AES and Attacks on Cryptographic Hashes

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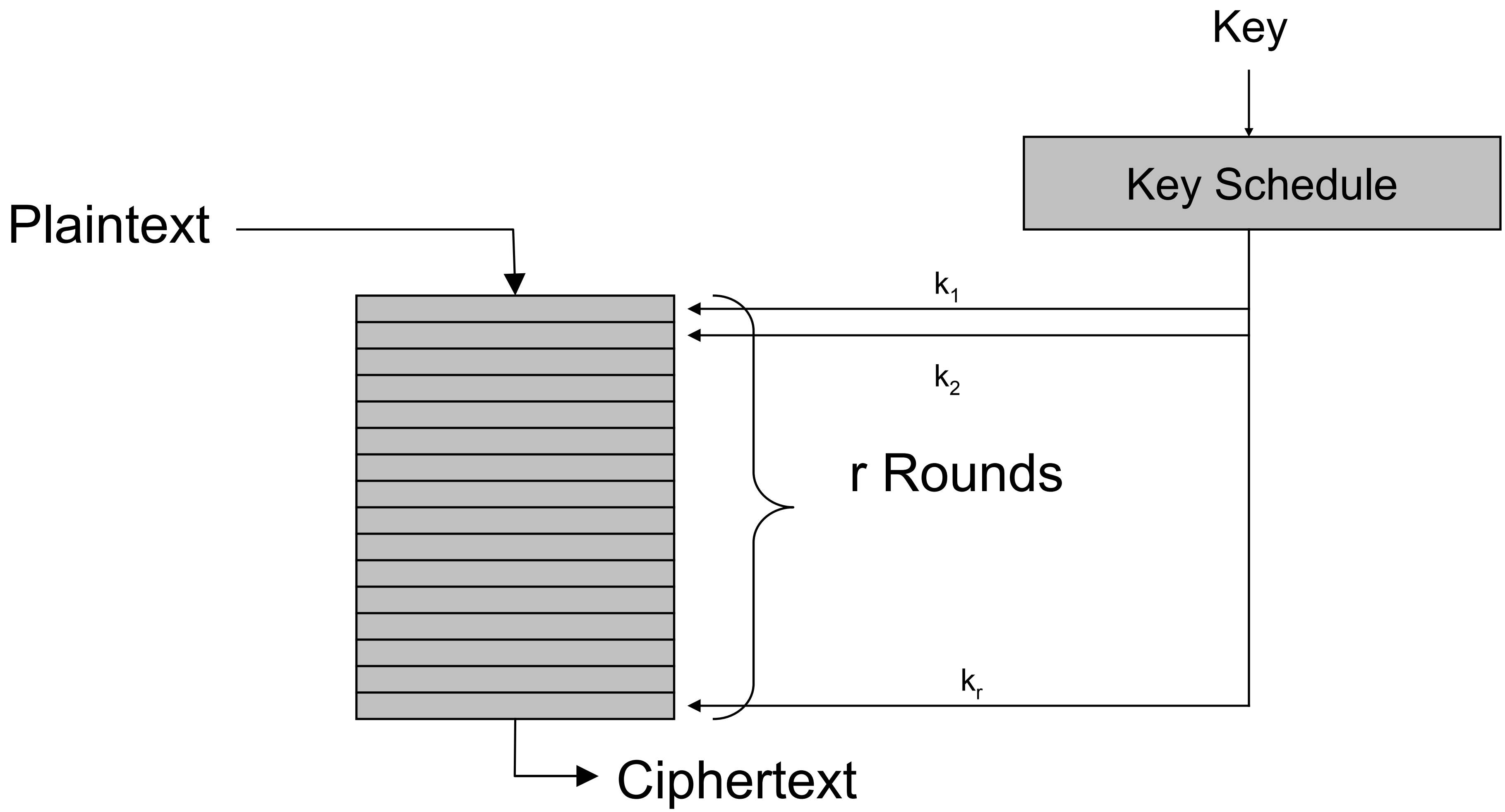
AES History

- Call for DES successor 1/97
- Square begets Rijndael (1998)
  - Rijndael Designers: Vincent Rijmen and Joan Daemen
- Nine Submissions
  - CAST-256, CRYPTON, DEAL, DFC (cipher), E2, FROG, HPC, LOKI97, MAGENTA, MARS, RC6, Rijndael, SAFER+, Serpent, and Twofish.
- Finalists
  - MARS, RC6, Rijndael, Serpent, and Twofish
- FIPS 197 published 11/2001

AES



AES Requirements

- 128, 192, 256 bit keys
- Algorithms will be judged on the following factors:
  - Actual security of the algorithm compared to other submitted algorithms (at the same key and block size).
  - The extent to which the algorithm output is indistinguishable from a random permutation on the input block.
  - Soundness of the mathematical basis for the algorithm’s security.
  - Other security factors raised by the public during the evaluation process, including any attacks which demonstrate that the actual security of the algorithm is less than the strength claimed by the submitter.
  - Claimed attacks will be evaluated for practicality.
- Key agility (NSA): “Two blocks encrypted with two different keys should not take much more time than two blocks encrypted with the same key.

Mars (Multiplication, Addition, Rotation and Substitution)

Basic Structure

1. Whiten
2. 8 rounds of key independent mixing
3. 16 rounds of keyed Feistel transforms (2 S-boxes)
4. 8 rounds of key independent mixing
5. Whiten

RC6 Design Philosophy

- Leverage our experience with RC5: use *data-dependent rotations* to achieve a high level of security.
- Adapt RC5 to meet AES requirements
- Take advantage of a new primitive for increased security and efficiency: *32x32 multiplication*, which executes quickly on modern processors, to compute rotation amounts.

Slide by Ron Rivest (Second AES Conference)

Security against differential attacks

Estimate of number of plaintext pairs required to mount a differential attack.

(Only 2^{128} such pairs are available.)

Rounds	Pairs
8	2^{56}
12	2^{117}
16	2^{190}
20 ← RC6 →	2^{238}
24	2^{299}

Infeasible

Slide by Ron Rivest (Second AES Conference)

Rijndael Overview

- Input
  - p consisting of N_b words
  - k with N_k words (N_k= 4,6,8)
- State
  - 4 rows, N_b columns
- Key
  - 4 rows, N_k columns
- Output
  - c consisting of N_b words

All tables filled first col first $s_{0,0}$, $s_{1,0}$, $s_{2,0}$, $s_{3,0}$, $s_{0,1}$, …

Rijndael Overview

- Design Philosophy
  - Wide Trails
- 32 bit word operations
- Non-linear substitution uses arithmetic over GF(2)
- Mixing uses polynomial arithmetic mod (x^4+1)

Rijndael Round Structure

$N_r = \max(N_k, N_b) + 6$

N_r	$N_b=4$	$N_b= 6$	$N_b=8$
$N_k=4$	10	12	14
$N_k=6$	12	12	14
$N_k=8$	14	14	14

Rijndael State Layout

State: $s_{i,j}$, i= Nb (mod 4), j= [Nb/4], Nb=4j+i

For Nb= 4

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

Rijndael Key Layout

Keys: $k_{i,j}$, i= Nk (mod 4), j= [Nk/4]

For Nk= 4

$k_{0,0}$	$k_{0,1}$	$k_{0,2}$	$k_{0,3}$
$k_{1,0}$	$k_{1,1}$	$k_{1,2}$	$k_{1,3}$
$k_{2,0}$	$k_{2,1}$	$k_{2,2}$	$k_{2,3}$
$k_{3,0}$	$k_{3,1}$	$k_{3,2}$	$k_{3,3}$

Rijndael Algorithm

```
Rijndael (p, k, Nb, Nk)  {
    ComputeRoundKeys(K, W[0…Nr])
    state= p
    AddRoundKey(0, state)
    for (i=1, i<=Nr, i++) {
        for each byte, b in state
            ByteSub(b)
        ShiftRow(state)
        if(i<Nr)
            MixCol(state)
        AddRoundKey(i, state)
        }
    c= state
    }
```

Inverse Rijndael Algorithm

```
InvRijndael (c, k, Nb, Nk)  {
    ComputeRoundKeys(K, W[0…Nr])
    state= c
    for (i=0, i<Nr, i++) {
        AddRoundKey(Nr-i, state)
        if(i>0)
            InvMixCol(state)
        InvShiftRow(state)
        for each byte, b in state
            InvByteSub(b)
        }
    AddRoundKey(0, state)
    p= state
    }
```

Review: Arithmetic of GF(2^n)

- Suppose m(x) is an irreducible polynomial of degree n over GF(2): $m(x)= x^n + m_{n-1} x^{n-1} + \ldots + m_0$.
- Let a(x) and b(x) be polynomials of degree <n. They form a vector space of dimension n over GF(2). Coefficients of like exponent "add": $(a_{n-1} x^{n-1} + \ldots + a_0)+ (b_{n-1} x^{n-1} + \ldots + b_0)= (a_{n-1}+ b_{n-1})x^{n-1} + \ldots + a_0 + b_0)$
- Euclidean algorithm: for a(x), b(x) polynomials of degrees $m \leqq n$, there are polynomials q(x), r(x), deg r(x) <n such that a(x)=q(x)b(x)+r(x)
- Polynomials over GF(2) modulo m(x) form a field (with 2^n elements). Multiplication is multiplication of polynomials mod m(x).
- Inverses exist by following theorem: If a(x) and b(x) are polynomials their greatest common denominator d(x) can be written as

  d(x)= a(x)u(x)+b(x)v(x) for some u(x), v(x).

Example of multiplication and inverse

- In particular if a(x) and b(x) are co-prime: $1= a(x)u(x)+b(x)v(x)$ for some u(x), v(x).
- Example
  - $m(x)= x^2 +x +1$. m(x) is irreducible (otherwise it would have a root in GF(2)
  - $x+(x+1) =1$, $1+(x+1)= x$
  - $(x+1)(x+1)= x^2+2x+1=x^2+1= (x) + (x^2 +x +1)= x \pmod{m(x)}$
  - (x+1) and m(x) are co-prime in fact,

    $1= (x+1) (x) + (x^2 +x +1)(1)$
  - So "x" is the multiplicative inverse of "x+1" in GF(4).
  - Usually elements of $GF(2^n)$ are written in place notation so $x^5+x^3 +x^2 +1=$ 101101.

ByteSub Primitive

```
ByteSub(b)
    if b==0
        t= 0
    else
        t= b^-1
return(Mt + a)
```

M= circ(1,0,0,0,1,1,1,1)
a= $(1,1,0,0,0,1,1,0)^T$
Arithmetic over GF(2) with m(x)= $x^8+x^4+x^3+x+1$.

ByteSub Data

M:

1	0	0	0	1	1	1	1
1	1	0	0	0	1	1	1
1	1	1	0	0	0	1	1
1	1	1	1	0	0	0	1
1	1	1	1	1	0	0	0
0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0
0	0	0	1	1	1	1	1

a:

1
1
0
0
0
1
1
0

Bytesub

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

$t_{0,0}$	$t_{0,1}$	$t_{0,2}$	$t_{0,3}$
$t_{1,0}$	$t_{1,1}$	$t_{1,2}$	$t_{1,3}$
$t_{2,0}$	$t_{2,1}$	$t_{2,2}$	$t_{2,3}$
$t_{3,0}$	$t_{3,1}$	$t_{3,2}$	$t_{3,3}$

Rijndael Primitives

ShiftRow(state)

- shift row 1 by 0.
- shift row 2 by 1.
- shift row 3 by 2 if Nb<8, 3 otherwise.
- shift row 3 by 3 if Nb<8, 4 otherwise.

MixCol(state)

- multiply each column of state by c(x) (mod $x^4 +1$)
- $c(x)= 0x03\ x^3 + 0x01\ x^2 + 0x01\ x + 0x02$

InvMixCol(state)

- multiply each column of state by d(x) (mod $x^4 +1$)
- $d(x)= 0x0b\ x^3 + 0x0d\ x^2 + 0x09\ x + 0x0e$

AddRoundKey(i,state)

- state= state + W[i]

ShiftRow

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,3}$	$s_{1,0}$	$s_{1,1}$	$s_{1,2}$
$s_{2,2}$	$s_{2,3}$	$s_{2,0}$	$s_{2,1}$
$s_{3,3}$	$s_{3,0}$	$s_{3,1}$	$s_{3,2}$

MixCol

$s_{0,0}$	$s_{0,1}$	$s_{0,3}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,3}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,3}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,3}$	$s_{3,3}$

$$t_{0,0}x^3+t_{1,0}x^2+t_{2,0}x+t_{3,0}=$$
$$(0x03x^3+0x01x^2+0x01x+0x02) \text{ x } (s_{0,0}x^3+s_{1,0}x^2+s_{2,0}x+s_{3,0})(\text{mod } x^4+1)$$

$t_{0,0}$	$s_{0,1}$	$s_{0,3}$	$s_{0,3}$
$t_{1,0}$	$s_{1,1}$	$s_{1,3}$	$s_{1,3}$
$t_{2,0}$	$s_{2,1}$	$s_{2,3}$	$s_{2,3}$
$t_{3,0}$	$s_{3,1}$	$s_{3,3}$	$s_{3,3}$

RoundKeys

```
ComputeRoundKeys(K[4*Nk], W[Nb*(Nr+1)]) {
    for(i=0; i<Nk; i++)
        W[i]= (K[4i], K[4i+1], K[4i+2], K[4i+3])
    for(i=Nk; i<Nb*Nr+1); i++) {
        t= W[i-1]
        if((i mod Nk)==0)
            t= SubByte(RotByte(t)) + RCon(i/Nk)
        else if(Nk>6 && (i mod Nk)==0)
                t=SubByte(t)
        W[i]= W[i-Nk] + t
        }
    }
```

RoundKeys Primitives

```
SubByte(w= (a,b,c,d)))
    w= (ByteSub(a), ByteSub(b), ByteSub(c), ByteSub(d))
    return(w)

RotByte(w= (a,b,c,d))
    w= (b,c,d,a)
    return(w)

RCon[i]= (RC[i], 0x00, 0x00, 0x00);
RC[1]= 0x01
RC[i+1]=  RC[i]*”0x2”                [multiply by “x”]
```

AES Finalist Bakeoff

	MARS	RC6	Rijndael (AES)	Serpent	Twofish
General Security	3	2	2	3	3
Implementation	1	1	3	3	2
SW Perf	2	2	3	1	1
Smart Card Perf	1	1	3	3	2
HW Perf	1	2	3	3	2
Design features	2	1	2	1	3

Score: 1 (low) to 3 (high). From NIST report 2 Oct 2000.

Algebraic Attacks - Preview

- XSL, Courtois, Pieprzyk, Murphy, Robshaw
  1. Generate equations of higher degree than the original equations by multiplying equation of an active S-box by passive S-box equations
  2. Solve the equations in the formal terms of the equations
- Estimate of linearly independent equations is necessary
- Claim that solving the equations for AES was possible because the estimated number of linearly independent equations was adequate generated excitement.
- Coppersmith cast doubt on the number of linearly independent equations.

Stream Ciphers

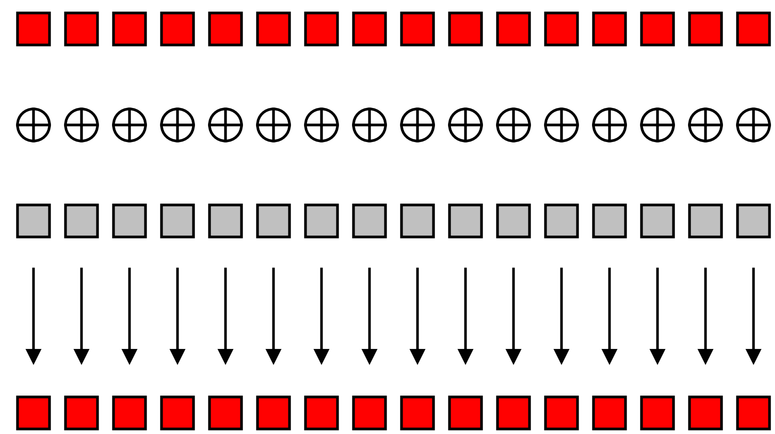
- Synchronous stream ciphers
  - The keystream is generated independently of the plaintext and the ciphertext
  - Using Keyed PRNG
- Asynchronous stream ciphers
  - The keystream is generated as a function of the key, K, and at most t previous ciphertext symbols.

Stream Cipher Encryption and PRNGs

Plaintext:

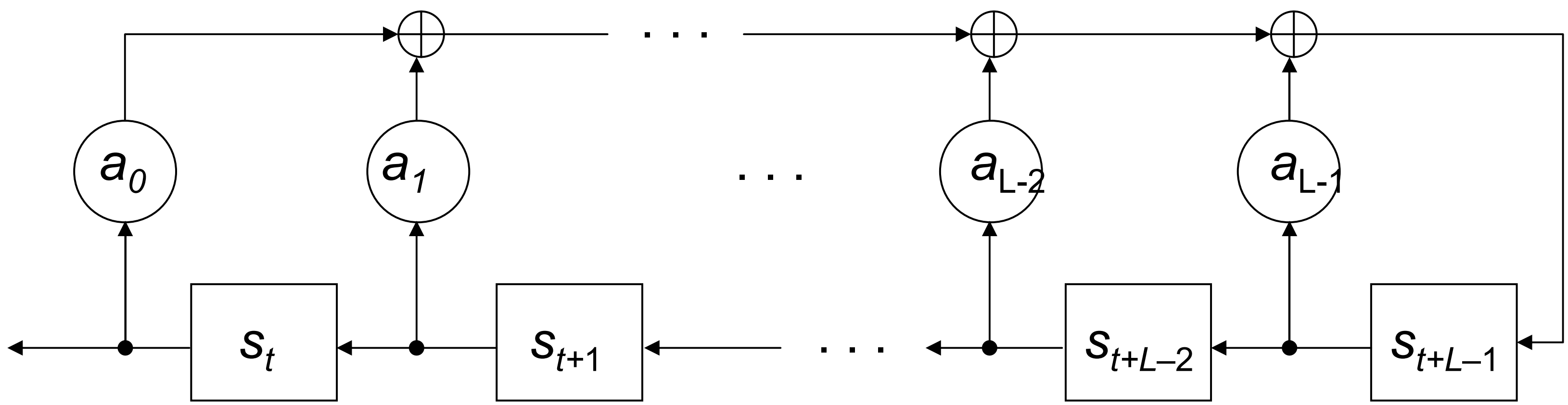
PRNG(seed):

Ciphertext:



Encryption Equation: $c_j = p_j \oplus k_j$

Synchronous Stream using Linear Feedback Shift Register (LFSR)



$a_i,\ s_j \in \mathrm{F}_q$

Recurrence: $s_{j+L} = \sum_{j=0,\,1\,\ldots\,L-1} a_j\, s_{j+L-1}$

Polynomial: $f(x) = \sum_{j=0,\,1\,\ldots\,L-1} a_j\, x^j - x^L$

LFSR-based keystream generator

- Nonlinear combination generators
- Nonlinear filter generators

RC4

<u>Initialization</u>

```
S[0..255] = 0,1,…,255
K[0..255] = Key,Key,Key,…
for i = 0 to 255
        j = (j + S[i] + K[i]) mod 256
        swap S[i] and S[j]
 i=j= 0
```

<u>Iteration</u>

```
i = (i + 1) mod 256
j = (j + S[i]) mod 256
swap S[i] and S[j]
t = (S[i] + S[j]) mod 256
Output S[t]
```

RC-4 Facts

- RC4 implements a permutation of the 2^N, $N=2^n$, where n=8 is word size.

- RC4 cannot enter states s: i= a, j=a+1, S[a+1]=1. There are N^{-2} of these.

- Notation
  - S_r, i_r, j_r and t_r denote the RC4 state during initialization after using key words [0, 1, …, r].
  - I(S), J(S), T(S), Z(S) are the state indicies, output index and first output word of RC4 (i.e.- just after initialization is complete).
  - T(S)=S[1]+S[S[1]], Z(S)= S[T(S)].
  - Key is l-words long

- References
  - Fluhrer, Mantin and Shamir. Attacks on RC4 and WEP.
  - Mantin, Master's Thesis.

Attacks on RC4 and FSRs

- Simple xor attack on stream without MAC or how to swindle your bank.

- Reproduce internal state
- Solve for “taps”
- Look for short cycles

- Alleged RC4 resists these.
  - RC4 is a good stream cipher if you throw away first bunch of bytes

Bias in Second Byte of RC4

Let S_i be the state at time i and let $<z_i>$ be the output sequence.

Theorem: $P(z_2=0)= 2/N$. (roughly twice what we expect from a random cipher)

Proof: Suppose $S_0[2]=0$, $S_0[1] =X\neq 2$, $S_0[X]= Y$.

Round 1:

i=1, $X=S_0[1]+0$. Exchange $S_0[1]$ and $S_0[Y]$.

Round 2:

i=2, $j= X+S_1[2]=X$. Output $S_1[S_1[2]+S_1[X]]= S_1[X]= 0$.

So $P(z_j = 0) \sim 1/N + 1/N (1-1/N) \sim 2/n$.

By Bayes, if $z_2= 0$, we can extract byte of state with probability 1/2.

Cryptographic Hashes

A cryptographic hash ("CH") is a "one way function," h, from all binary strings (of arbitrary length) into a fixed block of size n (called the size of the hash) with the following properties:

1. Given $y=h(x)$ it is infeasible to calculate a $x' \neq x$ such that $y=h(x')$. ("One way," "non-invertibility" or "pre-image" resistance). Functions satisfying this condition are called One Way Hash Functions (OWHF)
2. Given u, it is infeasible to find w such that $h(u)=h(w)$. (weak collision resistance, 2^{nd} pre-image resistance).
3. It is infeasible to find u, w such that $h(u)=h(w)$. (strong collision resistance). Note $3 \rightarrow 2$. Functions satisfying this condition are called Collision Resistant Functions (CRFs).

- Just like Symmetric ciphers ratio of work factor for computation of hash vs work factor to break hash should be very high.
- Adversary has complete information on computing hash and (obviously) can compute as many hashes from the target as she wants.

Observations on Cryptographic Hashes

- Hashes are a strong “checksum”
- OWHF and CRF conditions make CHs satisfy many of the properties of “random functions”
  - Small changes should create large changes (otherwise the pre-image of near neighbors are near neighbors making collisions easy to find)
  - Small input changes should be statistically unrelated (uncorrelated) to changes in a subset of the hash bits
  - Analysis of CHs very similar to Symmetric Cipher techniques

Popular practical cryptographic hashes

- MD4, MD5 (now “broken”)
- SHA-1, SHA-224, SHA-256, SHA-384, SHA-512 (last 4 are “SHA-2”)
- RIPEMD

Observations

- Collision Resistance → 2^{nd} pre-image resistance
- Let $f(x)= x^2-1 \pmod p$.
  - f(x) acts like a random function but is not a OWHF since square roots are easy to calculate mod p.
- Let $f(x)= x^2 \pmod{pq}$.
  - f(x) is a OWHF but is neither collision nor 2^{nd} pre-image resistant
- If either $h_1(x)$ or $h_2(x)$ is a CRHF so is $h(x)= h_1(x) \,||\, h_2(x)$
- MDC+signature & MAC+unknown Key require all three propertie

What are Hash Functions Good for?

- Modification Detection Codes (MDCs): This is a strong checksum (integrity check). Sometimes called “unkeyed” hashes.
- Message Authentication Code (MACs): If shared secret is part of the hash, two parties can determine authenticated integrity with CHs. Called “keyed hashes”.
- Message Digests (MDs): Encrypting (with private key) the CH of a message (its MD) acts as a certification that the message was “approved” by possessor of private key. This is called a Digital Signature. [Note: you could “sign’ the whole message rather than the hash but this would take oodles of time by comparison.]

What are Hash Functions Good for?

- Identity: Uniquely and securely identifies bit streams like programs. Hash is strong name for program.
- Entropy mixing: Since CHs are random functions into fixed size blocks with the properties of random functions, they are often used to “mix” biased input to produce a “seed” for a psuedo-random number generator.
- Password Protection: Store salted hash of password instead of password (Needham).
- Bit Commitment

One-Way Functions

Hashes come from two basic classes of one-way functions

- Mathematical
  - Multiplication: $Z=X \bullet Y$
  - Modular Exponentiation: $Z = Y^X \pmod{n}$ (Chaum vP Hash)
- Ad-hoc (Symmetric cipher-like constructions)
  - Custom Hash functions (MD4, SHA, MD5, RIPEMD)

Chaum-vanHeijst-Pfitzmann Compression Function

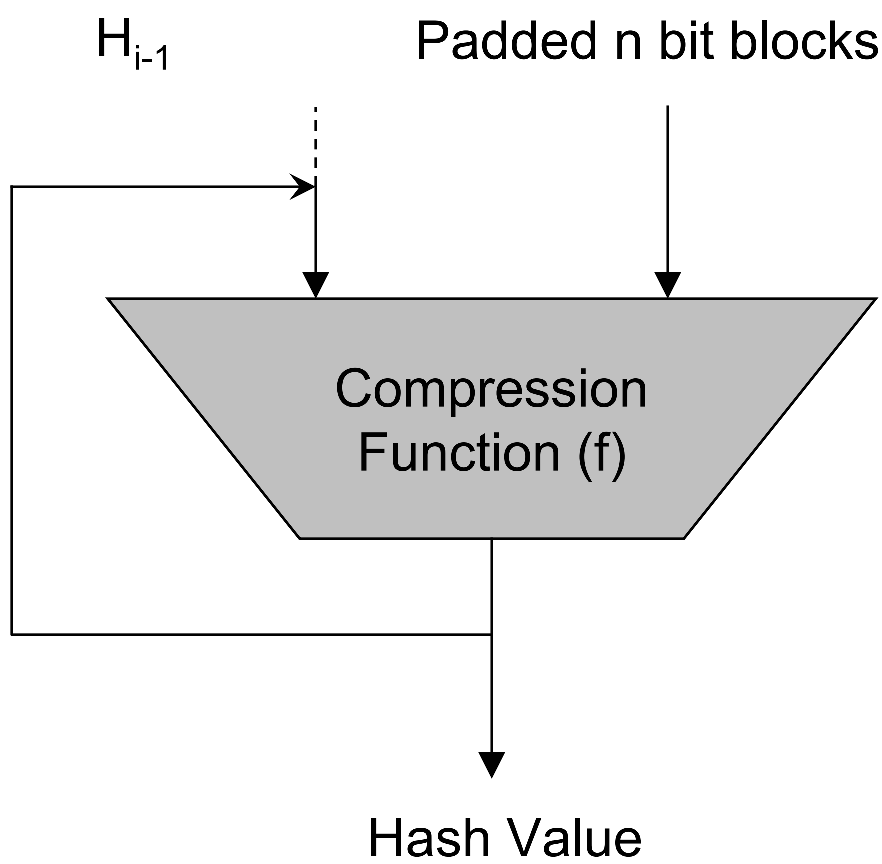
- Suppose p is prime, q=(p-1)/2 is prime, a is a primitive root in F_p, b is random.
- g: $\{1,2,\ldots,q-1\}^2 \rightarrow \{1,2,\ldots,p-1\}$, q=(p-1)/2 by:
  - $g(s, t) = a^s b^t \pmod p$
- Not used in practice: too slow.
- Reduction to discrete log:

  Suppose g(s, t)= g(u, v) can be found. Then $a^s b^t \pmod p = a^u b^v \pmod p$.

  So $a^{s-u} \pmod p = b^{v-t} \pmod p$. Let $b = a^x \pmod p$. Then $(s-u)=x(y-t) \pmod{p-1}$.

  But p-1= 2q so we can solve for x, thus determining the discrete log of b.

Merkle/Damgard Construction



Input: $x = x_1 || \ldots || x_t$
Input is usually padded

$H_0 = IV$
$H_i = f(H_{i-1}, x_i)$
$h(x) = g(h_t)$

Graphic by Josh Benaloh

Padding

- Standard technique
  - Let last message block have k bits. If k=n, make a new block and set k= 0.
  - Append a 1 to last block leaving r=n-k-1 remaining bits in block.
  - If r>=64, append r-64 0s then append bit length of input expressed as 64 bit unsigned integer
  - If r<64, append n-r 0’s (to fill out block), append n-64 0’s at beginning of next block then append bit length of input expressed as 64 bit unsigned integer

Technique for CHs from Block Ciphers

Let input be $x = x_1 \| x_2 \| \ldots \| x_t$ where each x_i is n bits long. Let g be a function taking an n bit input to an m bit input. Let E(k, x) be a block cipher with m bit keyspace and n bit block. Let H_0= IV.

<u>Construction 1</u>

$H_i = E(g(H_{i-1}), x_i) \oplus H_{i-1}$

<u>Construction 2</u>

$H_i = E(x_i, H_{i-1}) \oplus H_{i-1}$

<u>Construction 3</u>

$H_i = E(g(H_{i-1}), x_i) \oplus x_i \oplus H_{i-1}$

Note: Because of collisions n should be >64. Ideally, m=n and g= id. DES with n= 64 is too small. AES with n=m=128 is better.

Attacks on Cryptographic Hashes

- Birthday (Yuval) attacks
  - Probability of collision determined by “Birthday Paradox” calculation:
    - $(1- 1/n)\ (1- 2/n) \ldots (1-(k-1)/n)= (n!/k!)/n^k$
    - Probability of collision is >.5 when $k^2 > n$.
    - Need 2^{80} blocks for SHA.
    - $1+x \leqq e^x$, $\Pi_{i=1}^{i=k} (1-i/n) \leqq e^{-k(k-1)/(2n)}$
- Dobbertin Attacks on MD4
  - Collision attack based on compression function weakness
- Biham, Chen, Chabaud, Joux, Wang et al, Differential attacks on RIPEMD-128, HAVAL, MD4, MD5, SHA-0, SHA-1

Attacks on Cryptographic Hashes

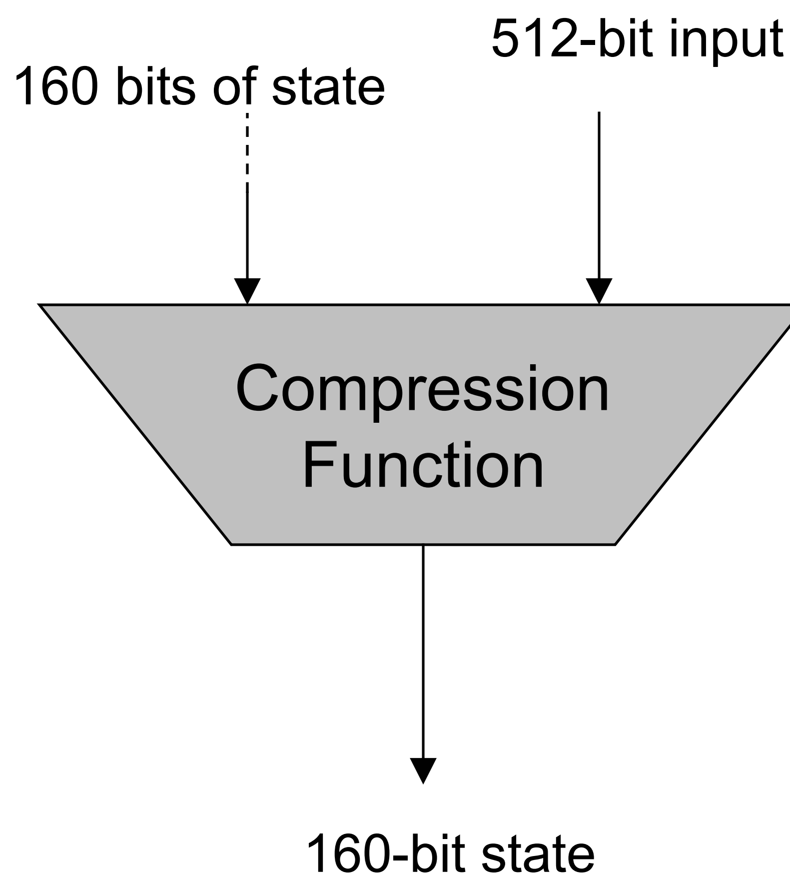
- Berson (1992) using differential cryptanalysis on 1 round MD-5.
- Boer and Bosselaers (1993), Pseudo collision in MD5.
- Dobbertin (1996), Collisions in compression function. Attacks inspired RIPEMD proposal.
- Biham and Chen (2004), Collisions in SHA-0.
- Chabaud and Joux (2004), Collisions in SHA-0 .
- Wang, Feng, Lai, Yu, (2004), MD4, MD5, RIPEMD
- Wang et al, (2004, 2005), SHA-1

- SHA-1 has stood up best: best known theoretical attack (11/05) requires 2^{64} operations.

Prefix attacks, and HMACs

- Prefix and suffix attacks
  - $Hash(m_1||m_2) = Hash(m_2)$, if internal state collides
  - To fix: $h_{DBL}(h(m)||m)$
- HMAC: keyed-hash message authentication code
- Two popular constructions
  - $HMAC_K(x) = Hash(k|p|m|k)$, p is a pad
  - $HMAC_K(x) = \text{SHA-1}(K \oplus opad \,||\, \text{SHA-1}(K \oplus ipad)||x)$

A Cryptographic Hash: SHA-1



Slide by Josh Benaloh

SHA-0/1

Absence of this term is only difference between SHA-0 and SHA-1

A= 0x67452301, B= 0xefcdab89,
C= 0x98badcfe, D= 0x10325476
E= 0xc3d2e1f0

$F_t(X,Y,Z) = (X \wedge Y) \vee ((\neg X) \wedge Z)$, $t = 0,\ldots,19$

$F_t(X,Y,Z) = X \oplus Y \oplus Z$, $t = 20,\ldots,39$

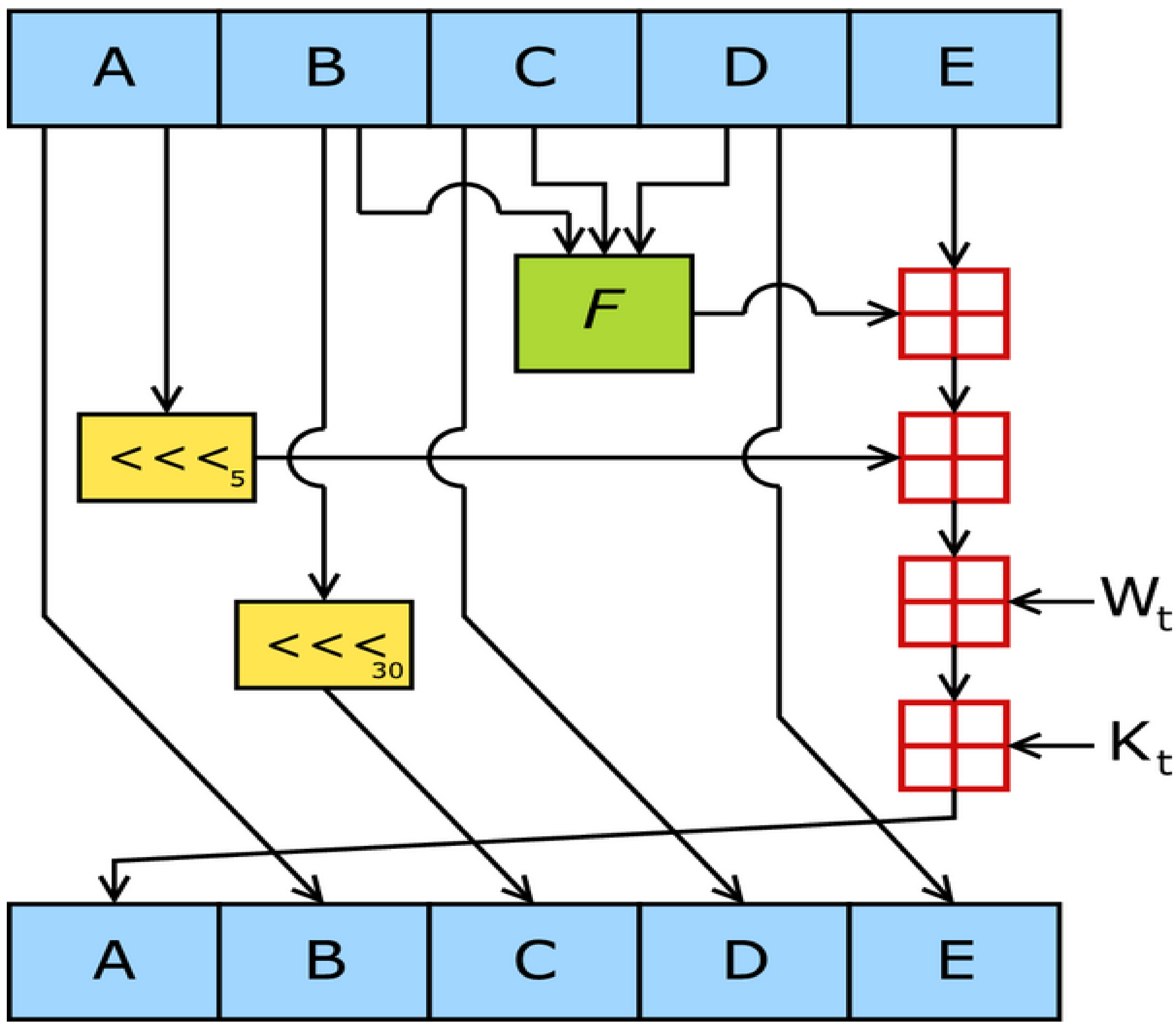
$F_t(X,Y,Z) = (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z)$, $t = 40,\ldots,59$

$F_t(X,Y,Z) = X \oplus Y \oplus Z$, $t = 60,\ldots,79$

K_t= 0x5a827999, t= 0,…,19
K_t= 0x6ed9eba1, t=20,…,39
K_t= 0x8f1bbcdc, t= 40,…,59
K_t= 0xca62c1d6, t=60,…,79

Do until no more input blocks {
    If last input block
        Pad to 512 bits by adding 1 then 0s then 64 bits of length.
    M_i= input block(32 bits) i= 0,…,15
    $W_t = M_t$, t= 0,…,15;
    $W_t = (W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16})$ $\boxed{<<<1,}$ t= 16,…,79
    a= A; b= B; c= C; d= D; e= E;
    for(t=0 to 79) {
        x= $(a<<<5)+f_t(b,c,d)+e+W_t+K_t$
        e= d; d=c; c= b<<<30;
        b=a; a= x;
        }
    A+= a; B+=b; C+= c; D+= d; E+= e;
}

A Cryptographic Hash: SHA-1



Picture from Wikipedia

A Cryptographic Hash: SHA-1

Depending on the round, the "non-linear" function f is one of the following.

$$f(X,Y,Z) = (X \wedge Y) \vee ((\neg X) \wedge Z)$$

$$f(X,Y,Z) = (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z)$$

$$f(X,Y,Z) = X \oplus Y \oplus Z$$

A Cryptographic Hash: SHA-1

What’s in the final 32-bit transform?

- Take the rightmost word.
- Add in the leftmost word rotated 5 bits.
- Add in a round-dependent function f of the middle three words.
- Add in a round-dependent constant.
- Add in a portion of the 512-bit message.

Breaking news on “Chinese” Attacks on Hashes

- Don’t use MD4 or you’ll look really really silly.
- Don’t use MD5.
- Don’t use RIPEMD-128
- SHA-1 appears to have collision attacks of the order 2^{64}
- Use SHA-2 functions
  - Truncate to provide legacy compatibility if you have to (i.e. – gun to head)
  - Required by “Suite B” Standards

SHA-2

- FIPS 180-2, 8/02.
  - Defines SHA-256, SHA-384, SHA-512.
  - SHA-224 (truncated) added 2/04
- Great increase in mixing between bits of the words compared to SHA-1.
- US Patent *6,829,355*
- Inventor: Glenn Lilly
- Assignee: NSA
- Can obtain source from
  - http://en.wikipedia.org/wiki/SHA-2

Other Cryptographic Hashes and Performance

Hash Name	Block Size	Relative Speed
MD4	128	1
MD5	128	.68
RIPEMD-128	128	.39
SHA-1	160	.28
RIPEMD-160	160	.24

What to take home

- Symmetric ciphers and hashes provide key ingredients for "distributed security"
  - Fast data transformation to provide confidentiality
  - Integrity
  - Public key crypto provides critical third component (trust negotiation, key distribution)
- It's important to know properties of cryptographic primitives and how likely possible attacks are, etc.
  - Most modern ciphers are designed so that knowing output of n-1 messages provides no useful information about n^{th} message.
  - This has an effect on some modes of operation.

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Web sites: www.rsa.com, www.counterpane.com, www.iacr.org has loads of preprints.

Homework 7

1. We saw that a typical round of AES consisted of the following operations:

```
for each byte, b in state
    ByteSub(b)
ShiftRow(state)
if(i<Nr)
    MixCol(state)
AddRoundKey(i, state)
```

For the 128 bit key, 128 bit block size version of Rijndael, using lookup tables to reduce the computations required and assuming basic operations (32 bit lookup, 32 bit xor, etc) all take about .001 microseconds and your code/data budget is under 16 MB, design a implementation of the round operations that is faster than implementing each of the primitive operations (ByteSub, ShiftRow, MixCol).

How long does each round take (about)?

Counter mode use of AES is used by selecting a nonce (n) and constructing cipher blocks $AES_K(n||ctr)$, $AES_K(n||ctr+1)$, $AES_K(n||ctr+2)$,…. The resulting bits are xored into the plaintext (as with the stream cipher). What properties of AES make this safe? Can the keystream be generated in parrallel and stored for later use? What performance properties does this mode have over ECB?

Homework 7

2. Show that $f(x)= x^2 \pmod{pq}$ is a One-Way Function but is not Collision Resistant, where p and q are prime.
3. Linear Feedback Shift Registers Cryptosystem: Suppose X is a cryptosystem implemented by a 5 element linear feedback shift register which generates a psuedo random stream $s_0, s_1, s_2, \ldots$ so

- $s_{n+5}= a_4 s_{n+4} \oplus a_3 s_{n+3} \oplus a_2 s_{n+2} \oplus a_1 s_{n+1} \oplus a_0 s_n$

If the first 10 output bits of the pseudo random generator are 1110100010, what are the next 3 bits? Assume n is the register length. About how many consecutive bits do you need to break a LFSR? How does this compare to a stream generator on an n bit state that is not linear?

Homework 7

4. Given i= 64, j= 245 and S is as stated below, what are the next 4 bytes of output of RC4? Estimate the speed of encrypting the next 4 bytes of output of an RC4 cipher on a computer in which assignment addition and logical AND requires .001 microseconds.

5. Suppose two parties share a secret key k and wish to communicate a series of “yes/no” answers over a public channel without disclosing the answers. Design a protocol to do this using a MAC. Be careful to make sure the adversary cannot figure out all the answers if they know whether the “code” for a few of the yes/no answers.

Homework 7

S[0…127]:

0x08	0xa5	0xe9	0x09	0x45	0xc0	0xed	0xf1
0x5d	0xfd	0x34	0xc3	0x4e	0x7b	0x9d	0x96
0x38	0x76	0x7c	0x49	0x8f	0xd9	0x35	0xcc
0x99	0xb0	0x2d	0x97	0xe7	0x1d	0xa9	0x16
0x7d	0x10	0x8c	0x89	0x51	0xa1	0xd7	0x5b
0x3d	0x1c	0x23	0x1e	0xe0	0xb2	0x84	0xa8
0xc5	0x24	0x86	0xb9	0x07	0xac	0xf0	0x52
0x32	0x92	0xda	0x06	0xe4	0xd4	0x82	0xd5
0xdb	0xae	0x04	0x4c	0x36	0xc6	0x19	0x2e
0xb4	0x2c	0x69	0xc7	0xce	0x71	0x91	0xa6
0xde	0x22	0x59	0xf4	0x54	0x25	0x42	0x0d
0xff	0x03	0x0a	0x44	0x87	0x37	0x8e	0x12
0x30	0x33	0x58	0x3a	0x81	0xf3	0x8d	0x9f
0xbd	0xc4	0x95	0x73	0x93	0x55	0x41	0xb6
0x90	0x63	0x9c	0x18	0x77	0xdd	0xe3	0xc9
0x8a	0xb1	0x7f	0xee	0xe5	0xad	0x05	0xa0

S[128…255]:

0x6d	0x15	0xc2	0xab	0x7a	0xa4	0x3f	0x00
0x48	0xa3	0xd1	0x4a	0x75	0xb7	0x85	0xd8
0xfb	0xfe	0xf2	0xe6	0x13	0x56	0xec	0xa7
0x9a	0xe2	0x64	0x53	0x5f	0x65	0xd3	0xc8
0x68	0x74	0x02	0xdc	0x6f	0x43	0xe1	0x8b
0xbf	0xa2	0x2a	0x80	0xbb	0x6a	0x28	0x78
0x17	0xf6	0xfc	0x67	0xb3	0x9e	0xcb	0x31
0xf9	0xaa	0x9b	0x2b	0xb8	0x1a	0x3e	0xf8
0xd2	0x5c	0x20	0x11	0x4b	0x3b	0x0b	0x6e
0xaf	0xca	0x6b	0x60	0x94	0x5a	0x61	0x27
0xb5	0x7e	0x4d	0xbe	0x57	0x26	0xcf	0xef
0xbc	0x40	0x72	0x14	0x83	0x47	0xf7	0x1b
0x79	0x50	0x1f	0x3c	0x5e	0x0f	0xf5	0x62
0x6c	0x21	0x70	0x4f	0xeb	0xea	0x98	0xfa
0xba	0x46	0x01	0xcd	0x88	0x0e	0x39	0xc1
0xd0	0xdf	0x2f	0x0c	0x29	0x66	0xd6	0xe8

Backup

Differential Cryptanalysis: Overview

Let $P=(P_L, P_R)$, $P^*=(P_L^*, P_R^*)$ and $C=(C_L, C_R)$, $C^*=(C_L^*, C_R^*)$ be pairs of inputs and outputs with prescribed xors

$P' = (P_L', P_R') = (P_L, P_R) \oplus (P_L^*, P_R^*)$

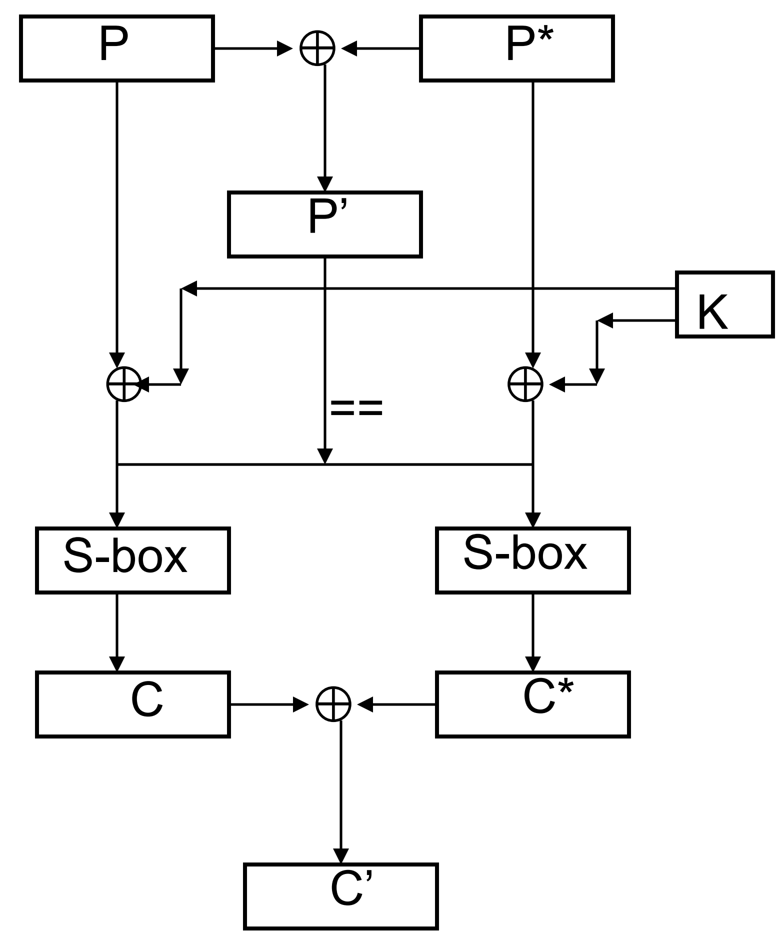
$C' = (C_L', C_R') = (C_L, C_R) \oplus (C_L^*, C_R^*)$

Output xor depends non uniformly on key bits.

Let non uniform distribution "vote" on set containing keys.

Uses chosen plaintext/ciphertext pairs to get enough compliant pairs by following the xor of two plaintexts through rounds of DES.

Examine last round to discover key



Differential Profile of single S-box

- For prescribed input and output differences x', y' set $D_j(x',y') = \{u: S_j(u \oplus x') \oplus S_j(u) = y'\}$, then
  - Note that u, $u \oplus x'$, $u \oplus k$, $u \oplus x' \oplus k$ will all appear in this set
  - $k \in x \oplus D_j(x',y')$, if x is an input (pre-key) to S_i.

- $|D_j(x', y')|$ has non uniform distribution.
- For given input difference about 80% of the output differences are possible.
- $p = |D_j(x', y')| / 2^m$, m is the dimension of the space of a'.
- Shamir and Biham denote this as $x' \rightarrow y'$, p.

S1 Differential Distribution

S box 1

In	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	6	0	2	4	4	0	10	12	4	10	6	2	4
2	0	0	0	8	0	4	4	4	0	6	8	6	12	6	4	2
3	14	4	2	2	10	6	4	2	6	4	4	0	2	2	2	0
4	0	0	0	6	0	10	10	6	0	4	6	4	2	8	6	2
5	4	8	6	2	2	4	4	2	0	4	4	0	12	2	4	6
6	0	4	2	4	8	2	6	2	8	4	4	2	4	2	0	12
7	2	4	10	4	0	4	8	4	2	4	8	2	2	2	4	4
8	0	0	0	12	0	8	8	4	0	6	2	8	8	2	2	4
9	10	2	4	0	2	4	6	0	2	2	8	0	10	0	2	12
a	0	8	6	2	2	8	6	0	6	4	6	0	4	0	2	10
b	2	4	0	10	2	2	4	0	2	6	2	6	6	4	2	12
c	0	0	0	8	0	6	6	0	0	6	6	4	6	6	**14**	2
d	6	6	4	8	4	8	2	6	0	6	4	6	0	2	0	2
e	0	4	8	8	6	6	4	0	6	6	4	0	0	4	0	8
f	2	0	2	4	4	6	4	2	4	8	2	2	2	6	8	8
10	0	0	0	0	0	0	2	14	0	6	6	12	4	6	8	6
11	6	8	2	4	6	4	8	6	4	0	6	6	0	4	0	0
12	0	8	4	2	6	6	4	6	6	4	2	6	6	0	4	0

S1 Differential Distribution: another view

<u>S box 1</u>

```
D1(00, 0d):   0 found
D1(01, 0d): (0a,0b) (0b,0a) (22,23) (23,22) (3e,3f) (3f,3e)   6 found
D1(02, 0d): (08,0a) (0a,08) (29,2b) (2b,29) (35,37) (37,35)   6 found
D1(03, 0d): (14,17) (17,14)   2 found
D1(04, 0d): (13,17) (17,13) (1b,1f) (1f,1b) (2a,2e) (2e,2a) (3b,3f)
   (3f,3b)   8 found
D1(05, 0d): (01,04) (04,01)   2 found
D1(06, 0d): (21,27) (27,21)   2 found
…            …               …
D1(33, 0d): (07,34) (0d,3e) (1a,29) (29,1a) (34,07)
            (3e,0d)   6 found
D1(34, 0d): (06,32) (10,24) (16,22) (1c,28) (22,16)
            (24,10) (28,1c) (32,06)   8 found
D1(35, 0d): (00,35) (35,00)   2 found
D1(36, 0d): (02,34) (0d,3b) (34,02) (3b,0d)   4 found
```

Example: Differential Cryptanalysis of S1 through a single round

Consider input texts and output xors from S1

P_1 = 0x01, P_1^* = 0x35 which produce output xor C_1'=0x0d. (So C_1' = 0x34).
P_2 = 0x22, P_2^* = 0x15 which produce output xor C_2'=0x03. (So C_2' = 0x34).

Then

D_1(0x34, 0xd)= {0x06, 0x10, 0x16, 0x1c, 0x22, 0x24, 0x28, 0x32}.
D_1(0x34, 0x3)= {0x01, 0x02, 0x15, 0x21, 0x35, 0x36}.

And

(1) $k \in P_1 \oplus D_1$(0x34, 0xd)
(2) $k \in P_2 \oplus D_1$(0x34, 0x3)

(1) reduces the possible key set to {0x07, 0x33, 0x11, 0x25, 0x17, 0x23, 0x1d, 0x29}
(2) reduces the possible key set to {0x20, 0x14, 0x23, 0x17, 0x34, 0x00}.

The intersection (and actual possibilities) are { 0x17, 0x23}

One Round Differential used to analyze 4 round DES

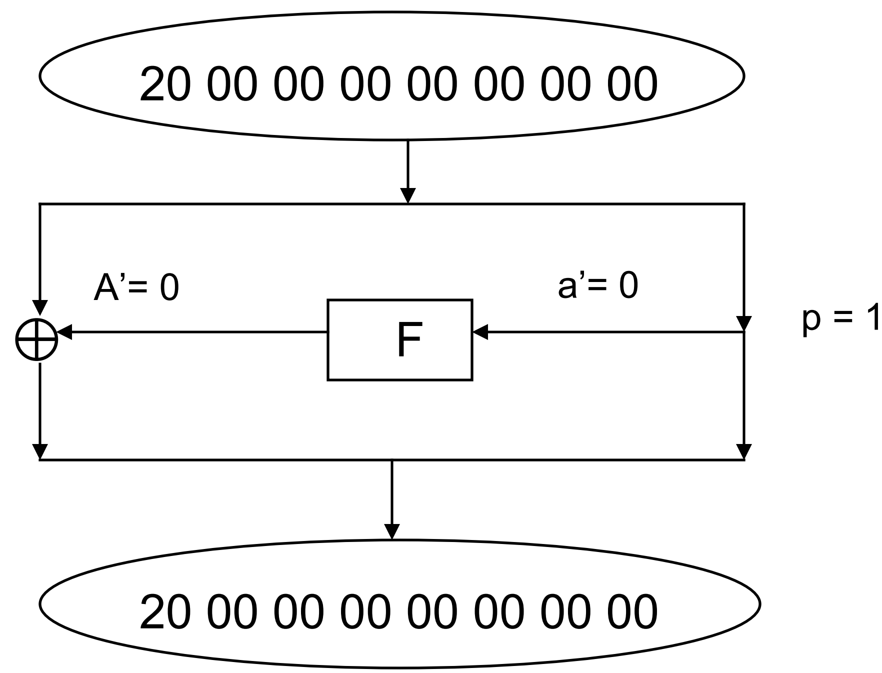
Method

Use 1 round characteristic to right.

Undo effect of permutation matrix and solve each S box separately.

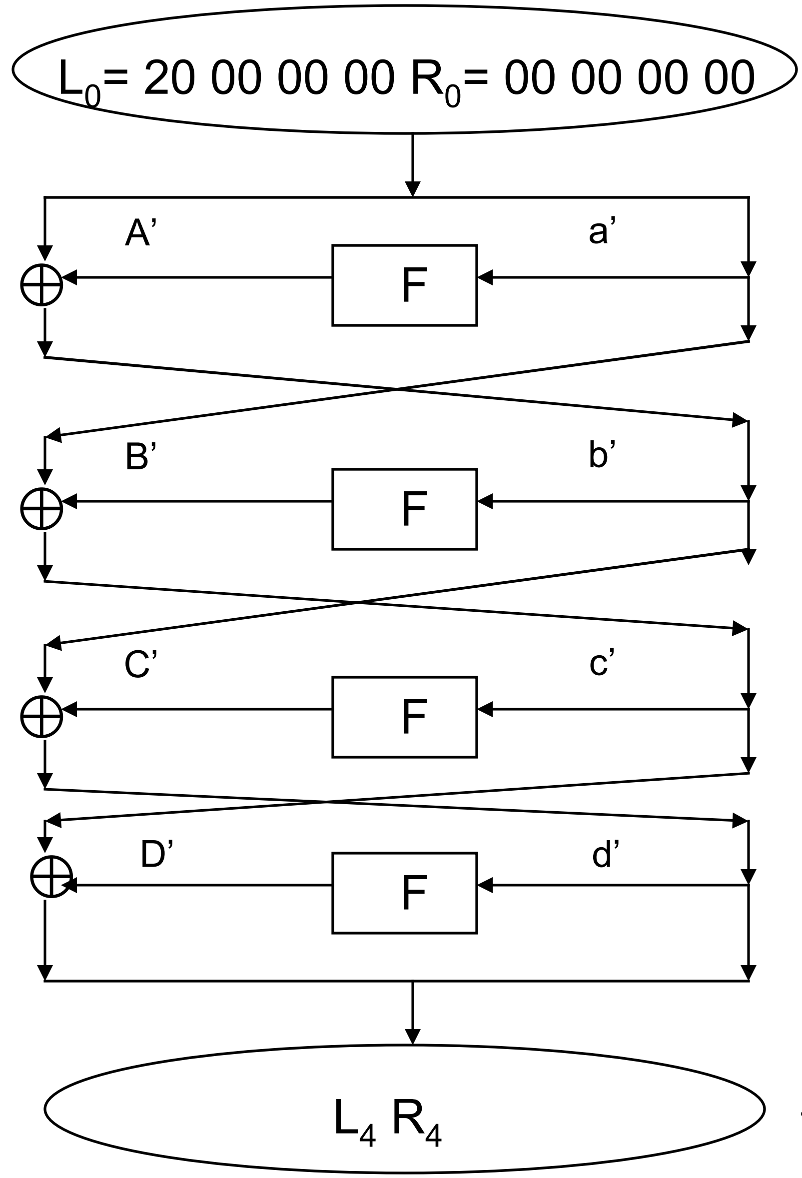
This allows us to solve for 48 key bits.

This 1 round characteristic will be used to estimate input xor in subsequent rounds.



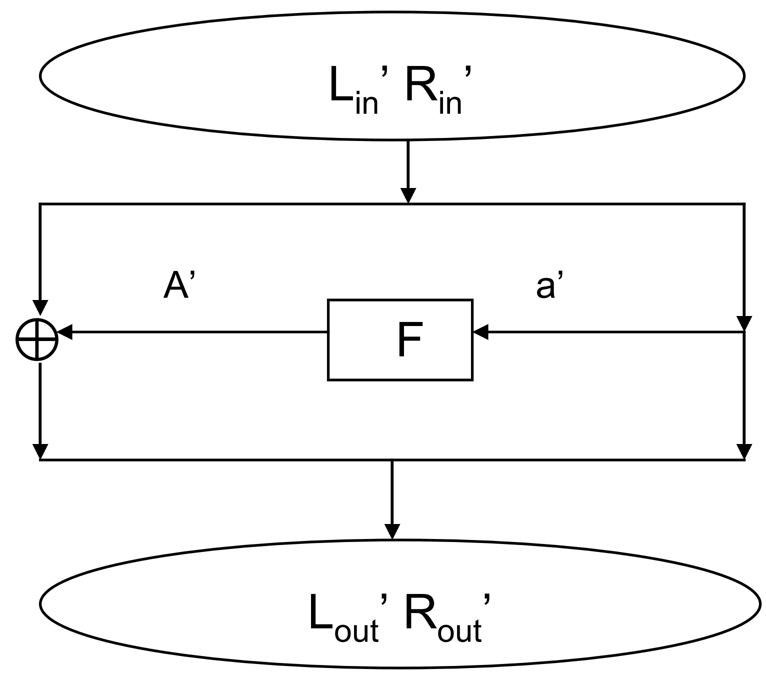
Differential Cryptanalysis of 4 rounds

- $D' = a' \oplus B' \oplus L_4'$
- $d' = R_4'$
- Because $b' = L_0'$, the output xor of $S_2, S_3, \ldots, S_8$ in round 2 is 0. This gives 28 bits of B' and hence 28 bits of D' is known.
- Since B' is known, we can calculate $D' = B' \oplus L_4'$ using 4 encrypted pairs for each of the 7 relevant S boxes. All key candidates are in this set which gives 7x6=42 bits of key with high probability.



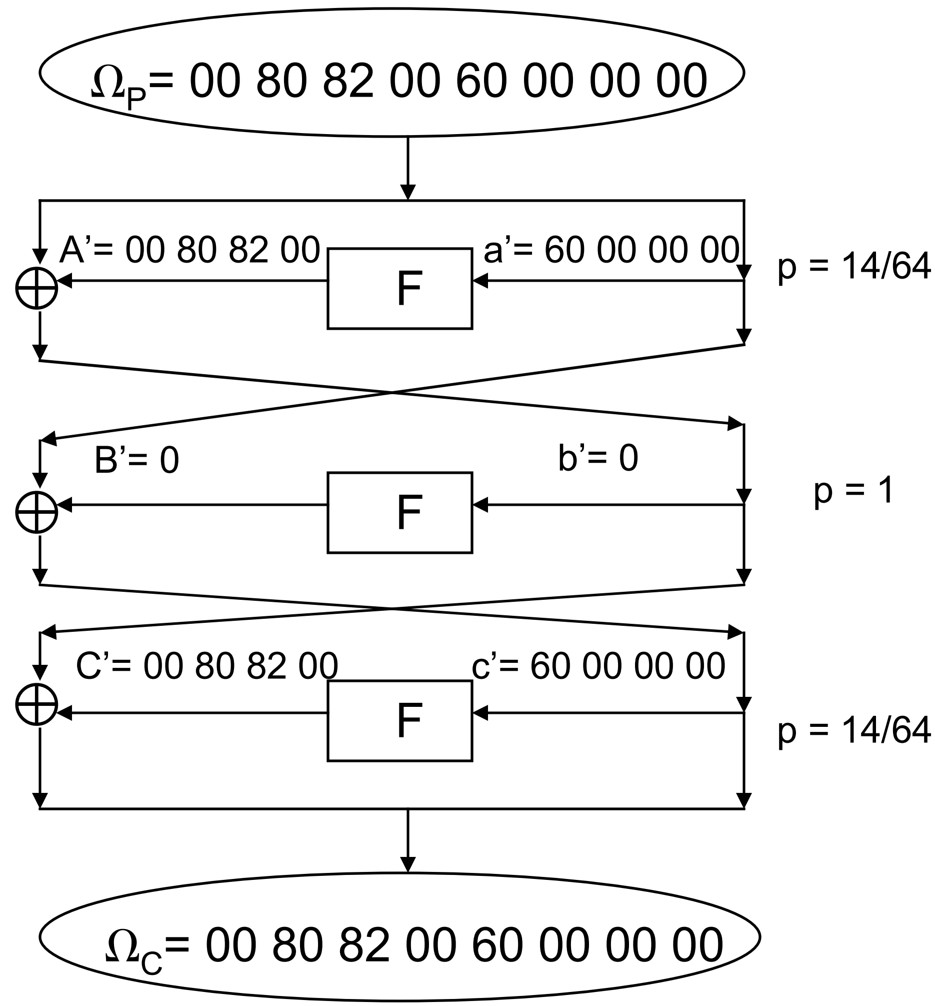
Computing a single characteristic

- The first and most important differential is (L',0) → (L',0), p=1.
- Another is (L',0x60000000) → (L' ⊕0x00808200,0x60000000), p=1/4.
- Construction:
  – E(0x60000000)= E(0110 0000 … 0000)= 001100 000000 … 000000
  – S_1(001100)' → 0xe with p=1/4, S_j(0)' → 0 with p=1, j>1 and P(0xe0000000)=0x00808200.



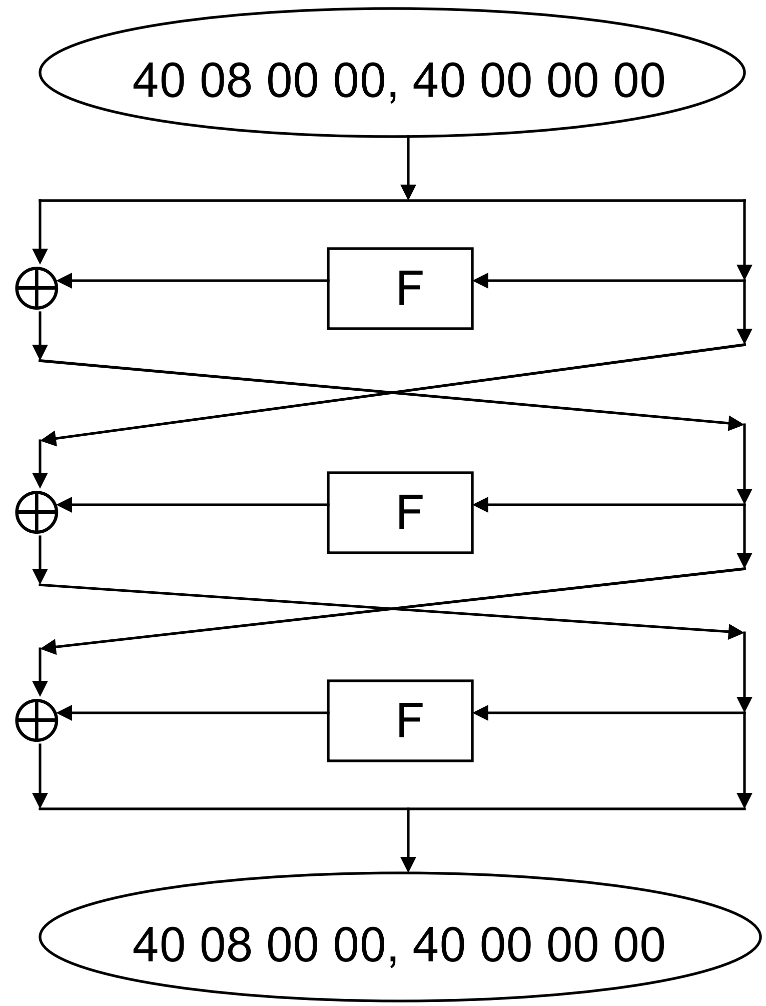
Multi-round Characteristics

- Sequence of Differentials with identified input and output xors. Each round differential occurs with probability p_i.
- Overall probability: $p=\prod p_i$
- Characteristic to the right is a three round characteristic with probability $(14/64)^2$
- Used to approximate differentials through multiple rounds.
- Each pair following the characteristic at each round is called a "right pair". Other pairs are "wrong pairs."
- Wrong pairs get distributed uniformly; right pairs follow overall characteristic probability.



Three Round Characteristic

- This characteristic occurs with probability p=1/16 and forms an estimate for the differential input of the 4^{th} round of the 6 rounds.
- (00 20 00 08 00 00 00 04) → (00 00 04 00 00 20 00 08) with p=1/16 is another such characteristic.



Differential Cryptanalysis of 6 rounds

- Suppose (L_{i-1}, R_{i-1}), k_i are the inputs to round i. $P_L = L_0$, $P_R = R_0$.
- $L_6 = R_4 \oplus f(k_6, R_6) = L_3 \oplus f(k_6, R_6) \oplus f(k_4, R_3)$
- $L_6' = L_3' \oplus f(k_6, R_6) \oplus f(k_6, R_6^*) \oplus f(k_4, R_3) \oplus f(k_4, R_3^*)$
- $L_6' = C_L$ and $R_6 = C_R$ are known.
- Estimate $L_3' = 40000000$, $R_3' = 40080000$, using the differential.
- Set $S = P^{-1}(C_L \oplus 40000000) = f(k_6, C_R) \oplus f(k_6, C_R^*) \oplus f(k_4, R_3) \oplus f(k_3, R_3^*) = S_1(E_1) \,||\, S_2(E_2) \,||\, \ldots \,||\, S_8(E_8)$ where $E_1 || E_2 || \ldots || E_8$ are the bits obtained by applying E to 40080000.
- $E_1 || E_2 || \ldots || E_8 = 001000000000000000101000..0 =$
  08||00||01||10||00||00||00||00.
- Since the input Xors to S2,S5,S6,S7,S8 are 0, $f(k_4, R_3) \oplus f(k_4, R_3^*)$ is 0 in the corresponding output bit positions and we are left with the simple differential: $P^{-1}(C_L \oplus 40000000) = f(k_6, C_R) \oplus f(k_6, C_R^*)$ for S2,S5,S6,S7,S8.

Differential Cryptanalysis of 6 rounds

- First characteristic yields 30 bits of key. Second one adds another 12 bits of key.
- Recall $P^{-1}(C_L \oplus 40000000) = f(k_6, C_R) \oplus f(k_6, C_R^*)$ for S2,S5,S6,S7,S8
- This occurs with p= 1/16.
- Straightforward implementation yielding 30 keybits:
  - Set up 2^{30} counters
  - Bump counter for suggested key for each pair of n chosen texts
  - Correct key be will “voted” at least 1/16 n time (“right pairs”)
  - Incorrect keys will be voted randomly each with probability $1/2^{30}$

Differential Cryptanalysis of 6 rounds

- Improving the “signal to noise” ratio by “filtering” pairs
  - For each of S2, S5, S6, S7, S8 with input xor x’ and output xor y’, look at $x \oplus D_j(x', y')$.
  - If this is empty, this must be wrong pair.
  - For any given S box the, this happens with probability .2.
  - The probability that all 5 S boxes have non-empty candidate key sets is $(.8)^5 = .33$. Call this set of pairs RP and the complement WP.
  - RP contains 1/3 of the pairs, WP contains 2/3
  - In RP, the probability of a “correct vote” is 3/16

Algebraic Attacks

- As we’ve seen, ciphertext can be expressed as algebraic function of keys and plaintext (Lagrange Interpolation Theorem).
- Sometimes key bits are expressible as functions of plain and cipher texts
- These are easy to solve if the equations are linear even for very large key spaces.
- These are very hard to solve if the equations are even quadratic (NP-hard in fact, see “General System of Quadratic Equations” slide).
- General problem is “Find one solution of a system of *m* equations in *n* variables of bounded degree, D, over K (usually finite):

$$\Sigma_{\mathbf{b}}\, a_b \boldsymbol{x}^{\boldsymbol{b}} + c_i = 0,\ \boldsymbol{x}^{\boldsymbol{b}} = x_1^{b1}\, x_2^{b2} \ldots X_n^{bn},\ \Sigma_i\, b_i \leqq D$$

- We refer to this problem as SolveAlgebraic(K,D,m,n) and often abbreviate equations as $l_j(\boldsymbol{x}) = 0$.

Solving SolveAlgebraic(K,D,m,n)

- Classic Technique is Grobner Basis, see
  - Lauritzen, Concrete Abstract Algebra. Cambridge.
  - Cox, Little, O'Shea, Using Algebraic Geometry. Springer.
- Grobner uses Buchberger's Algorithm which is doubly exponential time in the worst case since the monomial grow very rapidly and singly exponential time on average.
- This is not practical for $n>15$.
- However, we can do better with an overdefined set of equations ($m>n$).
- Note first that if we pick m random equations $m>n$ they will likely be inconsistent.
- Let's see how we might solve overdetermined systems by solving them as we do linear equations after we prove that solving even quadratic systems of equations is NP hard.

SHA-0 Strategy (Chabaud and Joux)

- Basic idea is to look for small differences that can be tracked through rounds like differential cryptanalysis.
- Consider three approximations to the SHA-0 compression function.
  - SHI-1
    - Use Xor instead of Add
    - Make $f^{(i)}$ linear
  - SHI-2
    - Use Xor instead of Add
    - Restore $f^{(i)}$ to original values
  - SHI-3
    - Restore Add
    - Make $f^{(i)}$ linear

SHI-1 Finding Collisions

- Assume the $W^{(i)}$ are unrelated and follow progress of a change to $W^{(1)}$.

	A	B	C	D	E
1	$W^1+ROL_5(A)+f(B,C,D)+E+K$	A	$ROL_{30}(B)$	C	D
2	$W^2+\ldots$				
3			$ROL_{30}(-)$		
4					
5					$ROL_{30}(W^1+ROL5(A)+f(B,C,D)+E+K)$
6	$W^6+\ldots$ - fixes W^1 perturbation				

SHI-1 Error Propagation in Hash

Perturbation On bit 1

Corrections defining masks

State \ Δ	$W_1^{(i)}$	$W_6^{(i+1)}$	$W_1^{(i+2)}$	$W_{31}^{(i+3)}$	$W_{31}^{(i+4)}$	$W_{31}^{(i+5)}$
$A^{(i)}$	$A_1^{(i)}$	$A^{(i+1)}$	$A^{(i+2)}$	$A^{(i+3)}$	$A^{(i+4)}$	$A^{(i+5)}$
$B^{(i)}$		$B_{31}^{(i+1)}$				$B^{(i+5)}$
$C^{(i)}$			$C_{31}^{(i+2)}$			$C^{(i+5)}$
$D^{(i)}$				$D_{31}^{(i+3)}$		$D^{(i+5)}$
$E^{(i)}$					$E_{31}^{(i+4)}$	$E^{(i+5)}$

Message Expansion

- Process of expanding from 16 32 bit words to 80 32 bit words in the compression function is called message expansion
  - MD5
    - Permutations
  - SHA-0
    - Linear code (LFSR)
  - SHA-1
    - Linear code with rotation
- Has profound effect on possible disturbance vectors in Differential attacks
- Being studied to provide greater protection
- Replace xor with modular addition to prevent codeword difference propagation
- Conditions on chaining variables for local collision (Prob between 2^{-39} and 2^{-42})

End