CSEP 590B
Spring Quarter 2006
Assignment
Due Friday, April 27, 2006

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) In this problem you will give the details of the reduction of the acceptance problem for Turing machines to the non-emptiness problem for two-headed finite automata, thereby showing its undecidablilty. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$ be a one-tape Turing machine and let $w \in \Sigma^*$ be an input. The goal is to construct a two-headed finite automaton $M' = (Q', \Sigma', \delta', F)$ such that M accepts w if and only if M' accepts some input. As described in class M' will accept only strings that represent accepting computation histories of M on input w. Such a string has the form $C_1 \# C_2 \# \cdots \# C_m \$$ where (i) $C_1 = q_0 w \sqcup^k$ for some k, (ii) C_{i+1} follows from C_i in one step of M for $1 \leq i < m$, (iii) C_m contains q_a, and (iv) $|C_1| = |C_i|$ for $1 \leq i \leq m$. Recall that M' runs in three phases. In the first phase, the first heads scans the input to the first $\#$ checking condition (i) above. In the second phase, the two heads scan the input in parallel checking condition (ii) above. At the end of the second phase, if all goes well, the first head is on the \$ and the second head is on the last $\#$. In the third phase, the second head scans the string between the last $\#$ and the \$ checking for condition (iii) above. If all goes well, condition (iv) holds.

   Recall that in the second phase, after a brief startup, M' uses states of the form $(a_1, a_2, a_3, b_1, b_2, b_3)$ which are six-tuples of symbols from $Q \cup \Gamma \cup \{\#, \$\}$. The meaning of this state is that $a_1 a_2 a_3$ are the three symbols just scanned by second head and $b_1 b_2 b_3$ are the three symbols just scanned by the first head. The relation $L(a_1, a_2, a_3, b_1, b_2, b_3)$ simply states that $a_1 a_2 a_3$ and $b_1 b_2 b_3$ are consistent with a move of M. For example, if $a_2 \in Q$, $a_3 \in \Gamma$, and $\delta(a_2, a_3) = (p, b, R)$ then $b_1 = a_1$, $b_2 = b$, and $b_3 = p$. There are several other conditions as well that define L. You can assume L is given for the rest of the problem.

   (a) Precisely define all of the states Q' of M'. Include the states for all three phases.

   (b) Precisely define all the input symbols Σ' of M'.

   (c) Precisely define the transition function δ' of M'. The function δ' maps $Q' \times \Sigma' \times \Sigma'$ to $Q' \times \{R, S\} \times \{R, S\}$. R means move the head right while S means keep it stationary.

   (d) Precisely define the set of final (accepting) states F.

2. (10 points) In this problem you will show that the non-empty intersection problem for context-free grammars is undecidable. The non-empty intersection problem is given to context-free grammars, do they generate a common terminal string. That is, given G_1 and G_2, is $L(G_1) \cap L(G_2) \neq \emptyset$. This can

be done by showing that PCP is reducible to this problem. Use the solution to problem 5.21 on page 212 of Sipser as a hint.