CSE 143

Computer Programming II

Recursive Backtracking
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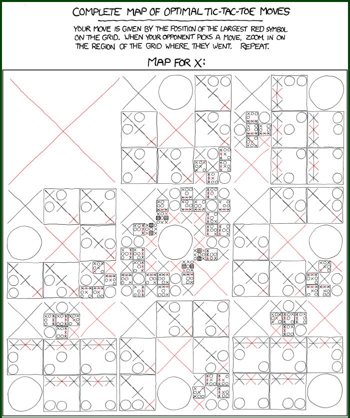


Outline

Definition (Recursive Backtracking)

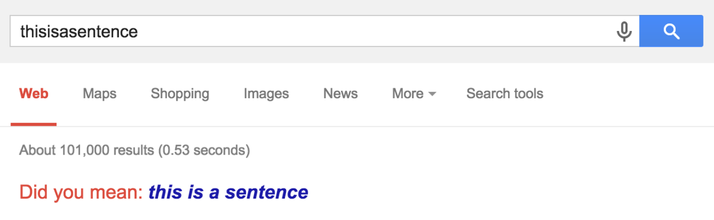
Recursive Backtracking is an attempt to find solution(s) by building up partial solutions and abandoning them if they don't work.

Recursive Backtracking Strategy

- If we found a solution, stop looking (e.g. return)
- Otherwise for each possible choice c...
  - Make the choice c
  - Recursively continue to make choices
  - Un-make the choice c (if we got back here, it means we need to continue looking)

Implementing a Tiny Piece of Google

When you enter a query with no spaces like **thisisasentence** into Google:



It fixes it into **this is a sentence** using recursive backtracking.

Sentence Splitting

Given an input string, `sentence`, containing **no spaces**, write a method:

```
public static String splitSentence(String sentence)
```

that returns `sentence` split up into words.

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- What are the choices we're making incrementally?
- How do we "undo" a choice?
- What are the base case(s)?

It helps to answer these questions for a particular input. So, pretend we're working with:

thisisasentence

Sentence Splitting

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that returns `sentence` split up into words.

To do recursive backtracking, we need to answer these questions:

- What are the choices we're making incrementally?
  ...which character to split at
- How do we "undo" a choice?
  ...re-combine a string by the char we split at
- What are the base case(s)?
  ...our left choice isn't a word **and** our right choice IS a word

It helps to answer these questions for a particular input. So, pretend we're working with:

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One More Important Choice

When doing recursive backtracking, we need to differentiate between:

- finding a result
- failing to find a result (e.g., backtracking)

Generally, we do this by treating `null` as a failure. For example:

- On the input, "**thisisasentence**", none of the recursive calls should return "**thisis**", because it isn't a word.
- If we get down to an empty string, that would indicate a failure; so, we'd return `null`

```java
public String splitSentence(String sentence) {
   // The entire sentence is a dictionary word!
   if (words.contains(sentence)) {
      return sentence;
   }

   // Try splitting at every character until we find one that works...
   for (int i = sentence.length() - 1; i > 0; i--){
      String left = sentence.substring(0, i);
      String right = sentence.substring(i, sentence.length());

      // If the left isn't a word, don't bother recursing.
      // If it is, split the remainder of the sentence recursively.
      if (words.contains(left)) {
         right = splitSentence(right);
         // Since the left was a word, if the right is also an answer,
         // then we found an answer to the whole thing!
         if (right != null) {
            return left + " " + right;
         }

         // Undo our choice by going back to sentence
      }
   }
   return null;
}
```

You may have noticed that many of the class examples I've been showing involve me using a class that I've already written. There are several reasons for this:

- Learning to read and use an API is a really important programming skill
- Switching between the client and implementor views is an important goal of this course
- The code I write is usually easy, but really tedious (so, it would be a waste of time to write in class)

Take-Away

Every time I print out an API for you, you should try to understand it from the comments. This will help you on the homework, on exams, and in any future programming endeavors.

Today's API is `BooleanExpression`.

What is a `BooleanExpression`?

The `BooleanExpression` class allows us to represent the conditions we write in `if` statements. For instance, to represent the following:

```
if (!(queue.size() > 0) && queue.peek() > 5) {
   ...
}
```

We would do

```
new BooleanExpression("(!a && b)");
```

Notice that we use single letter variable names instead of `queue.size() > 0`. This is a simplification for implementation.

Evaluating `BooleanExpressions`

Remember when we took $(1+2)*3$ and evaluated it to 9 recursively?
We can do a similar thing for `BooleanExpressions`:
Consider the `BooleanExpression` from above:

```
"(!a && b)"
```

Suppose we know the following:

- `a` is true.
- `b` is false.

What does this expression evaluate to?

`(!a && b)` $\longrightarrow$ `(!true && false)` $\longrightarrow$ `(false && false)` $\longrightarrow$ `false`

Suppose we wanted to write a method:

```
public static boolean evaluate(BooleanExpression e, ???  assn)
```

where `assn` represents the truth values of the variables.

What type would `assn` be? It's a **mapping** from variables to truth values.

Okay, so, we have:

```
public static boolean evaluate(BooleanExpression e,
          Map<String, Boolean> assignments)
```

Consider the following case:

evaluate return value?

- e is a && b
- assignments map is {a=true}.

What should evaluate return?

We can't answer the question. What seems like a good idea? **null**.

So, we change the return type to Boolean.

Who Writes `evaluate`?

- The implementor of `BooleanExpression`
  ...if so, it should be inside the `BooleanExpression` class
- The client of `BooleanExpression`
  ...if so, it should be outside the `BooleanExpression` class

The implementor of `BooleanExpression` should write the method, because then all the clients can use it.

That pesky `static`...

- If the implementor writes `evaluate`, then the method signature is:

```
public Boolean evaluate(Map<String, Boolean> assn)
```

- If the client writes `evaluate`, then the method signature is:

```
public static Boolean evaluate(
   BooleanExpression e,
   Map<String, Boolean> assn
)
```

canBeTrue

Write a method

```
public static boolean canBeTrue(BooleanExpression b)
```

that returns true if it is possible for the input to to **evaluate to true** and false otherwise.

Some examples:

- `a && b` ⟶ if we have `{a=true, b=true}`, then it is true.
- `a && !a` ⟶ no matter what a is, this will always be false.

To do recursive backtracking, we need to answer these questions:

- What are the choices we're making incrementally?
- How do we "undo" a choice?
- What are the base case(s)?

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To do recursive backtracking, we need to answer these questions:

- What are the choices we're making incrementally?
  ...assignments of each variable to true/false
- How do we "undo" a choice?
  ...remove the assignment from the map
- What are the base case(s)?
  ...the assignment must be true/false

We don't have a way of passing assignments through to the function. How can we fix this?

`public/private` pair!

Public/Private Recursive Pair

```
public static boolean canBeTrue(BooleanExpression b)

private static boolean canBeTrue(
   BooleanExpression b,
   Map<String, Boolean> m
)
```

canBeTrue Solution

```
public static Map<String, Boolean> canBeTrue(BooleanExpression b) {
   Map<String, Boolean> assignmentMap = new TreeMap<>();
   if (canBeTrue(b, assignmentMap)) { // Some assignment works...
      return assignmentMap;
   }
   return null;
}

private static boolean canBeTrue(BooleanExpression b, Map<String, Boolean> m) {
   Boolean result = b.evaluate(m); // See if the current assignment works
   if (result != null) {   // If we can’t answer, backtrack
      return result;
   }

   Set<String> variables = b.getVariables();
   variables.removeAll(m.keySet());
   for (String variable : variables) { // Try to assign any
      boolean[] choices = {true, false}; // variable we haven’t
      for (boolean assignment : choices) { // already assigned.
         m.put(variable, assignment);
         Boolean attempt = canBeTrue(b, m);
         if (attempt) {             // If we found an
            return true;            // assignment, we’re good.
         }
         m.remove(variable);        // Otherwise, backtrack
      }
   }

   return false;
}
```

BTW, Why does this problem matter?

Solving `canBeTrue` quickly is the **most important** open problem in Computer Science.

If you solve this problem in $\mathcal{O}(n^k)$ time for **any** k, the following happen:

- You get **one million** dollars.
- You get a PhD.
- You become the most famous Computer Scientist, pretty much ever
- You break all banks, credit cards, website encryption, etc.