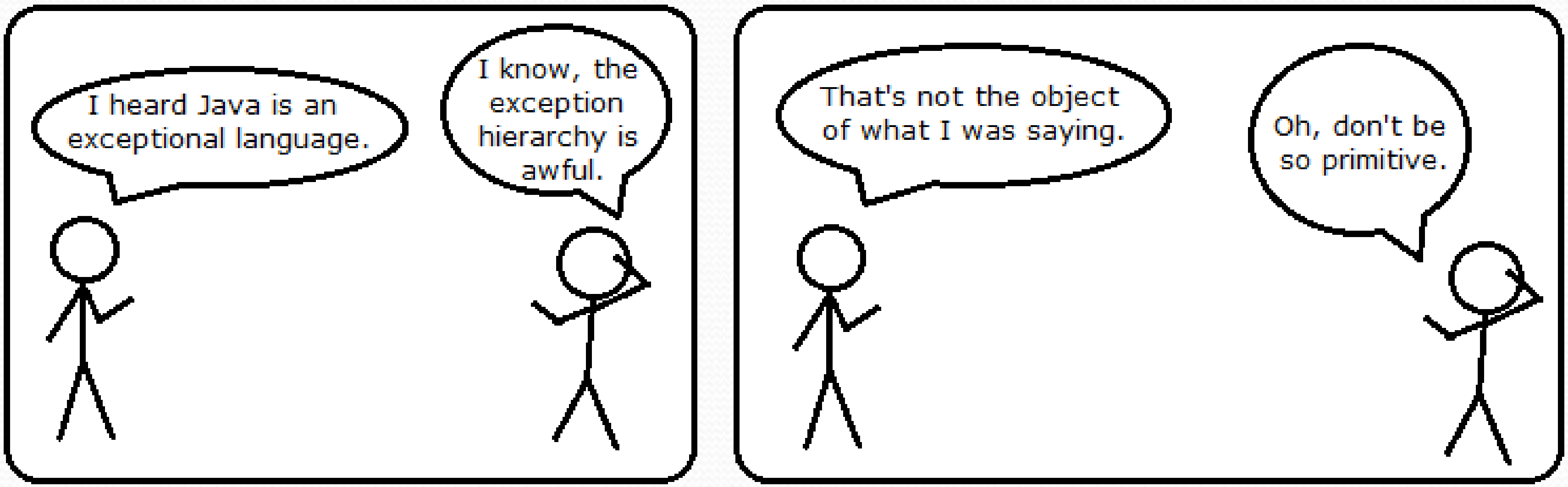
CSE 143

Lecture 14: binary search and complexity

reading: 13.1-13.2



http://www.alexsweet.co.uk/comics.php?comic=2

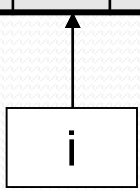
Searching methods

- Implement the following methods in `ArrayIntList`:
  - `indexOf` – returns first index of element, or -1 if not found
  - `contains` - returns true if the list contains the given int value

Sequential search

- **sequential search**: Locates a target value in an array / list by examining each element from start to finish. Used in `indexOf`.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



  - The array is sorted. Could we take advantage of this?

Sequential search

- What is its complexity class?

```
public int indexOf(int value) {
    for (int i = 0; i < size; i++) {
        if (elementData[i] == value) {
            return i;
        }
    }
    return -1;    // not found
}
```

N

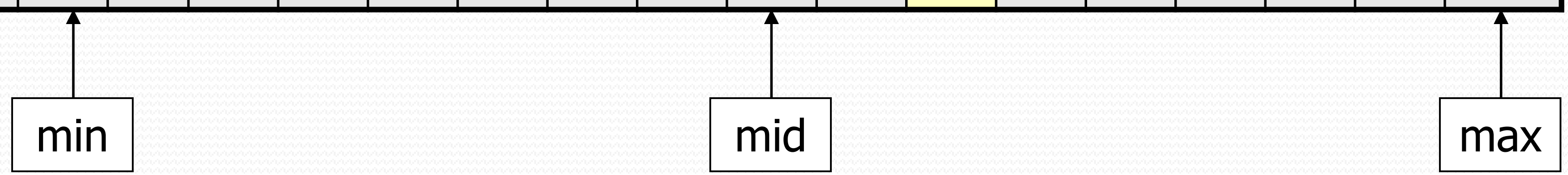
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

- On average, "only" N/2 elements are visited
  - 1/2 is a constant that can be ignored

Binary search (13.1)

- **binary search**: Locates a target value in a *sorted* array or list by successively eliminating half of the array from consideration.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



Binary search runtime

- For an array of size N, it eliminates ½ until 1 element remains.

  N, N/2, N/4, N/8, ..., 4, 2, 1

  - How many divisions does it take?

- Think of it from the other direction:
  - How many times do I have to multiply by 2 to reach N?

    1, 2, 4, 8, ..., N/4, N/2, N

  - Call this number of multiplications "x".

    $2^x = N$

    $\mathbf{x = \log_2 N}$

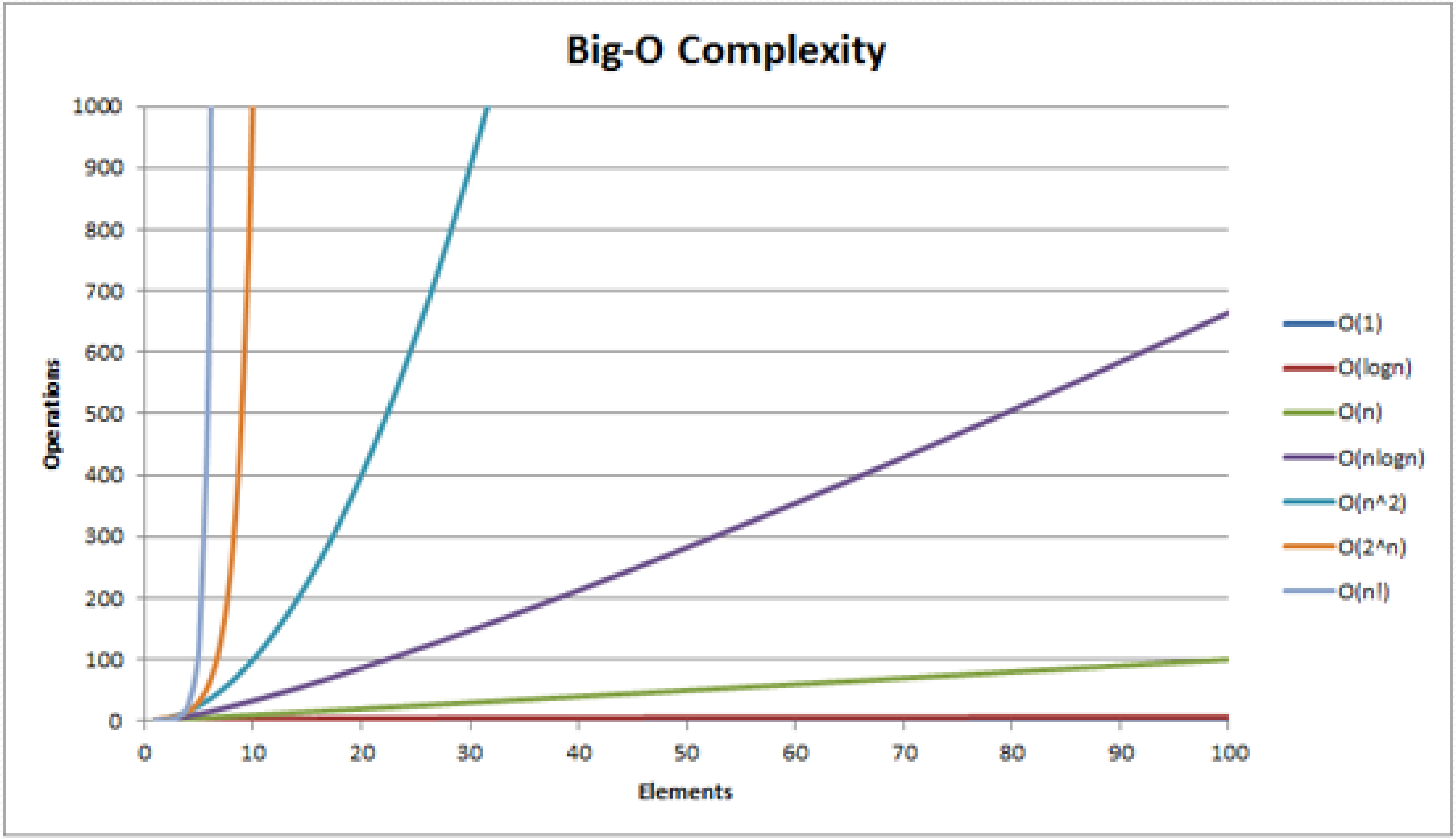
- Binary search is in the **logarithmic** complexity class.

Complexity classes

- **complexity class**: A category of algorithm efficiency based on the algorithm's relationship to the input size N.

Class	Big-Oh	If you double N, ...	Example
constant	$O(1)$	unchanged	10ms
logarithmic	$O(\log_2 N)$	increases slightly	175ms
linear	$O(N)$	doubles	3.2 sec
log-linear	$O(N \log_2 N)$	slightly more than doubles	6 sec
quadratic	$O(N^2)$	quadruples	1 min 42 sec
cubic	$O(N^3)$	multiplies by 8	55 min
...	...	...	...
exponential	$O(2^N)$	multiplies drastically	$5 * 10^{61}$ years

Complexity classes



http://recursive-design.com/blog/2010/12/07/comp-sci-101-big-o-notation/ - post about a Google interview

Arrays.binarySearch

```
// searches an entire sorted array for a given value
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, value)

// searches given portion of a sorted array for a given value
// examines minIndex (inclusive) through maxIndex (exclusive)
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, minIndex, maxIndex, value)
```

- The `binarySearch` method in the `Arrays` class searches an array very efficiently if the array is sorted.
  - You can search the entire array, or just a range of indexes (useful for "unfilled" arrays such as the one in `ArrayIntList`)

Using `binarySearch`

```
// index     0   1   2   3    4    5    6    7    8    9   10   11   12   13   14   15
int[] a = {-4, 2, 7, 9, 15, 19, 25, 28, 30, 36, 42, 50, 56, 68, 85, 92};

int index  = Arrays.binarySearch(a, 0, 16, 42);    // index1 is 10
int index2 = Arrays.binarySearch(a, 0, 16, 21);    // index2 is -7
```

- `binarySearch` returns the index where the value is found
- if the value is *not* found, `binarySearch` returns:

  `-(insertionPoint + 1)`

  - where `insertionPoint` is the index where the element *would* have been, if it had been in the array in sorted order.
  - To insert the value into the array, negate `insertionPoint` + 1

```
int indexToInsert21 = -(index2 + 1);   // 6
```

Binary search code

```
// Returns the index of an occurrence of target in a,
// or a negative number if the target is not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    int min = 0;
    int max = a.length - 1;

    while (min <= max) {
        int mid = (min + max) / 2;
        if (a[mid] < target) {
            min = mid + 1;
        } else if (a[mid] > target) {
            max = mid - 1;
        } else {
            return mid;    // target found
        }
    }

    return -(min + 1);    // target not found
}
```

Recursive binary search (13.3)

- Write a recursive `binarySearch` method.
  - If the target value is not found, return its negative insertion point.

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

```
int index  = binarySearch(data, 42);   // 10
int index2 = binarySearch(data, 66);   // -14
```

The `compareTo` method (10.2)

- The standard way for a Java class to define a comparison function for its objects is to define a `compareTo` method.
  - Example: in the `String` class, there is a method:

    `public int compareTo(String other)`

- A call of **A**`.compareTo(`**B**`)` will return:

a value <	0	if **A** comes "before" **B** in the ordering,
a value >	0	if **A** comes "after" **B** in the ordering,
or	0	if **A** and **B** are considered "equal" in the ordering.

Using `compareTo`

- `compareTo` can be used as a test in an `if` statement.

```
String a = "alice";
String b = "bob";
if (a.compareTo(b) < 0) {   // true
    ...
}
```

Primitives	Objects
`if (a < b) { ...`	`if (a.compareTo(b) < 0) { ...`
`if (a <= b) { ...`	`if (a.compareTo(b) <= 0) { ...`
`if (a == b) { ...`	`if (a.compareTo(b) == 0) { ...`
`if (a != b) { ...`	`if (a.compareTo(b) != 0) { ...`
`if (a >= b) { ...`	`if (a.compareTo(b) >= 0) { ...`
`if (a > b) { ...`	`if (a.compareTo(b) > 0) { ...`