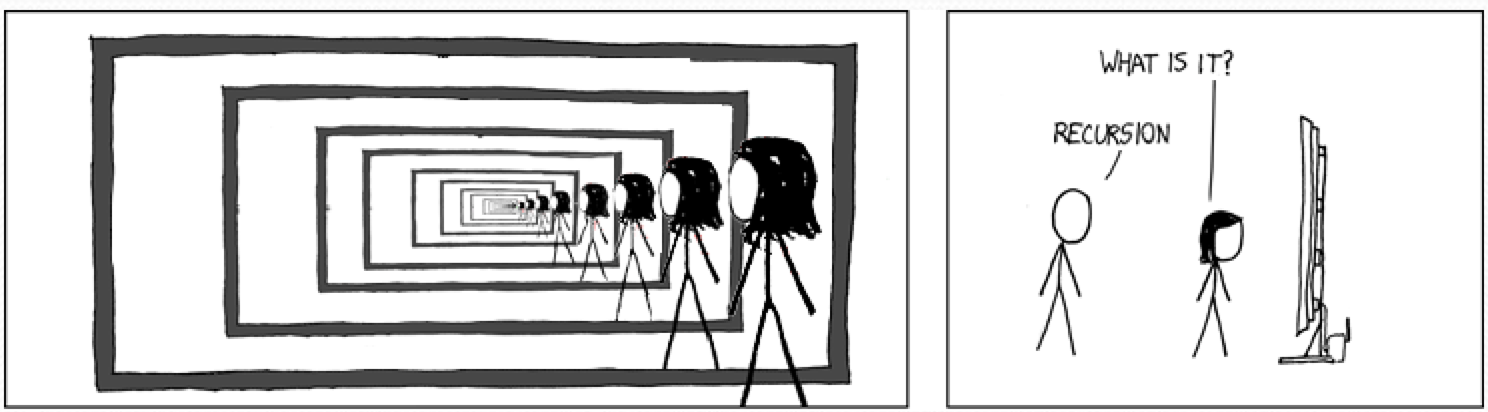
CSE 143

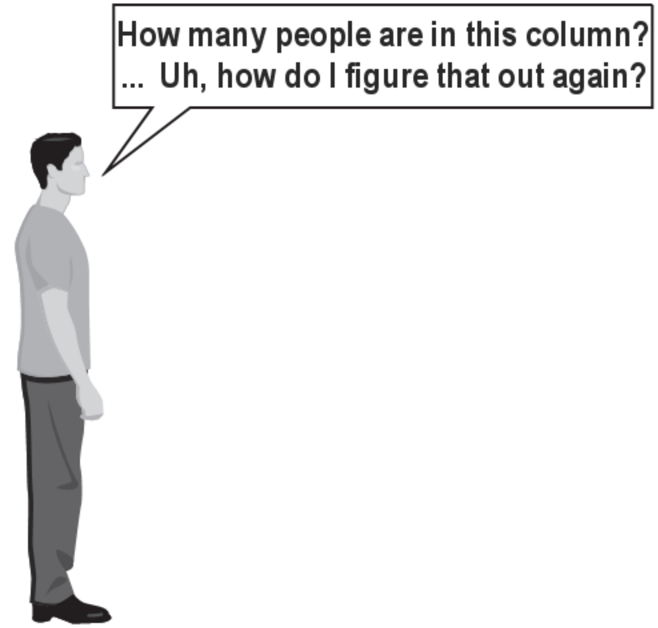
Lecture 9: introduction to recursion

reading: 12.1



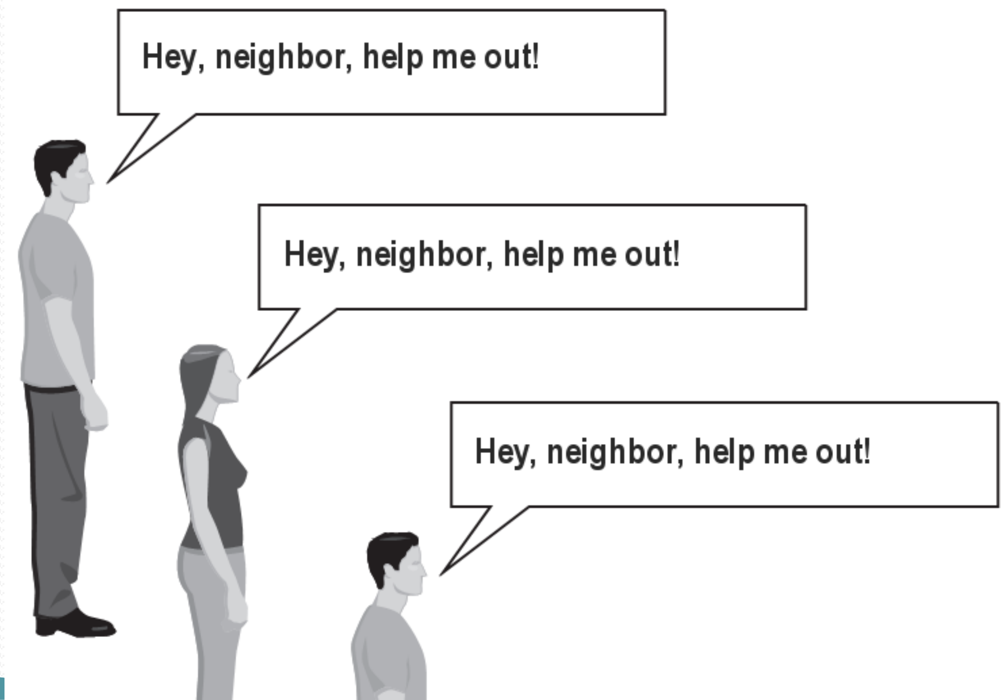
Exercise

- (To a student in the front row)
  How many students total are directly behind you in your "column" of the classroom?
  - You have poor vision, so you can see only the people right next to you. So you can't just look back and count.
  - But you are allowed to ask questions of the person next to you.
  - How can we solve this problem? (*recursively*)



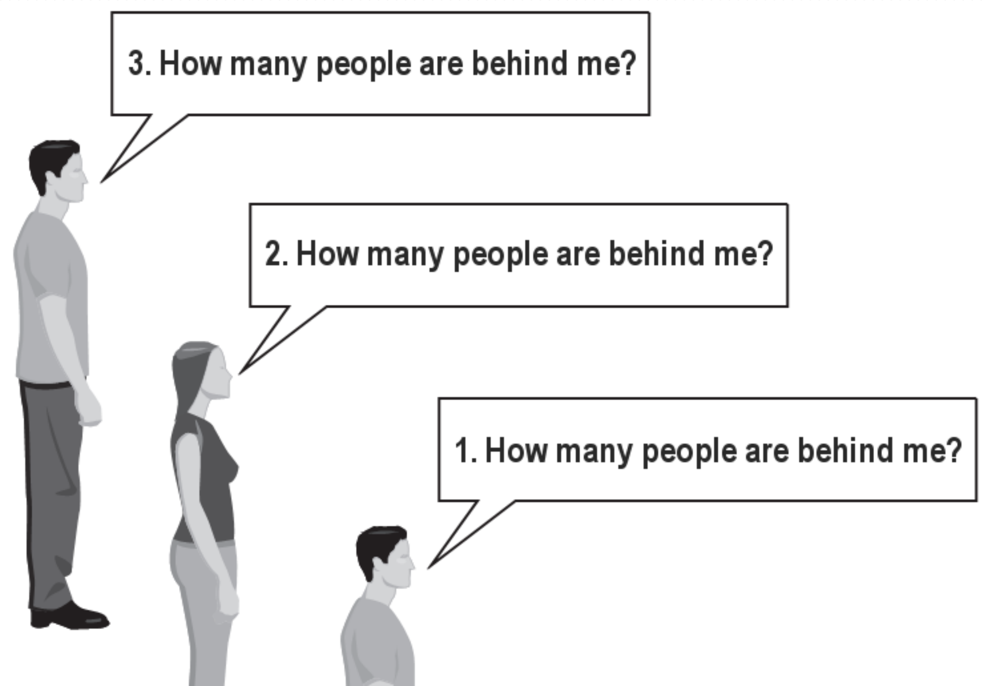
The idea

- Recursion is all about breaking a big problem into smaller occurrences of that same problem.
  - Each person can solve a small part of the problem.
    - What is a small version of the problem that would be easy to answer?
    - What information from a neighbor might help me?



Recursive algorithm

- Number of people behind me:
  - If there is someone behind me,
    ask him/her how many people are behind him/her.
    - When they respond with a value **N**, then I will answer **N + 1**.
  - If there is nobody behind me, I will answer **0**.



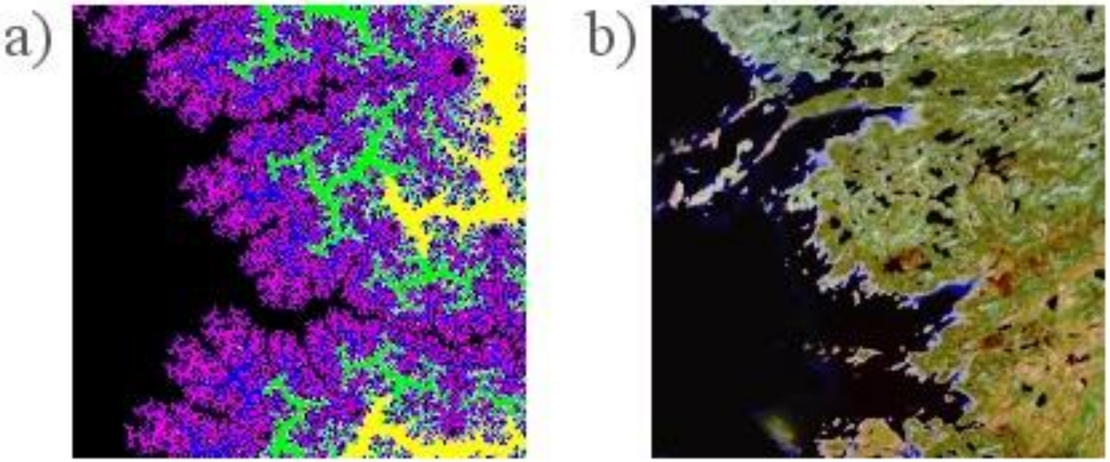
Recursion

- **recursion**: The definition of an operation in terms of itself.
  - Solving a problem using recursion depends on solving smaller occurrences of the same problem.

- **recursive programming**: Writing methods that call themselves to solve problems recursively.

  - An equally powerful substitute for *iteration* (loops)
  - Particularly well-suited to solving certain types of problems


ALOEACEAE
ALOE
POLYPHYLLA
SPIRAL ALOE
S. AFRICA



a) Part of the Mandelbrot set.

b) Part of the North American coastline near Hudson Bay.

Linked Lists are Self-Similar

- a linked list is:
  - null
  - a node whose `next` field references a list

- **recursive data structure**: a data structure partially composed of smaller or simpler instances of the same data structure

Why learn recursion?

- "Cultural experience" – think differently about problems
- Solves some problems more naturally than iteration
- Can lead to elegant, simplistic, short code (when used well)
- Many programming languages ("functional" languages such as Scheme, ML, and Haskell) use recursion exclusively (no loops)
- A key component of many of our assignments in CSE 143

Recursion and cases

- Every recursive algorithm involves at least 2 cases:
  - **base case**: A simple occurrence that can be answered directly.
  - **recursive case**: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.
  - Some recursive algorithms have more than one base or recursive case, but all have at least one of each.
  - A crucial part of recursive programming is identifying these cases.

Getting down stairs

- Need to know two things:
  - Getting down one stair
  - Recognizing the bottom

- Most code will look like:

```
if (simplest case) {
    compute and return solution
} else {
    divide into similar subproblem(s)
    solve each subproblem recursively
    assemble the overall solution
}
```

Another recursive task

- How can we remove exactly half of the M&M's in a large bowl, without dumping them all out or being able to count them?
  - What if multiple people help out with solving the problem? Can each person do a small part of the work?
  - What is a number of M&M's that it is easy to double, even if you can't count?
    - (What is a "base case"?)

Recursion in Java

- Consider the following method to print a line of `*` characters:

```
// Prints a line containing the given number of stars.
// Precondition: n >= 0
public static void printStars(int n) {
    for (int i = 0; i < n; i++) {
        System.out.print("*");
    }
    System.out.println();    // end the line of output
}
```

- Write a recursive version of this method (that calls itself).
  - Solve the problem without using any loops.
  - Hint: Your solution should print just one star at a time.

A basic case

- What are the cases to consider?
  - What is a very easy number of stars to print without a loop?

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        ...
    }
}
```

Handling more cases

- Handling additional cases, with no loops (in a bad way):

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        System.out.println("*");
    } else if (n == 3) {
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else if (n == 4) {
        System.out.print("*");
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else ...
}
```

Handling more cases 2

- Taking advantage of the repeated pattern (somewhat better):

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        printStars(1);     // prints "*"
    } else if (n == 3) {
        System.out.print("*");
        printStars(2);     // prints "**"
    } else if (n == 4) {
        System.out.print("*");
        printStars(3);     // prints "***"
    } else ...
}
```

Using recursion properly

- Condensing the recursive cases into a single case:

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

"Recursion Zen"

- The real, even simpler, base case is an `n` of 0, not 1:

```
public static void printStars(int n) {
    if (n == 0) {
        // base case; just end the line of output
        System.out.println();
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

- **Recursion Zen**: The art of properly identifying the best set of cases for a recursive algorithm and expressing them elegantly.

  (A CSE 143 informal term)

Recursive tracing

- Consider the following recursive method:

```
public static int mystery(int n) {
    if (n < 10) {
        return n;
    } else {
        int a = n / 10;
        int b = n % 10;
        return mystery(a + b);
    }
}
```

  - What is the result of the following call?

    `mystery(648)`

A recursive trace

```
mystery(648):
  ▪ int a = 648 / 10;          // 64
  ▪ int b = 648 % 10;          //  8
  ▪ return mystery(a + b);     // mystery(72)

    mystery(72):
    ▪ int a = 72 / 10;         // 7
    ▪ int b = 72 % 10;         // 2
    ▪ return mystery(a + b);   // mystery(9)

      mystery(9):
      ▪ return 9;
```

Recursive tracing 2

- Consider the following recursive method:

```
public static int mystery(int n) {
    if (n < 10) {
        return (10 * n) + n;
    } else {
        int a = mystery(n / 10);
        int b = mystery(n % 10);
        return (100 * a) + b;
    }
}
```

  - What is the result of the following call?

    `mystery(348)`

A recursive trace 2

```
mystery(348)
  int a = mystery(34);
    int a = mystery(3);
      return (10 * 3) + 3;    // 33
    int b = mystery(4);
      return (10 * 4) + 4;    // 44
    return (100 * 33) + 44;    // 3344

  int b = mystery(8);
    return (10 * 8) + 8;    // 88

  return (100 * 3344) + 88;    // 334488
```

- What is this method really doing?

Exercise

- Write a recursive method `reverseLines` that accepts a file `Scanner` and prints the lines of the file in reverse order.
  - Example input file:

```
I have eaten
the plums
that were in
the icebox
```

Expected console output:

```
the icebox
that were in
the plums
I have eaten
```

  - What are the cases to consider?
    - How can we solve a small part of the problem at a time?
    - What is a file that is very easy to reverse?

Reversal pseudocode

- Reversing the lines of a file:
  - Read a line L from the file.
  - Print the rest of the lines in reverse order.
  - Print the line L.

- If only we had a way to reverse the rest of the lines of the file....

Reversal solution

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        // recursive case
        String line = input.nextLine();
        reverseLines(input);
        System.out.println(line);
    }
}
```

- Where is the base case?

Tracing our algorithm

- **call stack**: The method invocations currently running

```
    reverseLines(new Scanner("poem.txt"));
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();   // "I have eaten"
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();   // "the plums"
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();   // "that were in"
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();   // "the icebox"
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {    // false
        ...
    }
}
```

```
I have eaten
the plums
that were in
the icebox
```

```
the icebox
that were in
the plums
I have eaten
```