CSEp 590B
Spring Quarter 2006
Assignment 6
Due Friday, May 11, 2006

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) It is well known that the Hamiltonian Path problem is NP-complete (cf. 286-291 of Sipser). The Hamiltonian Path problem is defined by:

Input:	An undirected graph $G = (V, E)$ and vertices $s, t \in V$.
Property:	There is a path in G from s to t that visits every vertex of G exactly once.

   Show that the problem of Bounded Degree Spanning Tree is also NP-complete. Bounded Degree Spanning Tree problem is defined by:

Input:	A connected undirected graph $G = (V, E)$ and number k.
Property:	There is a connected subgraph T of G such that T contains all the vertices of G, contains no cycles, and each vertex of T has degree $\leq k$.

   Part of your proof should be the construction of a polynomial time reduction of Hamiltonian Path to Bounded Degree Spanning Tree.

2. (10 points) Consider the following scheduling problem with release times and deadlines, called *Job Scheduling*.

   - Input: n jobs of lengths $L_1, \ldots, L_n$, with release times $R_1, \ldots R_n$ and deadlines $D_1, \ldots D_n$, all integers.
   - Property: All jobs can be scheduled after their release times, finished before their deadlines, without overlapping. That is, the jobs have start times $S_1, \ldots S_n$, such that, for all i, $R_i \leq S_i$, $S_i + L_i \leq D_i$, and for $j \neq i$, either $S_i + L_i \leq S_j$ or $S_j + L_j \leq S_i$

   Show that Job Scheduling is NP-complete. As part of your proof show that Subset Sum is polynomial Time reducible to Job Scheduling.

3. (10 points) Problem 7.24 on page 296 of Sipser.