}(x,y))$$

Examples: many amusing warps of images

$g(x,y) = f(2x, y)$



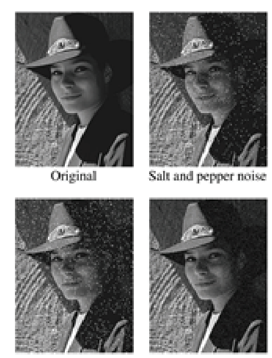
$\tilde{x} = 2x$

$\tilde{y} = y$

[Show image sequence.]

Noise

Image processing is also useful for noise reduction and edge enhancement. We will focus on these applications for the remainder of the lecture…



Original — Salt and pepper noise — Impulse noise — Gaussian noise

Common types of noise:

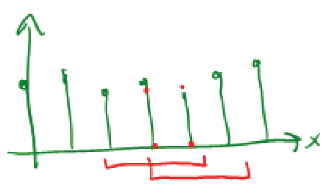
- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Ideal noise reduction

Ideal noise reduction

Practical noise reduction

How can we "smooth" away noise in a single image?



Is there a more abstract way to represent this sort of operation? *Of course there is!*

Convolution

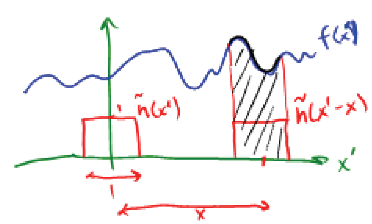
One of the most common methods for filtering an image is called **convolution**.

In 1D, convolution is defined as:

$$
\begin{aligned}
g(x) &= f(x) * h(x) \\
&= \int_{-\infty}^{\infty} f(x')h(x-x')dx' \\
&= \int_{-\infty}^{\infty} f(x')\tilde{h}(x'-x)dx'
\end{aligned}
$$

where $\tilde{h}(x) = h(-x)$.

Example:



Discrete convolution

For a digital signal, we define **discrete convolution** as:

$$
\begin{aligned}
g[i] &= f[i] * h[i] \\
&= \sum_{i'} f[i']h[i-i'] \\
&= \sum_{i'} f[i']\tilde{h}[i'-i]
\end{aligned}
$$

where $\tilde{h}[i] = h[-i]$.

Aside:

One can show that convolution has some convenient properties. Given functions a, b, c:

$$
\begin{aligned}
a * b &= b * a \\
(a * b) * c &= a * (b * c) \\
a * (b + c) &= a * b + a * c
\end{aligned}
$$

We'll make use of these properties later…

Convolution in 2D

In two dimensions, convolution becomes:

$$
\begin{aligned}
g(x,y) &= f(x,y) * h(x,y) \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')h(x-x',y-y')dx'dy' \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\tilde{h}(x'-x,y'-y)dx'dy'
\end{aligned}
$$

where $\tilde{h}(x,y) = h(-x,-y)$.

Discrete convolution in 2D

Similarly, discrete convolution in 2D becomes:

$$
\begin{aligned}
g[i,j] &= f[i,j] * h[i,j] \\
&= \sum_{j'}\sum_{i'} f[i',j']h[i-i',j-j'] \\
&= \sum_{j'}\sum_{i'} f[i',j']\tilde{h}[i'-i,j'-j]
\end{aligned}
$$

where $\tilde{h}[i,j] = h[-i,-j]$.

Note: convolution (continuous or discrete, in any dimension) is a *linear* operation. Here is one of the consequences of that linearity:

$$
\begin{aligned}
g[i,j] &= f[i,j] * h[i,j] + f[i,j] * e[i,j] \\
&= f[i,j] * (h[i,j] + e[i,j])
\end{aligned}
$$

Convolution representation

Since f and h are defined over finite regions, we can write them out in two-dimensional arrays:

128	54	9	78	100
145	98	240	233	86
89	177	246	228	127
67	90	255	237	95
106	111	128	167	20
221	154	97	123	0

X 0.1	X 0.1	X 0.1
X 0.1	X 0.2	X 0.1
X 0.1	X 0.1	X 0.1

support

filter kernel

Note: *This is not matrix multiplication*!

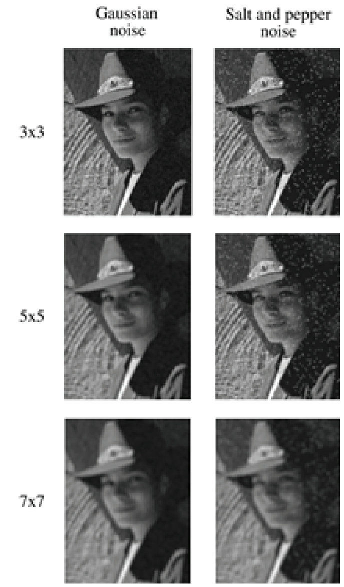
Q: What happens at the edges?

Mean filters

How can we represent our noise-reducing averaging filter as a convolution diagram (know as a **mean filter**)?

$$\frac{1}{m^2}\begin{array}{cc} m & \\ \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} & m \end{array}$$

Effect of mean filters



Gaussian filters

Gaussian filters weight pixels based on their distance from the center of the convolution filter. In particular:

$$h[i,j] = \frac{e^{-(i^2+j^2)/(2\sigma^2)}}{C}$$

This does a decent job of blurring noise while preserving features of the image.

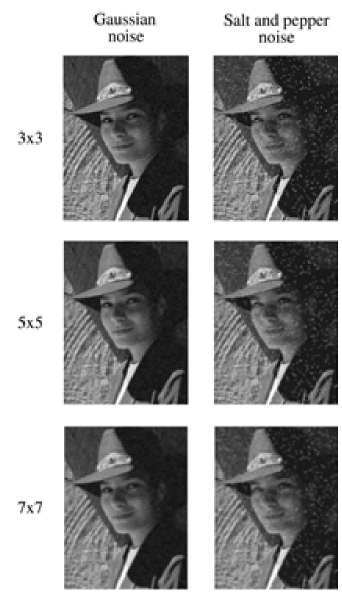
What parameter controls the width of the Gaussian? σ

What happens to the image as the Gaussian filter kernel gets wider? gets blurrier

What is the constant C? What should we set it to?

$$C = \sum e^{-(i^2+j^2)/2\sigma^2}$$

Effect of Gaussian filters

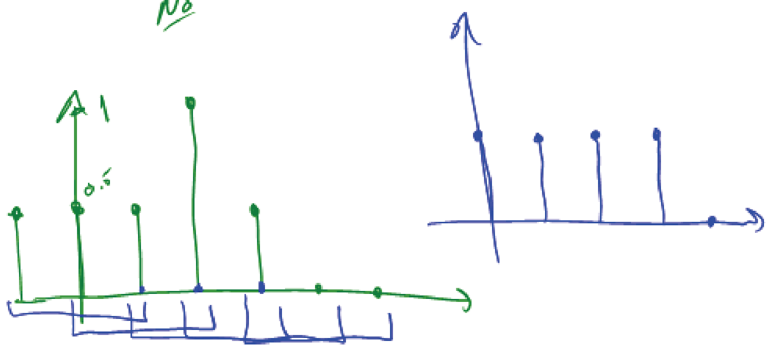


Median filters

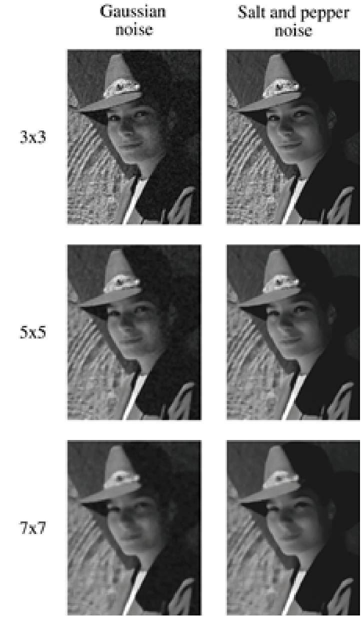
A **median filter** operates over an *mxm* region by selecting the median intensity in the region.

What advantage does a median filter have over a mean filter? acts like an outlier rejection filter

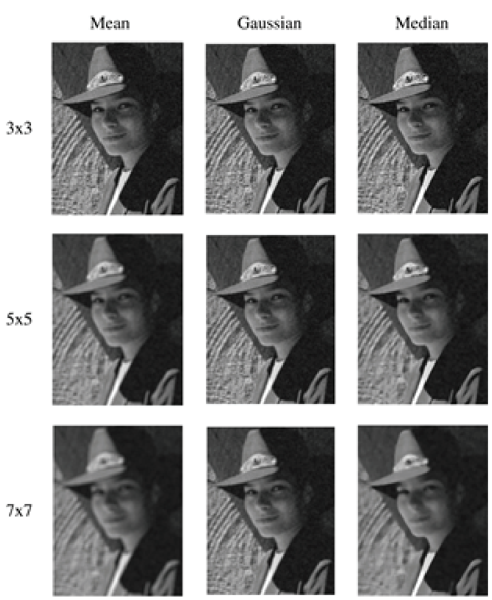
Is a median filter a kind of convolution? No



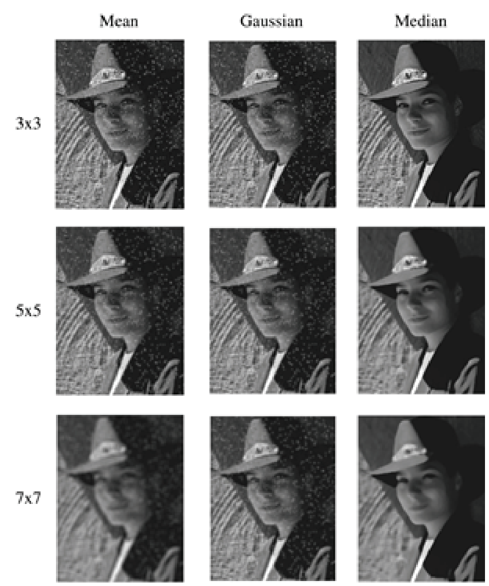
Effect of median filters



Comparison: Gaussian noise



Comparison: salt and pepper noise



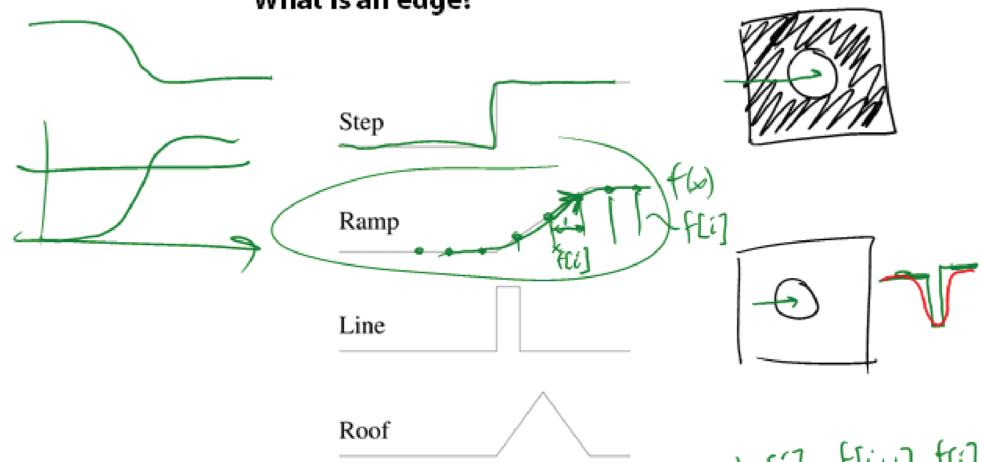
Edge detection

One of the most important uses of image processing is **edge detection:**

- Really easy for humans
- Really difficult for computers

- Fundamental in computer vision
- Important in many graphics applications

What is an edge?



Q: How might you detect an edge in 1D?

$\left|\frac{df}{dx}\right| >$ thresh. $\Rightarrow$ edge

$g[i] = f[i+1] - f[i]$

$g = h * f$

$\tilde{h} = [0 \ -1 \ 1]$

$h = [1 \ -1 \ 0]$

Gradients

$\frac{df}{dx} \approx h * f$

The **gradient** is the 2D equivalent of the derivative: $\frac{\partial f}{\partial x} \approx h_x * f$

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

Properties of the gradient

$\theta = \tan^{-1}\left(\frac{\partial f/\partial y}{\partial f/\partial x}\right)$

- It's a vector
- Points in the direction of maximum increase of f
- Magnitude is rate of increase

$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$

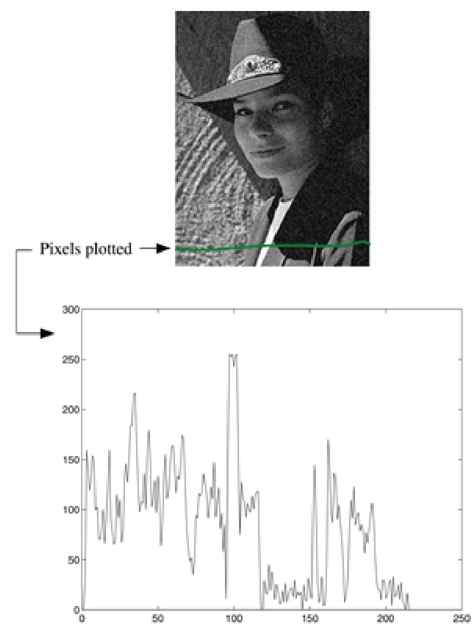
How can we approximate the gradient in a discrete image?

$g_x[i,j] = g[i+1,j] - g[i,j]$

$g_y[i,j] = g[i,j+1] - g[i,j]$

$\tilde{h}_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \tilde{h}_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Less than ideal edges



Steps in edge detection

Edge detection algorithms typically proceed in three or four steps:

- **Filtering**: cut down on noise
- **Enhancement**: amplify the difference between edges and non-edges
- **Detection**: use a threshold operation
- **Localization** (optional): estimate geometry of edges, which generally pass between pixels

Edge enhancement

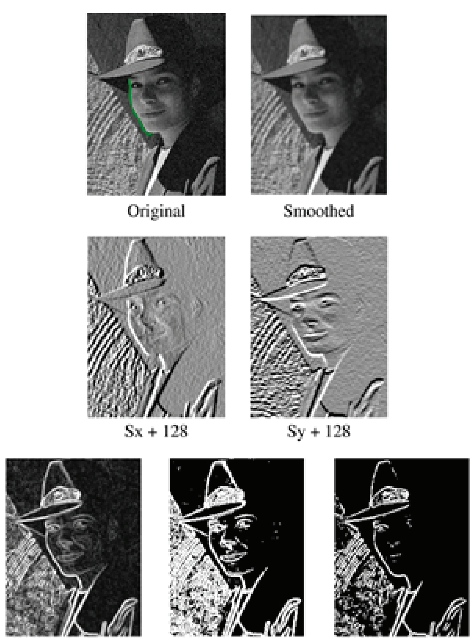
A popular gradient magnitude computation is the **Sobel operator**:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{(pre-flipped)}$$

$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

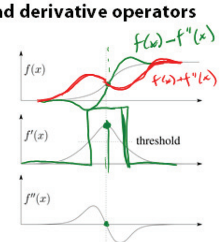
We can then compute the magnitude of the vector (s_x, s_y).

Results of Sobel edge detection



Original · Smoothed · Sx + 128 · Sy + 128 · Magnitude · Threshold = 64 · Threshold = 128

Second derivative operators



The Sobel operator can produce thick edges. Ideally, we're looking for infinitely thin boundaries.

An alternative approach is to look for local extrema in the first derivative: places where the change in the gradient is highest.

Q: A peak in the first derivative corresponds to what in the second derivative? 0

Q: How might we write this as a convolution filter?

$f'(x) \equiv \frac{df}{dx}$

$f_x(x) \equiv$

$\frac{d}{dx}\frac{df}{dx} \approx h*(h*f) = (h*h)*f$

$h = [1\ -1\ 0] \qquad \tilde{h} = [0\ -1\ 1]$

$[0\ -1\ 1]$

$[0\ -1\ 1]$

$[0\ -1\ 1]$

$h*h = [1\ -2\ 1]$

Localization with the Laplacian

An equivalent measure of the second derivative in 2D is the **Laplacian**:

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$h_x^2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Using the same arguments we used to compute the gradient filters, we can derive a Laplacian filter to be:

$$h_y^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

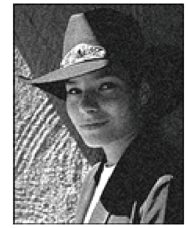
$$\Delta = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

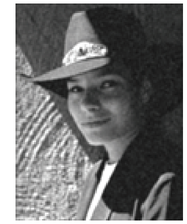
$$\Delta f = h_x^2 * f + h_y^2 * f$$
$$= (h_x^2 + h_y^2) * f$$

(The symbol Δ is often used to refer to the *discrete* Laplacian filter.)

Zero crossings of this filter(ed image) correspond to positions of maximum gradient. These zero crossings can be used to localize edges.

Localization with the Laplacian

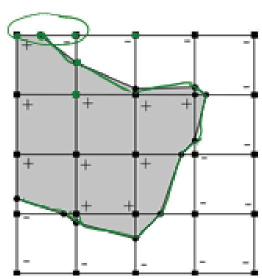

Original

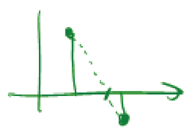

Smoothed


Laplacian (+128)

Marching squares

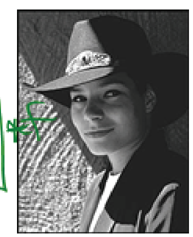
We can convert these signed values into edge contours using a "marching squares" technique:





Sharpening with the Laplacian

$$s = f - \Delta * f$$
$$= [1] * f - \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} * f$$
$$= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{I} * f - \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{\Delta} * f$$
$$= (I - \Delta) * f$$
$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} * f$$


Original


Laplacian (+128)


Original + Laplacian


Original - Laplacian

$$\text{Sharpen} = (I - \lambda\Delta)$$

$$\lambda = \tfrac{1}{2} \Rightarrow \begin{bmatrix} & -1/2 & \\ -1/2 & 3 & -1/2 \\ & -1/2 & \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} & -1 & \\ -1 & 6 & -1 \\ & -1 & \end{bmatrix}$$

Why does the sign make a difference?

How can you write each filter that makes each bottom image?

Summary

What you should take away from this lecture:

- The meanings of all the boldfaced terms.
- How noise reduction is done
- How discrete convolution filtering works
- The effect of mean, Gaussian, and median filters
- What an image gradient is and how it can be computed
- How edge detection is done
- What the Laplacian image is and how it is used in either edge detection or image sharpening