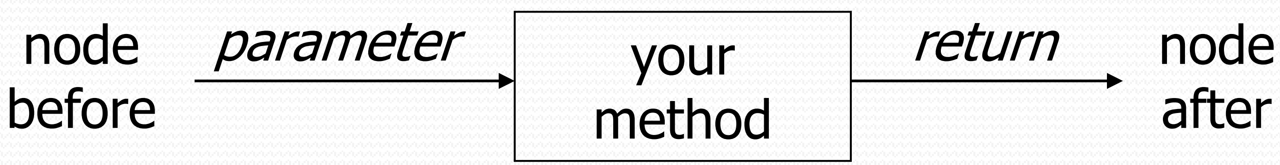
CSE 143

Lecture 21: Binary Search Trees; TreeSet

Recall: x = change(x)

- Methods that modify a tree should have the following pattern:
  - input (parameter): old state of the node
  - output (return): new state of the node



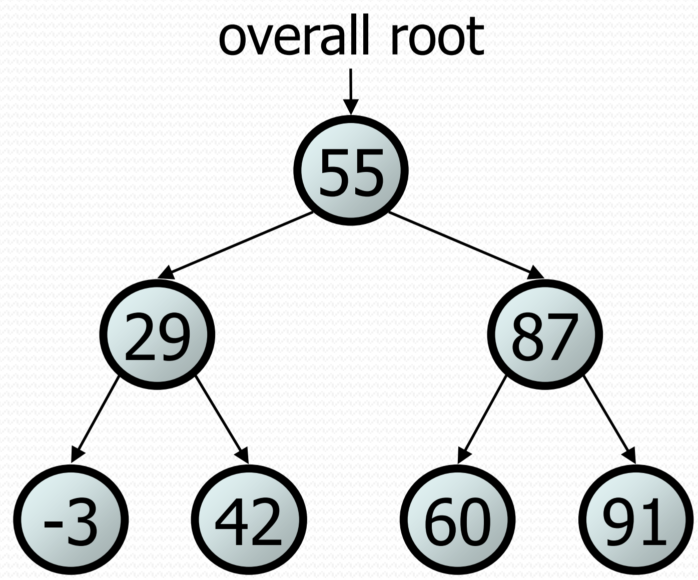
- In order to actually change the tree, you must reassign:

```
node        = change(node, parameters);
node.left   = change(node.left, parameters);
node.right  = change(node.right, parameters);
overallRoot = change(overallRoot, parameters);
```

Exercise

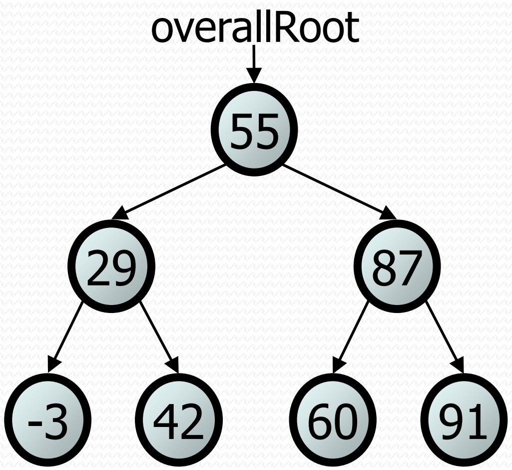
- Add a method `getMin` to the `IntTree` class that returns the minimum integer value from the tree. Assume that the elements of the `IntTree` constitute a legal binary search tree. Throw a `NoSuchElementException` if the tree is empty.

```
int min = tree.getMin();   // -3
```



Exercise solution

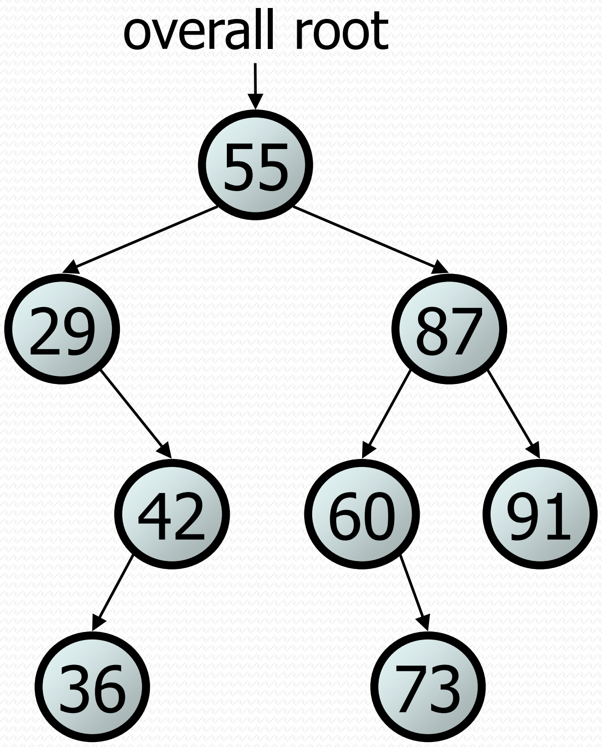
```
// Returns the minimum value from this BST.
// Throws a NoSuchElementException if the tree is empty.
public int getMin() {
    if (overallRoot == null) {
        throw new NoSuchElementException();
    }
    return getMin(overallRoot);
}

private int getMin(IntTreeNode root) {
    if (root.left == null) {
        return root.data;
    } else {
        return getMin(root.left);
    }
}
```



Exercise

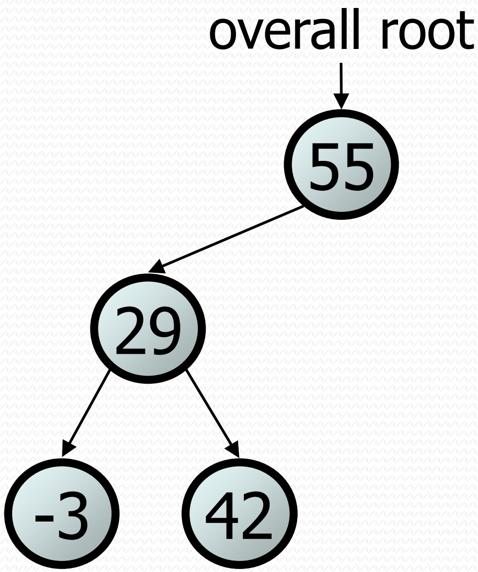
- Add a method `remove` to the `IntTree` class that removes a given integer value from the tree, if present. Remove the value in such a way as to maintain BST ordering.

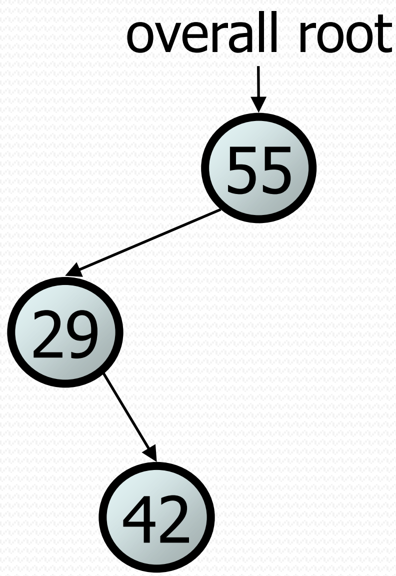
  - `tree.remove(73);`
  - `tree.remove(29);`
  - `tree.remove(87);`
  - `tree.remove(55);`

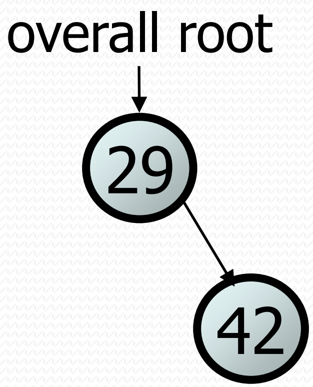


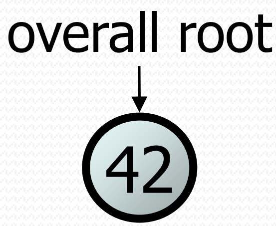
Cases for removal 1

1. a **leaf**: replace with `null`
2. a node with a **left child only**: replace with left child
3. a node with a **right child only**: replace with right child







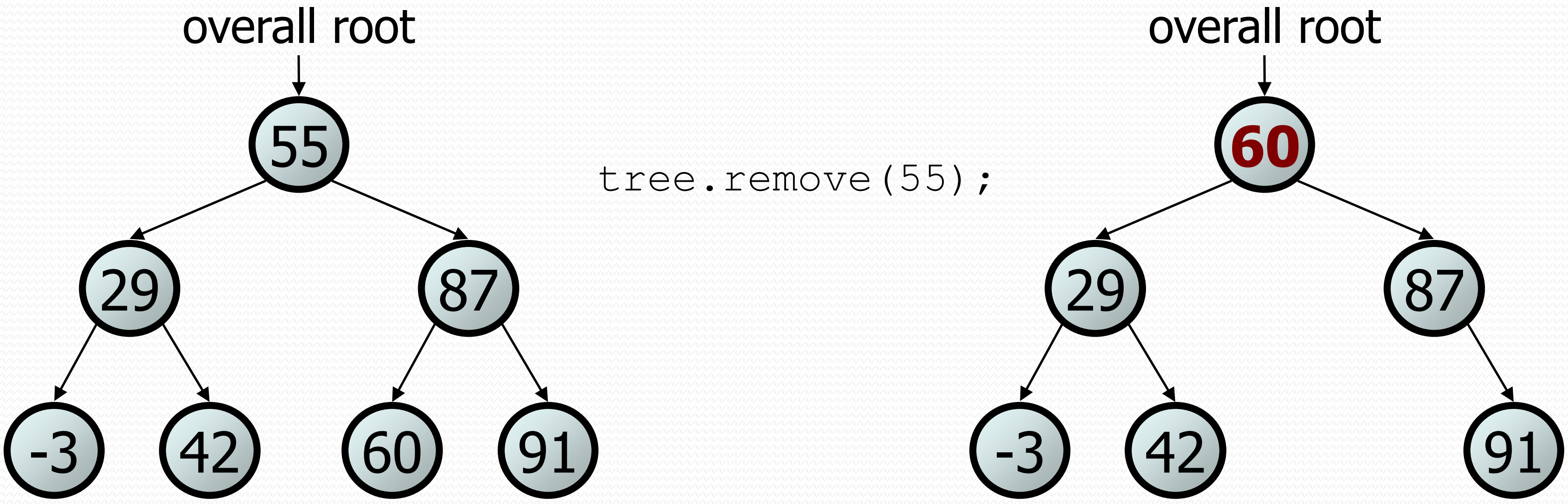


`tree.remove(29);`

`tree.remove(-3);`   `tree.remove(55);`

Cases for removal 2

4. a node with **both** children: replace with **min from right**
   - (replacing with max from left would also work)



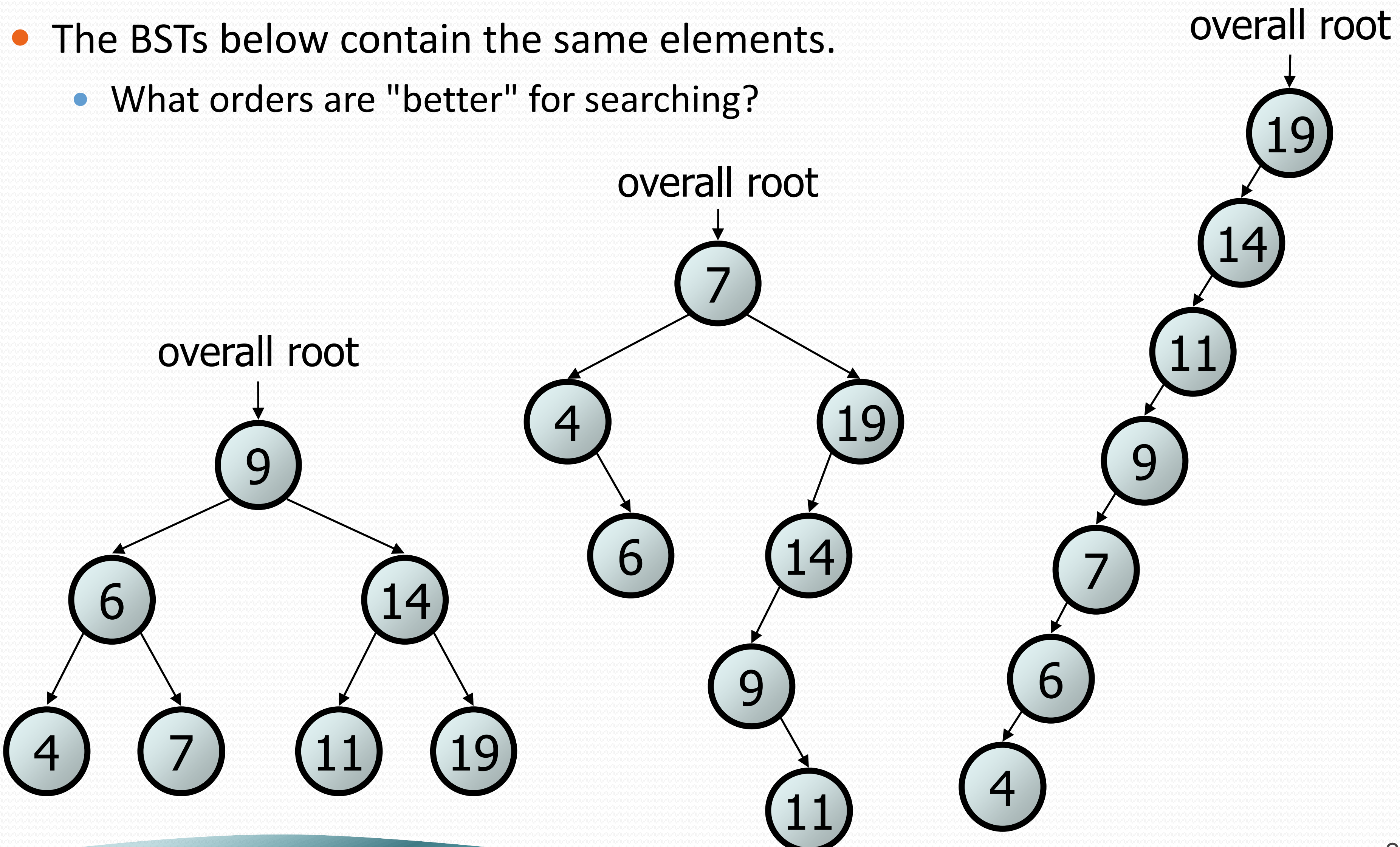

```
tree.remove(55);
```

Exercise solution

```
// Removes the given value from this BST, if it exists.
public void remove(int value) {
    overallRoot = remove(overallRoot, value);
}

private IntTreeNode remove(IntTreeNode root, int value) {
    if (root == null) {
        return null;
    } else if (root.data > value) {
        root.left = remove(root.left, value);
    } else if (root.data < value) {
        root.right = remove(root.right, value);
    } else {  // root.data == value; remove this node
        if (root.right == null) {
            return root.left;     // no R child; replace w/ L
        } else if (root.left == null) {
            return root.right;    // no L child; replace w/ R
        } else {
            // both children; replace w/ min from R
            root.data = getMin(root.right);
            root.right = remove(root.right, root.data);
        }
    }
    return root;
}
```

Searching BSTs

- The BSTs below contain the same elements.
  - What orders are "better" for searching?



Trees and balance

- **balanced tree**: One whose subtrees differ in height by at most 1 and are themselves balanced.
  - A balanced tree of N nodes has a height of ~ $\log_2$ N.
  - A very unbalanced tree can have a height close to N.

  - The runtime of adding to / searching a BST is closely related to height.

  - Some tree collections (e.g. `TreeSet`) contain code to balance themselves as new nodes are added.