University of Washington
CSEP 590TU – Practical Aspects of Modern Cryptography

Instructors: Josh Benaloh, Brian LaMacchia, John Manferdelli

Tuesdays: 6:30-9:30, Allen Center 305

Webpage: http://www.cs.washington.edu/education/courses/csep590/06wi/

Recommended texts:

Stinson, Cryptography, Theory and Practice. 2nd Edition, CRC Press, 2002.

Menezes, vanOrtshot, Vanstone. Handbook of Applied Cryptography.

Ferguson and Schneier, Practical Cryptography.

New Lecture Schedule

	Date	Topic	Lecturer
1	1/3	Practical Aspects of Cryptography	Josh
2	1/10	Symmetric Key Ciphers and Hashes	John
3	1/17	Public Key Ciphers	Josh
4	1/24	Cryptographic Protocols I	Brian
5	1/31	Cryptographic Protocols II	Brian
6	2/7	Security of Block Ciphers	John
7	2/14	AES and Cryptographic Hashes	John
8	2/21	Trust, PKI, Key Management [Last HW Assignment)	Brian
9	3/1	Random Numbers/Elliptic Curve Crypto	Josh
10	3/8	Three topics: Elections, ITAR/Politics, Side Channels/Timing Attacks, DRM, BigNum Implementation	All

Analysis of Block Ciphers

John Manferdelli
jlm@cs.washington.edu
jmanfer@microsoft.com

Digital Block Ciphers

Complicated keyed invertible functions constructed from iterated elementary rounds.

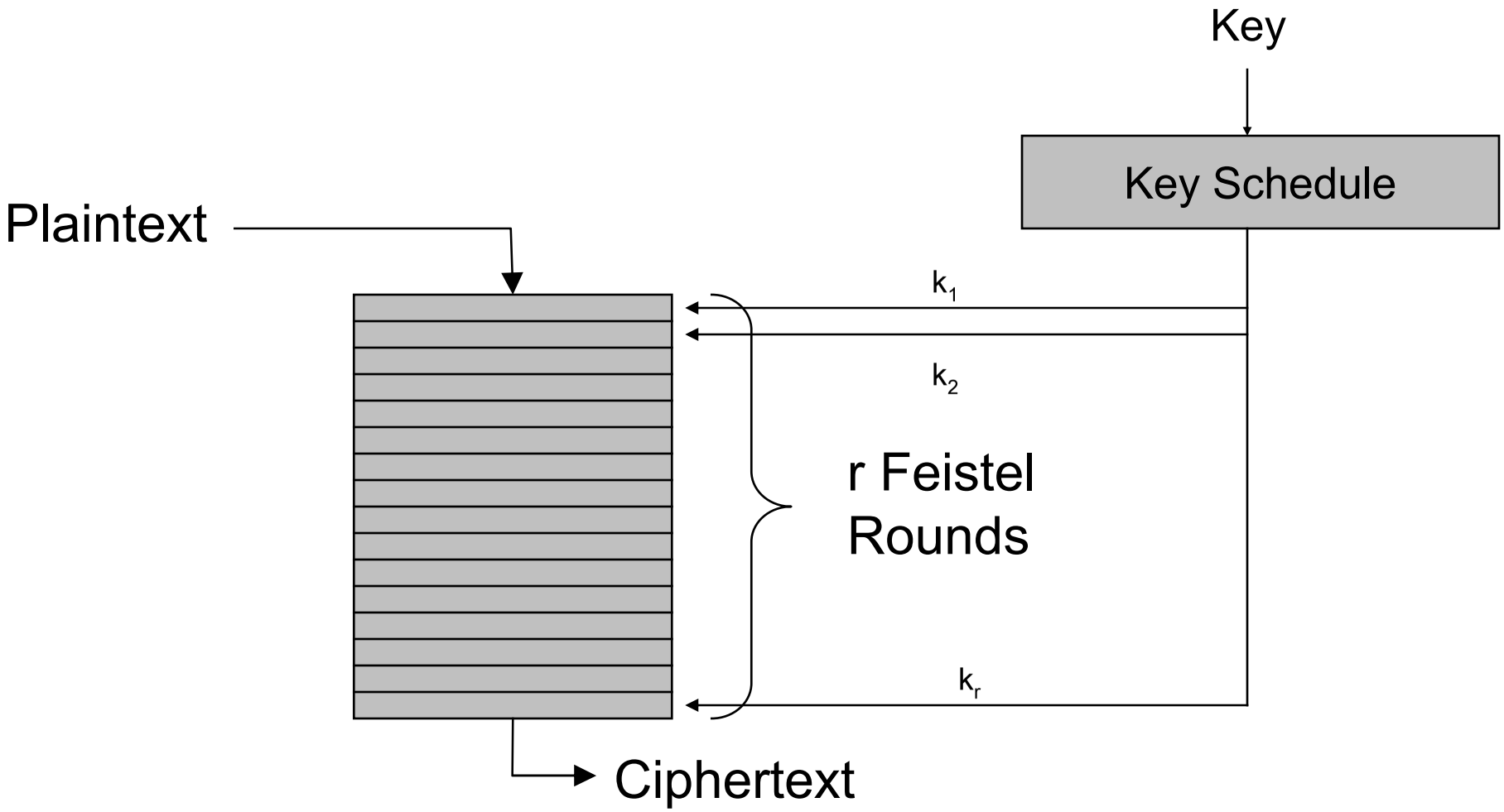
- Confusion: non-linear functions (ROM lookup)
- Diffusion: permute round output bits
- Key mixing: xor “key schedule” at beginning of round

Characteristics:

- *Fast*
- *Data encrypted in fixed “block sizes” (64,128,256 bit blocks are common).*
- *Key and message bits non-linearly mixed in ciphertext*

DES (1974) design was watershed in public symmetric key crypto.

Iterated Feistel Cipher



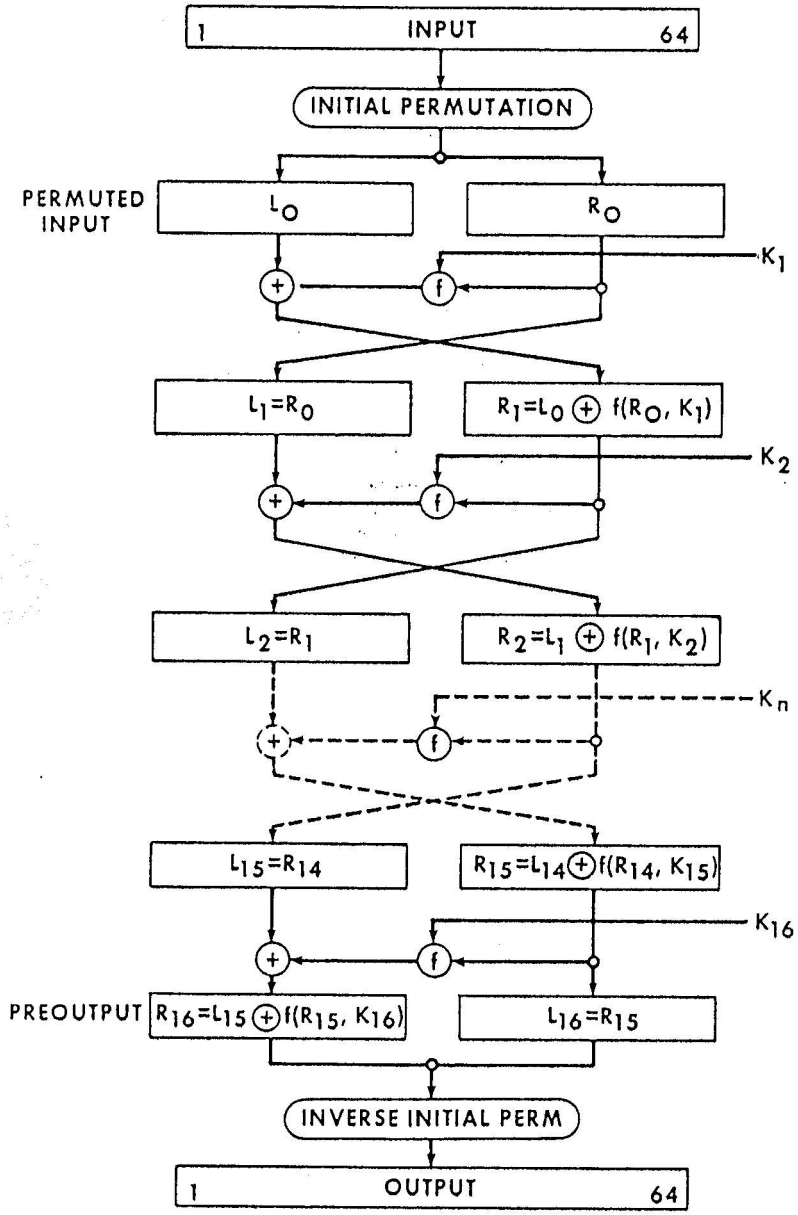


Figure 5.1. Electronic Codebook (ECB) Mode—Enciphering Computation.

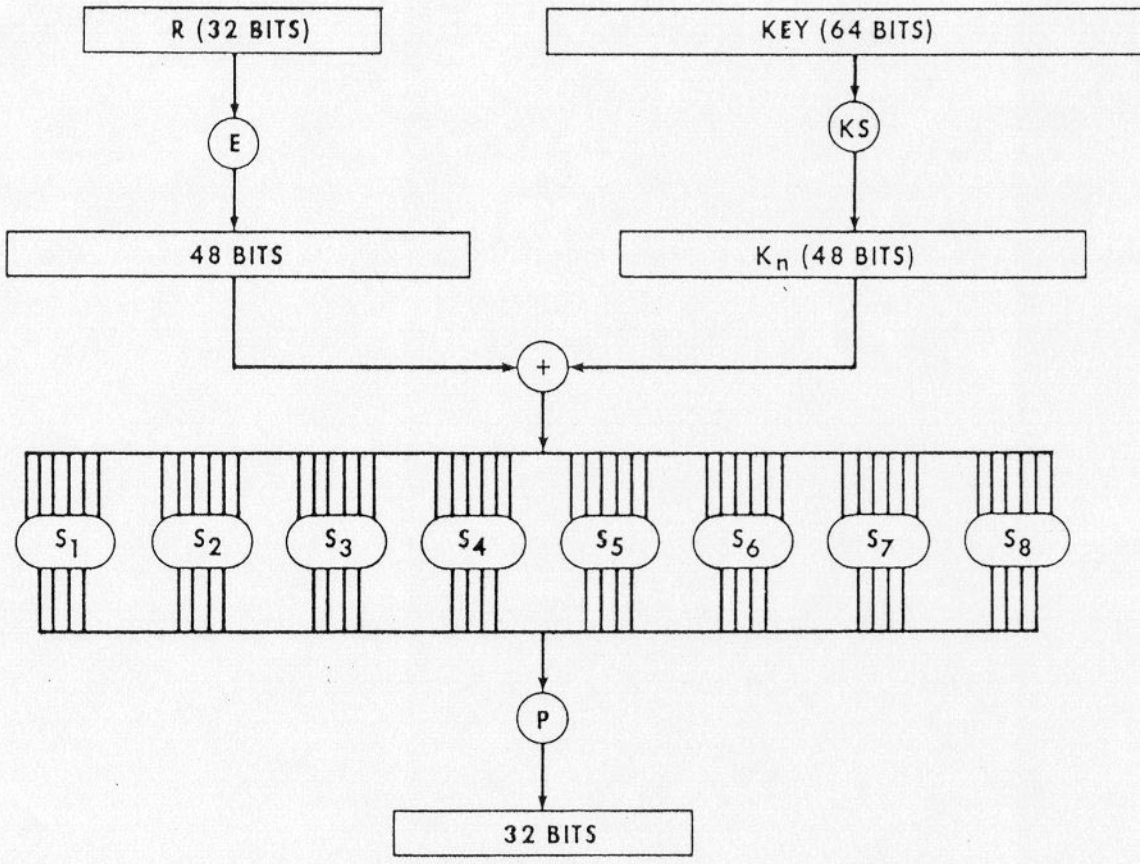


Figure 5.2. Electronic Codebook (ECB) Mode—Calculation of f(R,K).

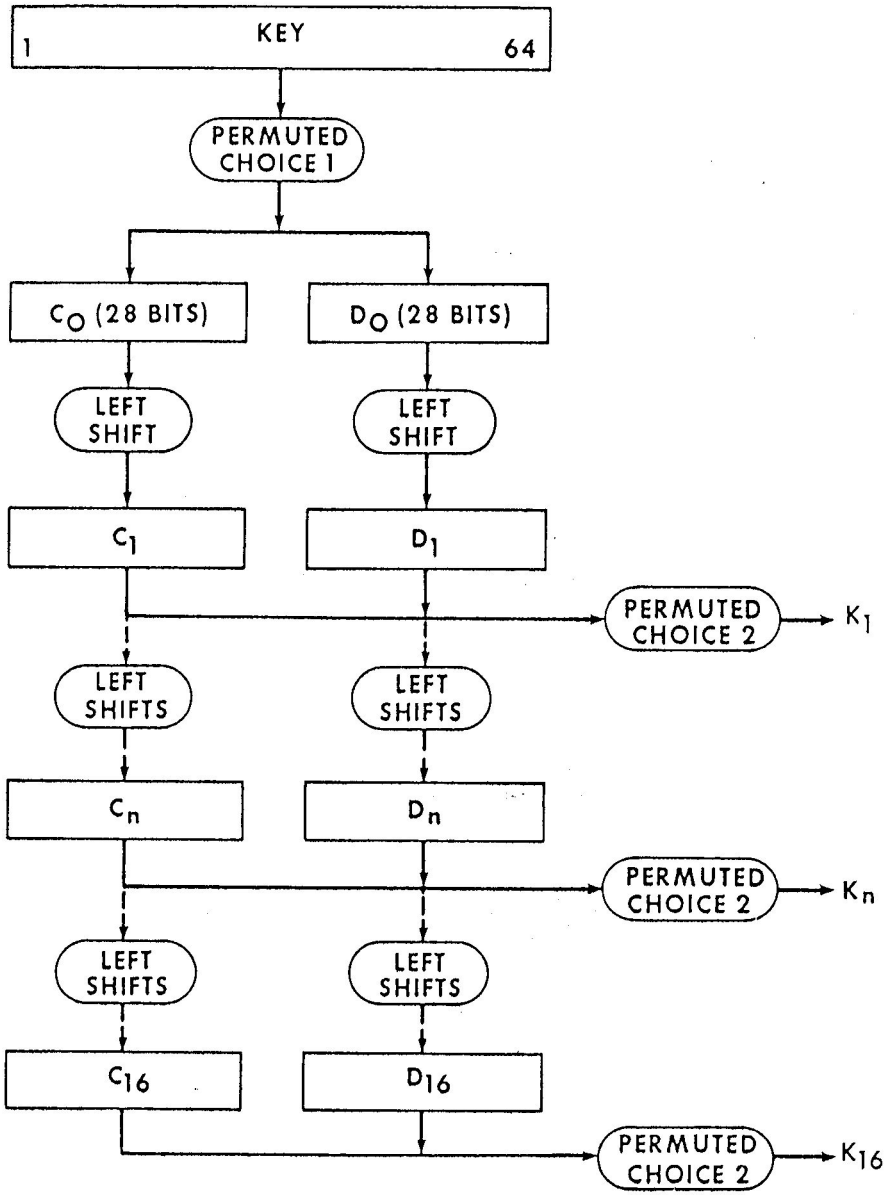


Figure 5.3. Electronic Codebook (ECB) Mode—Key Schedule (KS) Calculation.

DES Attacks: Exhaustive Search

- Symmetry DES(**k** $\oplus$ **1**, **x** $\oplus$ **1**)=DES(**k, x**) $\oplus$ **1**
- Suppose we know plain/cipher text pair (p,c)

```
for(k=0;k<2^56;k++) {
  if(DES(k,p)==c) {
       printf("Key is %x\n", k);
       break;
       }
}
```

- Expected number of trials (if k was chosen at random) before success: 2^{55}

DES Attacks: Exhaustive Search

- Poor random number generator: 20 bits of entropy
- How long does it take?
- 2^{20} vs 2^{56}
- Second biggest real problem
- First biggest: bad key management

- Symmetric ciphers are said to be secure in practice if no known attack works more efficiently than exhaustive search. Note that the barrier is computational not information theoretic.

DES Described Algebraically

$\sigma_i(L,R) = (L \oplus f(E(R) \oplus k_i), R)$, $\tau(L,R) = (R,L)$.

k_i is 48 bit sub-key for round i and

$f(x) = P(S_1 S_2 S_3 \ldots S_8(x))$. Here, each S –box operates on 6 bit quantities and outputs 4 bit quantities. P permutes the resulting 32 output bits.

Each round (except last) is $\tau \sigma_i$. Note that $\tau \tau = \tau^2 = 1 = \sigma_i \sigma_i = \sigma_i^2$.

Full DES is:

$DES_K(x) = IP^{-1} \sigma_{16} \tau \ldots \sigma_3 \tau \sigma_2 \tau \sigma_1 IP(x)$

So its inverse is

$DES_K^{-1}(x) = IP^{-1} \sigma_1 \tau \ldots \sigma_{14} \tau \sigma_{15} \tau \sigma_{16} IP(x)$

Theorem (Coppersmith and Grossman): If $\sigma_K(L,R) = (L \oplus f(E(R) \oplus K, R)$,
$\langle \tau, \sigma_K \rangle = A_N$, $N = 2^n$.

Note: If a and b are chosen at random from S_n there is a good chance that $\langle a,b \rangle = A_n$ or S_n

DES Key Schedule

$C_0D_0 = PC_1(K)$

$C_{i+1} = \text{LeftShift}(\text{Shift}_i, C_i)$, $D_{i+1} = \text{LeftShift}(\text{Shift}_i, D_i)$,

$K_i = PC_2(C_i \,||D_i)$

$\text{Shift}_i = <1,2,2,2,2,2,2,1,2,2,2,2,2,2,1,1>$

Note: Irregular Key schedule protects against related key attacks. [Biham, New Types of Cryptanalytic Attacks using Related Keys, TR-753, Technion]

Weak Keys

- DES has:
  - Four weak keys k for which $E_k(E_k(m)) = m$.
  - Twelve semi-weak keys which come in pairs k_1 and k_2 and are such that $E_{k1}(E_{k2}(m)) = m$.
  - Weak keys are due to “key schedule” algorithm

How Weak Keys Arise

- A 28 bit quantity has potential symmetries of period 1,2,4,7, and 14
- Suppose each of C_0 and D_0 has a symmetry of period 1; for example C_0 =0x0000000, D_0= 0x1111111. We can easily figure out a master key (K) that produces such a C_0 and D_0.
- Then $DES_K(DES_K(x))=x$.

Birthday Attacks

- Probability of collision determined by “Birthday Paradox” calculation:
  - $(1-1/n)\ (1-2/n)\ \ldots\ (1-(k-1)/n) = (n!/k!)/n^k$
  - Probability of collision is >.5 when $k^2 > n$.
  - $1+x \leqq e^x$, $\Pi_{i=1}^{i=k}\ (1-i/n) \leqq e^{-k(k-1)/(2n)}$

Meet in the Middle

Does double encryption double effective key size? Suppose:

$E'_{(a,b)}(p)=E_b(E_a(p))$

Attack if DES is a group. [$E_k(p)=E_b(E_a(p))$].

But it isn't: Campbell and Weiner, DES is not a Group, Crypto '92.

Time Memory tradeoff [Known plaintext (p_1, c_1), (p_2,c_2)]

Store $E_a(p)=x$, Calculate $E^{-1}_b(c)=y$.

The Linear Attack (Full Monty)

- Suppose Λ is a cipher, the ciphertext and plaintext both come from $GF(2)^n$, $\mathbf{c}=K\mathbf{p}$, where K is an n x n invertible matrix, $K=(k_{ij})$ over GF(2). How can we "break" Λ?

- Answer: Get (about) n corresponding plaintext-ciphertext pairs ($\mathbf{p}_i$, $\mathbf{c}_i$). Solve resulting linear equations to invert K.

The Linear Attack (Partial Monty)

- Suppose Λ is a cipher, the ciphertext and plaintext both come from $GF(2)^n$, with key bits k_i i= 1,2,…,n. How can we "break" Λ?

- Answer: Obtain m (linearly independent) linear relations
  - $F_i(c) + G_i(p) = a_{i1} k_1 + \ldots + a_{in} k_n$, i= 1,2,…,m
  - Guess n-m-1bits of the key, solve for the other m bits
  - Time: 2^{n-m-1} + O(m) vs 2^n
  - Much faster than exhaustive search $O(2^n)$

Almost True Linear Constraints

- Usually finding linear relations is hard or impossible
- Look for linear relations that hold with probability p>1/2
- Collect a number of samples
- Vote for the maximally *consistent* subset of equations
- Suppose you have n (linear) equations hold with probability p, what is the probability of m of them being correct.
  - ${}_nC_m\ p^m\ (1-p)^{n-m}$
- Large consistent subsets very likely to be correct even if p is only slightly bigger than .5

Almost Linear Attacks

- Sparse Representation
- Decomposition
- Transformation
- “Almost Linear” constraints
- Differential Cryptanalysis
- Linear Cryptanalysis

Algebraic Representations

- *Algebraic normal form* (ANF):

$$f(X) = a_0 \oplus (\bigoplus_{i=1}^{i=n} a_i x_i)$$
$$\oplus (\bigoplus_{1 \le i \ne j \le n} a_{ij} x_i x_j) \oplus \ldots \oplus a_{12\ldots n} x_1 x_2 \ldots x_n$$

- *Degree*: deg(f),the highest degree term in ANF.
  - Example
    f(X)= x_1+x_2, deg(f)=1
    g(X)=x_1x_2, deg(g)=2
- Functions of degree 1 are called “affine”. Composition of Affine functions is affine.

- Lagrange Interpolation Theorem. Every function in n variables can be expressed as a polynomial (hence ANF).

Boolean Functions

- For a set of Boolean functions Δ, d(f,g)=#{X|f(X) $\neq$ g(X)}.
- *Distance*: For Boolean function f(X) and g(X), d(f,Δ)= $\min_{[g(X)\epsilon\Delta]}$ d(f,g)
- *Affine function*: h(x)= $a_1x_1 \oplus a_2x_2 \oplus \ldots \oplus a_nx_n+c$
- *nl(f)* denotes the minimum distance between f(X) and the set of affine functions Δ_{affine}. *nl(f)= d(f, Δ_{affine})*, *Δ_{affine}= RM(1,n).*
- *Balance*: f(X) is balanced iff there is an equal number of 0's and 1's in the output of f(X).

S Boxes as Polynomials over GF(2)

```
1,1:
   56+4+35+2+26+25+246+245+236+2356+16+15+156+14+146+145+13+135+134+
   1346+1345+13456+125+1256+1245+123+12356+1234+12346

1,2:
   C+6+5+4+45+456+36+35+34+346+26+25+24+246+2456+23+236+235+234+2346
   +1+15+156+134+13456+12+126+1256+124+1246+1245+12456+123+1236+1235
   +12356+1234+12346

1,3:
   C+6+56+46+45+3+35+356+346+3456+2+26+24+246+245+236+16+15+145+13+1
   356+134+13456+12+126+125+12456+123+1236+1235+12356+1234+12346

1,4:
   C+6+5+456+3+34+346+345+2+23+234+1+15+14+146+135+134+1346+1345+125
   6+124+1246+1245+123+12356+1234+12346
```

Legend: C+6+56+46 means $1 \oplus x_6 \oplus x_5 x_6 \oplus x_4 x_6$

Decomposable Systems

$E_{k1||k2}(x) = E'_{k1}(x) \,||\, E''_{k2}(x)$

m	t	2^{mt}	$m2^{t}$
2	32	2^{64}	2^{33}
4	16	2^{64}	2^{18}

Linear Attacks on sparse coefficients

$c_1 = f[1,0](k_1,\ldots,k_m)\, p_1^0 p_2^0 \ldots p_n^0 + f[1,1](k_1,\ldots,k_m)\, p_1^1 p_2^0 \ldots p_n^0 + \ldots + f[1,2^m](k_1,\ldots,k_m)\, p_1 p_2 \ldots p_n$

… … ….

$c_n = f[n,1](k_1,\ldots,k_m)\, p_1 p_2 \ldots p_n + f[1,1](k_1,\ldots,k_m)\, p_1^1 p_2^0 \ldots p_n^0 \ldots + f[1,2^m](k_1,\ldots,k_m)\, p_1 p_2 \ldots p_n$

Suppose most of the f[i,j] are 0. Can solve for these with different $p_1,p_2,\ldots,p_n$, even it the F[i,j] are complex and non-linear in the $k_1,\ldots,k_m$

Mathematics of Boolean Functions

- Correlation
  - $c(f,g) = P[f(x)=g(x)] - P[f(x) \neq g(x)]$. So
    $P[f(x)=g(x)] = .5(1+c(f,g))$
- Hadamard
  - $S_f(w) = 2^{-n}\Sigma_x (-1)^{f(x)+w.x} = c(f(x), w\ x)$
- Bent functions

  Furthest from linear (all Hadamard coefficients are equal)

Differential Cryptanalysis of Biham and Shamir (1990)

- Was known to IBM team whose design rules provided some resistance
- Breaks Khafre with 1500 corresponding plain/cipher texts in an hour
- Breaks 8 round Lucifer in 2^{21} steps with 24 texts
- Breaks FEAL.
- Breaks 8 round DES.
- Useful in breaking cryptographic hashes
- DES Results: 2^{47} Chosen plaintext attack.

Differential Cryptanalysis: Overview

Let (P_L, P_R), (P_L^*, P_R^*) and (C_L^*, C_R^*), (C_L^*, C_R^*) be pairs of inputs and outputs with prescribed xors

$(P_L', P_R') = (P_L, P_R) \oplus (P_L^*, P_R^*)$

$(C_L', C_R') = (C_L, C_R) \oplus (C_L^*, C_R^*)$

Follow the xor of two plaintexts through rounds of DES.

Output xor depends non uniformly on key bits.

Take lots of chosen plaintext/ciphertext pairs.

Let non uniform distribution “vote” on set containing keys.

Key Observation

Consider a round of DES: σ_K: (L,R)→(L $\oplus$F(E(R)$\oplus$K),R) where if
$X=X_1||X_2||\ldots||X_8$ with each X_i a six bit quantity.
$F(X)= P(S_1(X_1)|| \ldots ||S_8(X_8))$

Let (L, R) and (L*, R*) be inputs to a round of DES and
(L', R')= (L, R) $\oplus$(L*, R*).

At each S-box position, an input xor is independent of key and the distribution of the output xors is very non-linear and not all output xors are possible. Keys choose among output xors. Aside from S-box, all operations are linear.

Differential Profile of single S-box

- If $x' \rightarrow y'$ and $D_j(x',y')= \{u: S_j(u \oplus x') \oplus S_j(u)= y'\}$, then
  $k \in x \oplus D_j(x',y')$.
- $|D_j(x', y')|$ has non uniform distribution.
- The propagation ratio, $R_p(x', y') = |D_j(x', y')| /2^m$, where m is the dimension of the space of a'.
  - Shamir and Biham denote this as $x' \rightarrow y'$, p.
    ($p= |D_j(x', y')| /2^m$).

S1 Differential Distribution

S box 1

In	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	6	0	2	4	4	0	10	12	4	10	6	2	4
2	0	0	0	8	0	4	4	4	0	6	8	6	12	6	4	2
3	14	4	2	2	10	6	4	2	6	4	4	0	2	2	2	0
4	0	0	0	6	0	10	10	6	0	4	6	4	2	8	6	2
5	4	8	6	2	2	4	4	2	0	4	4	0	12	2	4	6
6	0	4	2	4	8	2	6	2	8	4	4	2	4	2	0	12
7	2	4	10	4	0	4	8	4	2	4	8	2	2	2	4	4
8	0	0	0	12	0	8	8	4	0	6	2	8	8	2	2	4
9	10	2	4	0	2	4	6	0	2	2	8	0	10	0	2	12
a	0	8	6	2	2	8	6	0	6	4	6	0	4	0	2	10
b	2	4	0	10	2	2	4	0	2	6	2	6	6	4	2	12
c	0	0	0	8	0	6	6	0	0	6	6	4	6	6	14	2
d	6	6	4	8	4	8	2	6	0	6	4	6	0	2	0	2
e	0	4	8	8	6	6	4	0	6	6	4	0	0	4	0	8
f	2	0	2	4	4	6	4	2	4	8	2	2	2	6	8	8
10	0	0	0	0	0	0	2	14	0	6	6	12	4	6	8	6
11	6	8	2	4	6	4	8	6	4	0	6	6	0	4	0	0
12	0	8	4	2	6	6	4	6	6	4	2	6	6	0	4	0

S1 Differential Distribution

S box 1

In	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
13	2	4	4	6	2	0	4	6	2	0	6	8	4	6	4	6
14	0	8	8	0	10	0	4	2	8	2	2	4	4	8	4	0
15	0	4	6	4	2	2	4	10	6	2	0	10	0	4	6	4
16	0	8	10	8	0	2	2	6	10	2	0	2	0	6	2	6
17	4	4	6	0	10	6	0	2	4	4	4	6	6	6	2	0
18	0	6	6	0	8	4	2	2	2	4	6	8	6	6	2	2
19	2	6	2	4	0	8	4	6	10	4	0	4	2	8	4	0
1a	0	6	4	0	4	6	6	6	6	2	2	0	4	4	6	8
1b	4	4	2	4	10	6	6	4	6	2	2	4	2	2	4	2
1c	0	10	10	6	6	0	0	12	6	4	0	0	2	4	4	0
1d	4	2	4	0	8	0	0	2	10	0	2	6	6	6	14	0
1e	0	2	6	0	14	2	0	0	6	4	10	8	2	2	6	2
1f	2	4	10	6	2	2	2	8	6	8	0	0	0	4	6	4
20	0	0	0	10	0	12	8	2	0	6	4	4	4	2	0	12
21	0	4	2	4	4	8	10	0	4	4	10	0	4	0	2	8
22	10	4	6	2	2	8	2	2	2	2	6	0	4	0	4	10
23	0	4	4	8	0	2	6	0	6	6	2	10	2	4	0	10
24	12	0	0	2	2	2	2	0	14	14	2	0	2	6	2	4

S1 Differential Distribution

S box 1

In	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
25	6	4	4	12	4	4	4	10	2	2	2	0	4	2	2	2
26	0	0	4	10	10	10	2	4	0	4	6	4	4	4	2	0
27	10	4	2	0	2	4	2	0	4	8	0	4	8	8	4	4
28	12	2	2	8	2	6	12	0	0	2	6	0	4	0	6	2
29	4	2	2	10	0	2	4	0	0	14	10	2	4	6	0	4
2a	4	2	4	6	0	2	8	2	2	14	2	6	2	6	2	2
2b	12	2	2	2	4	6	6	2	0	2	6	2	6	0	8	4
2c	4	2	2	4	0	2	10	4	2	2	4	8	8	4	2	6
2d	6	2	6	2	8	4	4	4	2	4	6	0	8	2	0	6
2e	6	6	2	2	0	2	4	6	4	0	6	2	12	2	6	4
2f	2	2	2	2	2	6	8	8	2	4	4	6	8	2	4	2
30	0	4	6	0	12	6	2	2	8	2	4	4	6	2	2	4
31	4	8	2	10	2	2	2	2	6	0	0	2	2	4	10	8
32	4	2	6	4	4	2	2	4	6	6	4	8	2	2	8	0
33	4	4	6	2	10	8	4	2	4	0	2	2	4	6	2	4
34	0	8	16	6	2	0	0	12	6	0	0	0	0	8	0	6
35	2	2	4	0	8	0	0	0	14	4	6	8	0	2	14	0
36	2	6	2	2	8	0	2	2	4	2	6	8	6	4	10	0

S1 Differential Distribution

S box 1

In	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
37	2	2	12	4	2	4	4	10	4	4	2	6	0	2	2	4
38	0	6	2	2	2	0	2	2	4	6	4	4	4	6	10	10
39	6	2	2	4	12	6	4	8	4	0	2	4	2	4	4	0
3a	6	4	6	4	6	8	0	6	2	2	6	2	2	6	4	0
3b	2	6	4	0	0	2	4	6	4	6	8	6	4	4	6	2
3c	0	10	4	0	12	0	4	2	6	0	4	12	4	4	2	0
3d	0	8	6	2	2	6	0	8	4	4	0	4	0	12	4	4
3e	4	8	2	2	2	4	4	14	4	2	0	2	0	8	4	4
3f	4	8	4	2	4	0	2	4	4	2	4	8	8	6	2	2

Example: Differential Cryptanalysis of S1 through a single round

$S1_{E^*}$ = 0x01, $S1_E$ = 0x35 → 0x0d=S_O'.
$S1_{E^*}$ = 0x21, $S1_E$ = 0x15 → 0x03=S_O'.

0x34 → 0x0d *means input difference 0x34 produced output difference 0x0d*
(1) For 0x34→0xd, $S1_I$'= {0x06, 0x10, 0x16, 0x1c, 0x22, 0x24, 0x28, 0x32}.
(2) For 0x34→0x3, $S1_I$'= {0x01, 0x02, 0x15, 0x21, 0x35, 0x36}.

Since $S1_E$ + $S1_I$= $S1_K$,

(1) Reduces the possible key set to {0x07, 0x33, 0x11, 0x25, 0x17, 0x23, 0x1d, 0x29}
(2) Reduces the possible key set to {0x20, 0x14, 0x23, 0x17, 0x34, 0x00}.

The intersection (and actual possibilities) are { 0x17, 0x23}

One Round Differential used to analyze 4 round DES

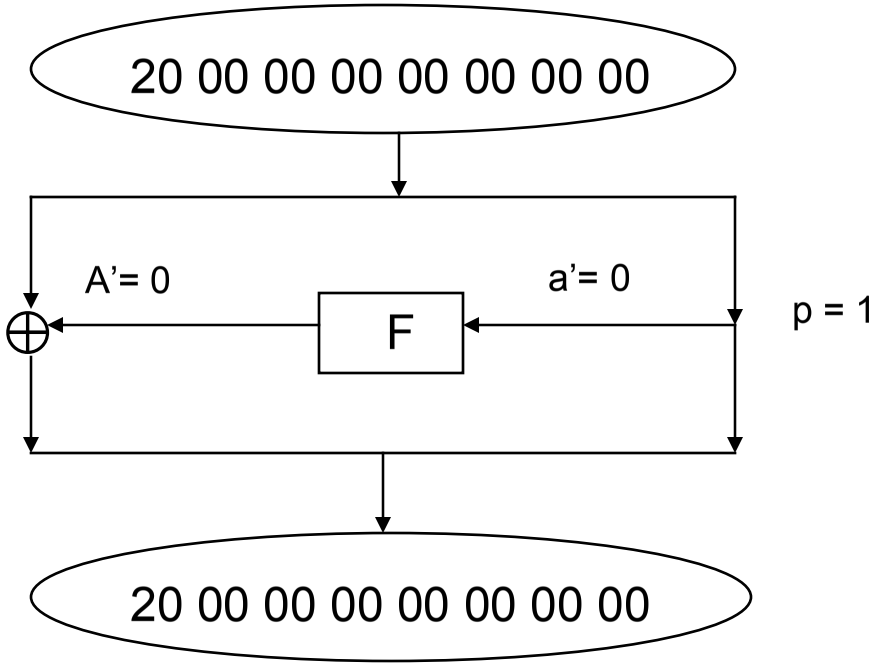
Method

Use 1 round characteristic to right.

Undo effect of permutation matrix and solve each S box separately.

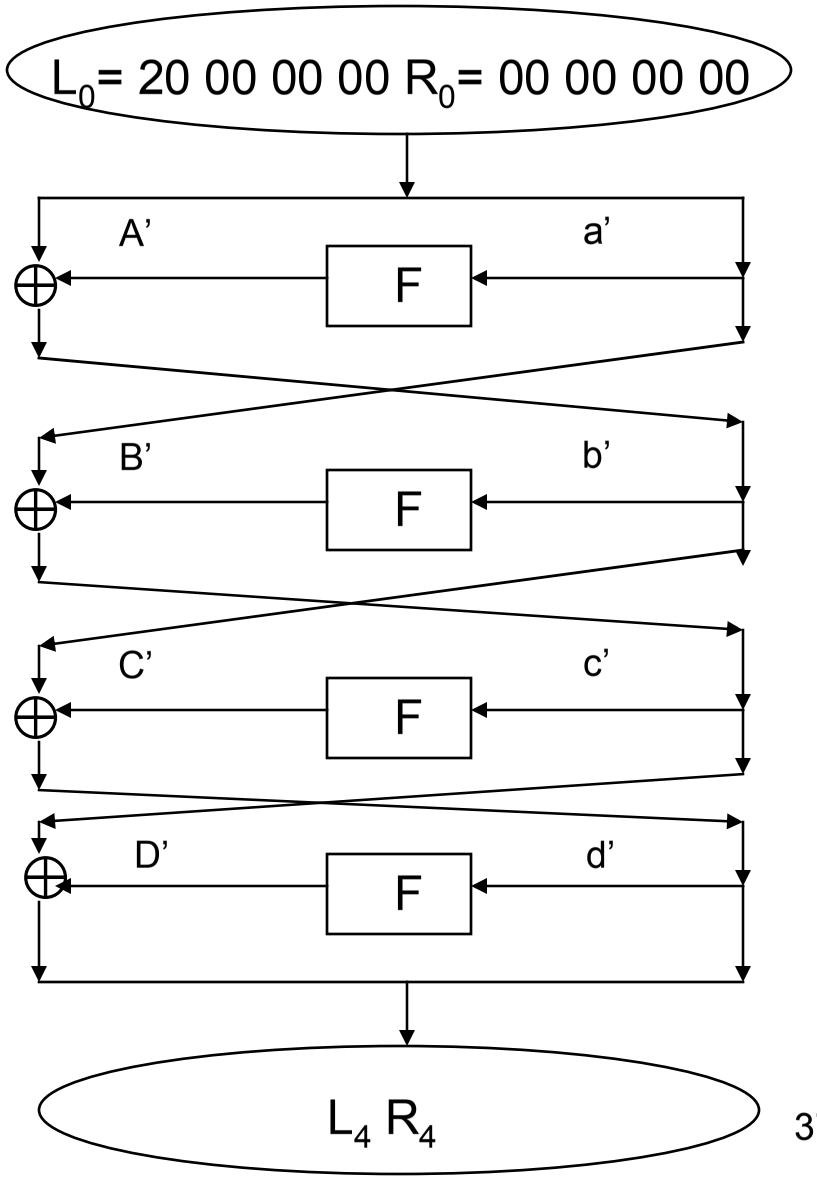
This allows us to solve for 48 key bits.

This 1 round characteristic will be used to estimate input xor in subsequent rounds.



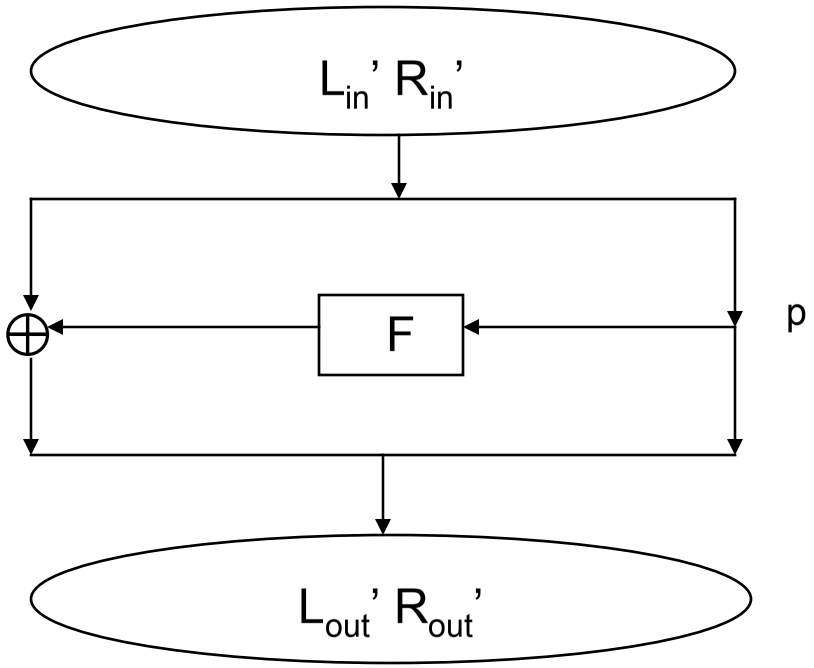
Differential Cryptanalysis of 4 rounds

- D'=a' $\oplus$ B' $\oplus$ L_4'
- d' = R_4'
- Because b'=L_0', the output xor of S_2, S_3, … , S_8 in round 2 is 0. This gives 28 bits of B' and hence 28 bits of D' is known.
- Since B' is known, we can calculate D'= B' $\oplus L_4$' using 4 encrypted pairs for each of the 7 relevant S boxes. All key candidates are in this set which gives 7x6=42 bits of key with high probability.



Extension to Full Round: One Round Characteristic

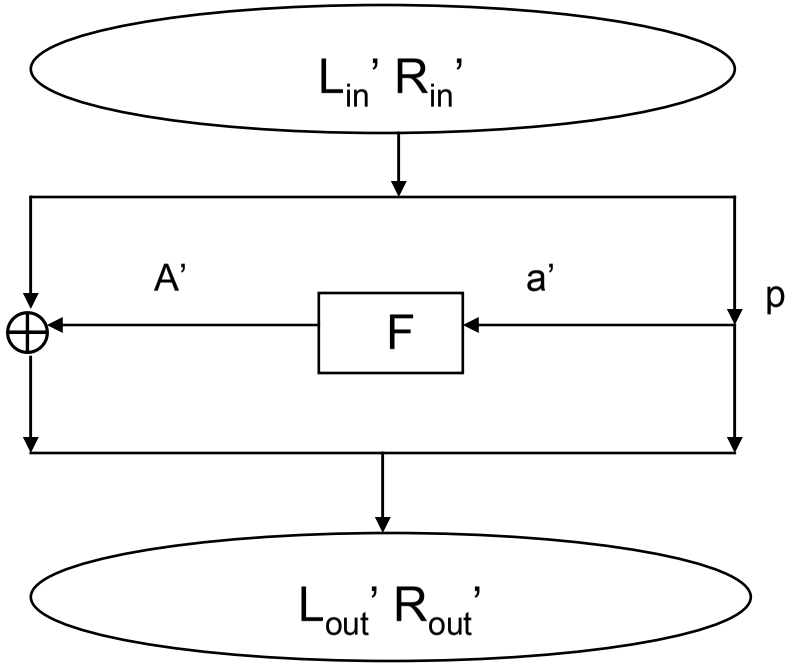
A *one round characteristic* is a triple $[(L_{in}', R_{in}'), (L_{out}', R_{out}'), p]$ such that for all pairs (L_{in}, R_{in}), (L_{in}^*, R_{in}^*) with $(L_{in}, R_{in}) \oplus (L_{in}^*, R_{in}^*) = (L_{in}', R_{in}')$ subjected to a single cipher round (depicted on the right) producing corresponding pairs (L_{out}, R_{out}), (L_{out}^*, R_{out}^*), the ratio of output pairs with $(L_{out}, R_{out}) \oplus (L_{out}^*, R_{out}^*) = (L_{out}', R_{out}')$ is p.



Characteristics can be extended through multiple rounds.

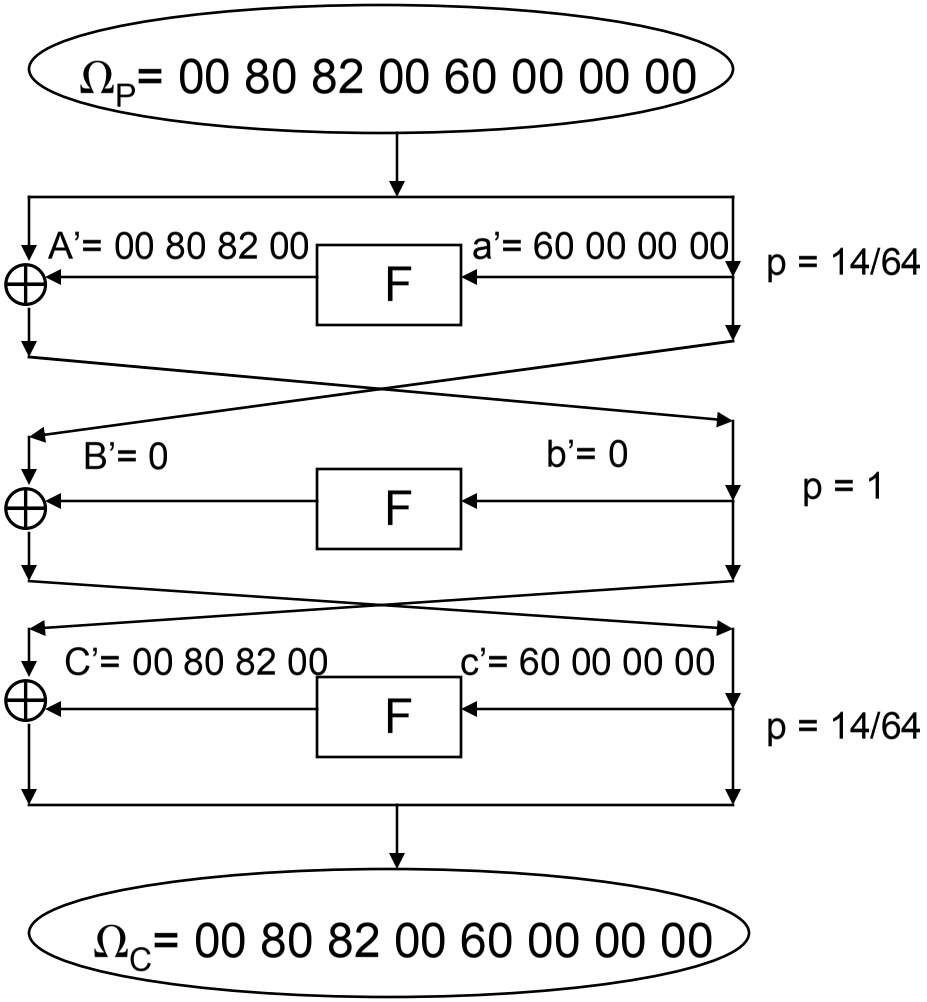
Computing a single characteristic

- The first and most important differential is (L',0) → (L',0), p=1.
- Another is (L',0x60000000) → (L' ⊕0x00808200,0x60000000), p=1/4.
- Construction:
  - E(0x60000000)= E(0110 0000 … 0000)= 001100 000000 … 000000
  - $S_1(001100)'$ → 0xe with p=1/4, $S_j(0)'$ → 0 with p=1, j>1 and P(0xe0000000)=0x00808200.



Multi-round Characteristics

- Sequence of Differentials with identified input and output xors. Each round differential occurs with probability p_i.
- Overall probability: $p=\Pi\, p_i$
- Characteristic to the right is a three round characteristic with probability $(14/64)^2$
- Used to approximate differentials through multiple rounds.
- Each pair following the characteristic at each round is called a “right pair”. Other pairs are “wrong pairs.”
- Wrong pairs get distributed uniformly; right pairs follow overall characteristic probability.



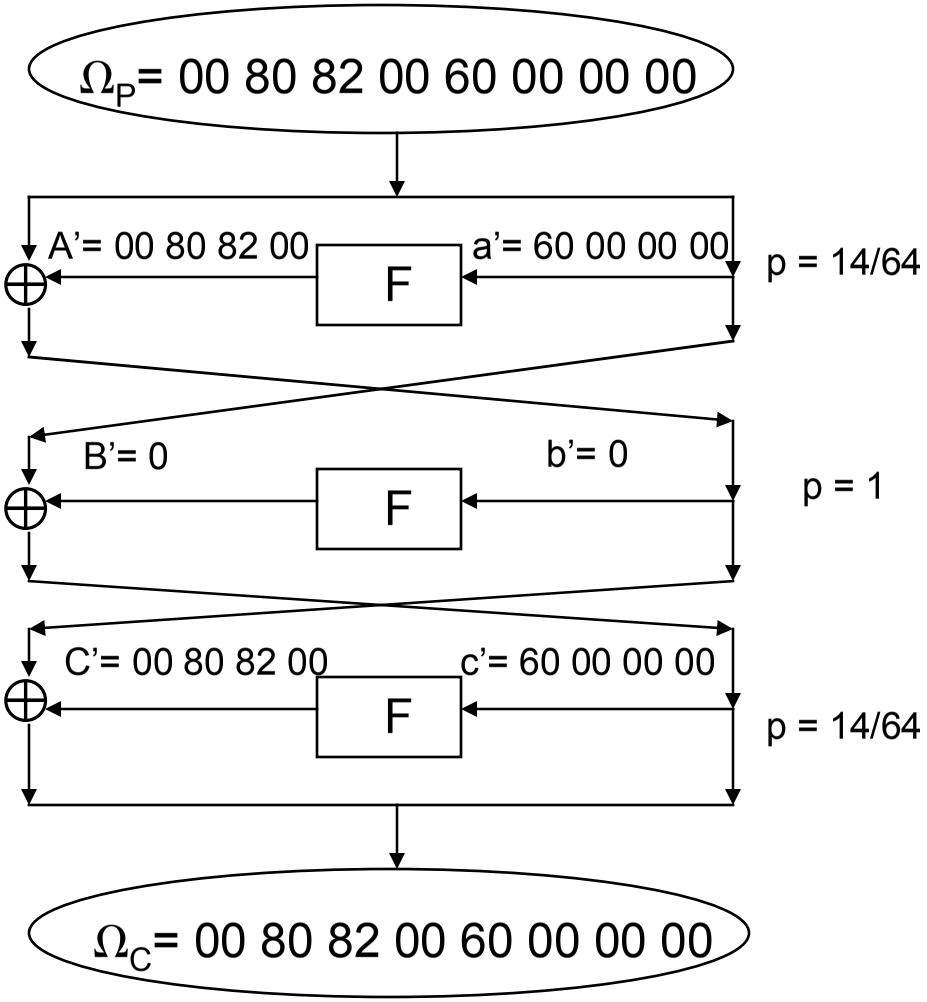
Multi-round Characteristics - Formal

- An n-round characteristic is a triple $(\Omega_P, \Lambda, \Omega_T)$, with $\Lambda=(\Lambda_1, \ldots, \Lambda_n)$ where each Λ_1 is a one round differential that holds with probability p_i, with first round input xor (to Λ_1) is Ω_P, and last round output xor (form Λ_n) is Ω_T and rounds "match up".
- Probability of combined n-round characteristic is $p^{\Omega}=\prod p_i$

- Each pair following the characteristic at each round is called a "right pair". Other pairs are "wrong pairs."
- Wrong pairs get distributed uniformly; right pairs follow overall characteristic probability.

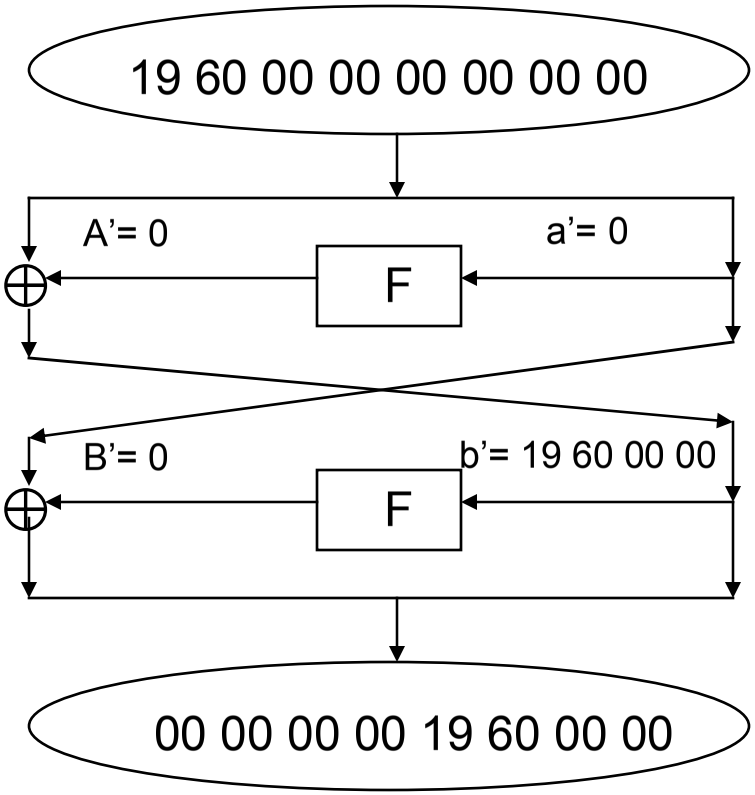
Constructing 3 Round Characteristic

- Use
- (L',0) → (L',0), p=1.
- (0x00808200, 0x60000000) → (0x00808200, 0x60000000), p=1/4.



Two round iterative characteristic

- p= 1/234
- Obviously useful for multi-round estimates



Differential Cryptanalysis of 3 Round DES

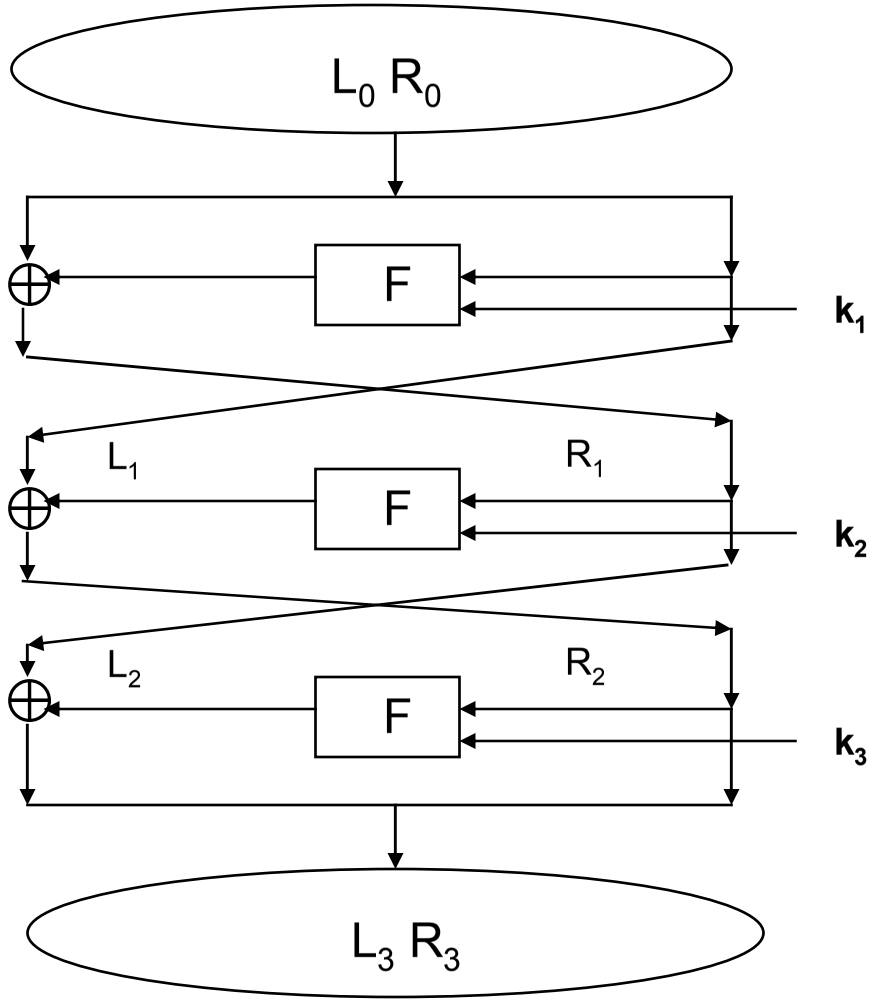
$R_3 = L_2 + f(E(R_2)+k_2)$
$= R_1 + f(E(R_2)+k_2)$
$= L_0 + f(E(R_0)+k_1)) + f(E(R_2)+k_3)$

Pick pair (R_0, R_0^*) with
$R_0 + R_0^* = R_0' = 0$.

$R_3' + L_0' = f(E(R_2)+k_3) + f(E(R_2^*)+k_3)$

$P^{-1}(R_3' + L_1') = f(E(R_2)+k_3) + f(E(R_2^*)+k_3)$.

Note $R_2 = L_3$.



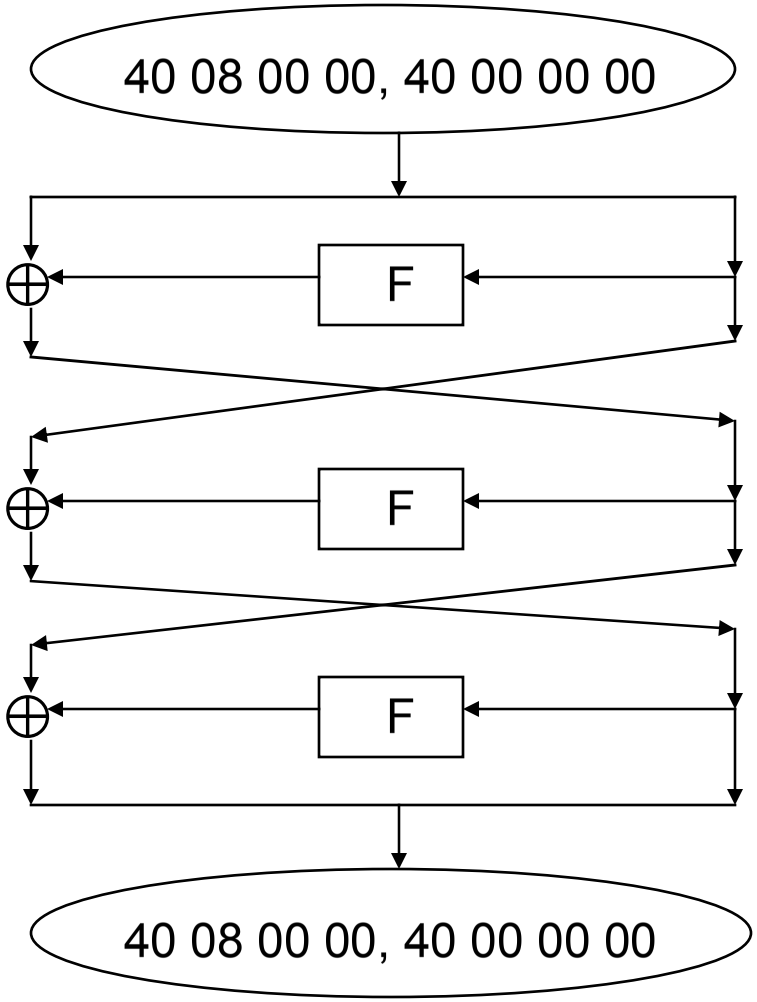
Data for Differential Cryptanalysis of 3 Round DES

Plaintext	**Ciphertext**
0x748502cd38451097	0x03c70306d8a09f10
0x387475643845 1097	0x78560a0960e6d4cb
0x486911026acdff31	0x45fa285be5adc730
0x375bd31f6acdff31	0x134f7915ac253457
0x357418da013fec86	0xd8a31b2f28bbc5cf
0x12549847013fec86	0x0f317ac2b23cb944

Only one counter in each 6 bit set will be possible for all 3 pairs.

Three Round Characteristic

- This characteristic occurs with probability p=1/16 and forms an estimate for the differential input of the 4th round of the 6 rounds.
- (00 20 00 08 00 00 00 04) → (00 00 04 00 00 20 00 08) with p=1/16 is another such characteristic.



Differential Cryptanalysis of 6 rounds

- Suppose (L_{i-1}, R_{i-1}), k_i are the inputs to round i. $P_L = L_0$, $P_R = R_0$.
- $L_6 = R_4 \oplus f(k_6, R_6) = L_3 \oplus f(k_6, R_6) \oplus f(k_4, R_3)$
- $L_6' = L_3' \oplus f(k_6, R_6) \oplus f(k_6, R_6^*) \oplus f(k_4, R_3) \oplus f(k_4, R_3^*)$
- L_6' = C_L and $R_6 = C_R$ are known.
- Estimate L_3' = 40000000, R_3' = 40080000, using the differential.
- Set $S = P^{-1}(C_L \oplus 40000000) = f(k_6, C_R) \oplus f(k_6, C_R^*) \oplus f(k_4, R_3) \oplus f(k_3, R_3^*) = S_1(E_1) \,||\, S_2(E_2) \,||\, \ldots \,||\, S_8(E_8)$ where $E_1 || E_2 || \ldots || E_8$ are the bits obtained by applying E to 40080000.
- $E_1 || E_2 || \ldots || E_8$ =00100000000000000101000..0=

  08||00||01||10||00||00||00||00.
- Since the input Xors to S2,S5,S6,S7,S8 are 0, $f(k_4, R_3) \oplus f(k_4, R_3^*)$ is 0 in the corresponding output bit positions and we are left with the simple differential: $P^{-1}(C_L \oplus 40000000) = f(k_6, C_R) \oplus f(k_6, C_R^*)$ for S2,S5,S6,S7,S8.

Differential Cryptanalysis of 6 rounds

- First characteristic yields 30 bits of key. Second one adds another 12 bits of key.
- Recall $P^{-1}(C_L \oplus 40000000) = f(k_6, C_R) \oplus f(k_6, C_R^*)$ for S2,S5,S6,S7,S8
- This occurs with p= 1/16.
- Straightforward implementation yielding 30 keybits:
  - Set up 2^{30} counters
  - Bump counter for suggested key for each pair of n chosen texts
  - Correct key be will “voted” at least 1/16 n time (“right pairs”)
  - Incorrect keys will be voted randomly each with probability $1/2^{30}$

Differential Cryptanalysis of 6 rounds

- Improving the "signal to noise" ratio by "filtering" pairs
  - For each of S2, S5, S6, S7,S8 with input xor x' and output xor y', look at $x \oplus D_j(x',y')$.
  - If this is empty, this must be wrong pair.
  - For any given S box the, this happens with probability .2.
  - The probability that all 5 S boxes have non-empty candidate key sets is $(.8)^5$=.33. Call this set of pairs RP and the complement WP.
  - RP contains 1/3 of the pairs, WP contains 2/3
  - In RP, the probability of a "correct vote" is 3/16

“Signal to Noise”

- S/N= right_pairs/average_count
  - α= average_counted/counted_pairs
  - β=ratio of_counted_to_all
  - m is the number of pairs
- S/N= mp / (m $\alpha\beta$ /2^k)= (2^k p)/($\alpha\beta$)

- S/N affects number of right pairs needed:
  - S/N 1-2, 20-40 right pairs required
  - S/N 16, 7-8 right pairs required

Differential Cryptanalysis of DES

- Best 16 rounds attack uses 13 round approximation
  - Requires 2^{47} texts
  - Not much better than exhaustive search
- Converting Chosen Plaintext to Corresponding plaintext attack
  - If m pairs are required for Chosen sqrt(2m) 2^{32} are required for corresponding plaintext

Comments on Differential Cryptanalysis of full DES

# Rounds	Needed pairs	Analyzed Pairs	Bits Found	# Char rounds	Char prob	S/N	Chosen Plain
4	2^3	2^3	42	1	1	16	2^4
6	2^7	2^7	30	3	1/16	2^{16}	2^8
8	2^{15}	2^{13}	30	5	1/1048 6	15.6	2^{16}
16	2^{57}	2^5	18	15	$2^{-55.1}$	16	2^{58}

DES S-Box Design Criteria

- No S-box is linear or affine function of its input.
- Changing one bit in the input of an S-Box changes at least two output bits.
- S-boxes were chosen to minimize the difference between the number of 1's and 0's when any input bit is held constant.
- S(X) and S(X$\oplus$001100) differ in at least 2 bits
- S(X) $\neq$ S(X$\oplus$11xy00)

Comments on effect of components on Differential Cryptanalysis

- E
  - Without expansion, there is a 4 round iterative characteristic with p= 1/256
- P
  - Major influence. If P=I, there is a 10 round characteristic with p= $2^{-14.5}$ (but other attacks would be worse).
- S order
  - If S1, S7 and S4 were in order, there would be a 2 round iterative characteristic with p= 1/73. However, Matsui found an order (24673158) that is better and also better against Linear crypto. Optimum order for LC resistance: 27643158.
- S properties
  - S boxes are nearly optimum against differential crypto

Linear Cryptanalysis

- Invented by Mitsuru Matsui in 1993.
- 16-round DES can be attacked using 2^{43} known plaintexts - get 26 bits, brute force the remaining 30 bits
  - $2^{43} = 9 \times 10^{12} = 9$ trillion known plaintext blocks
- Also exploits biases in S-boxes, which were not designed against the attack
- A DES key was recovered in 50 days using 12 HP9735 workstations in a lab setting

Linear Cryptanalysis

Basic idea:

Suppose $\alpha_i(P) \oplus \beta_i(C) = \gamma_i(k)$ holds with γ_i, linear, for i= 1,2,…,m.

Each equation imposes a linear constraint and reduces key search by a factor of 2.

Guess (n-m-1) bits of key. There are $2^{(n-m-1)}$. Use the constraints to get the remaining keys.

Can we find linear constraints in the "per round" functions and knit them together?

No! Per Round functions do not have linear constraints.

Linear Cryptanalysis

Next idea

Can we find α(P) $\oplus$ β(C) = γ(k) which holds with γ, linear, with probability p?

Suppose α(P) $\oplus$ β(C) = γ(k), with probability p>.5.

Collect a lot of plain/cipher pairs.

Each will "vote" for γ(k)=0 or γ(k)=1.

Pick the winner.

p= 1/2+ϵ requires cϵ^{-2} texts (we'll see why later). ϵ is called "bias".

Breaks 16 round DES with 2^{47} work factor (actually a little more complicated than simple voting over whole cipher).

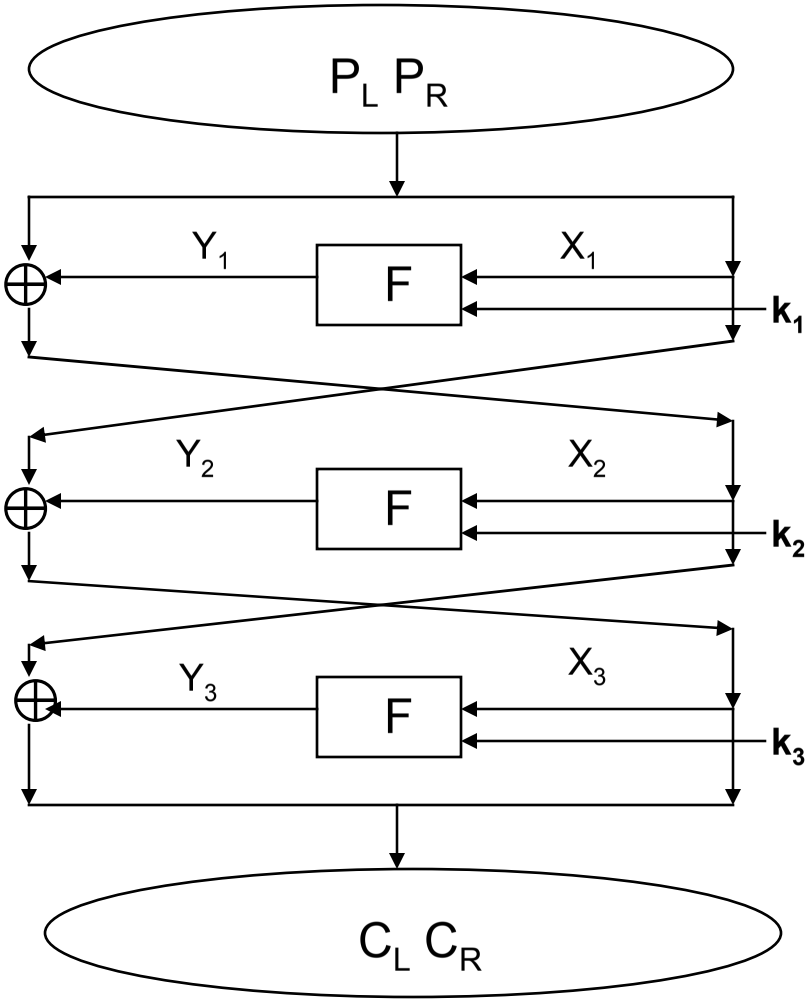
Linear Cryptanalysis Notation

Matsui numbers bits from right to left, rightmost bit is bit 0. FIPS (and everyone else) goes from left to right starting at 1. I will use the FIPS conventions. To map Matsui positions to Everyone else:

M(i)= 64-EE(i). For 32 bits make the obvious change.

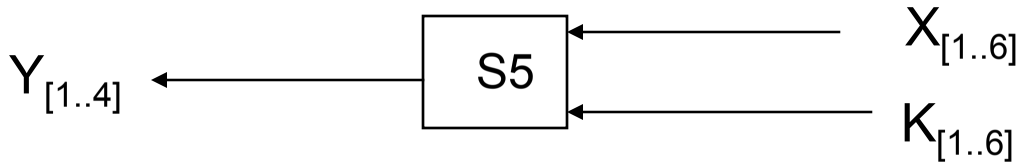
Matsui also refers to the two potions of the Plan and Ciphertext as

(P_H, P_L), (C_H, C_L) we'll stick with (P_L, P_R), (C_L, C_R).



Linear and near linear dependence

Here is a linear relationship over GF(2) in S5 that holds with probability 52/64 (from $NS_5(010000,1111)= 12$:



$X[2] \oplus Y[1] \oplus Y[2] \oplus Y[3] \oplus Y[4]=K[2] \oplus 1$,

Sometime written: $X[2] \oplus Y[1,2,3,4]=K[2] \oplus 1$

You can find relations like this using the "Boolean Function" techniques we describe a little later

Inside full round (after applying P), this becomes

$X[17] \oplus F(X,K)[3,8,14,25]= K[26] \oplus 1$

Linear Cryptanalysis of 3 round DES

$X[17] \oplus Y[3,8,14,25] = K[26] \oplus 1$, p= 52/64

- Round 1

$X_1[17] \oplus Y_1[3,8,14,25] = K_1[26] \oplus 1$

$P_R[17] \oplus P_L[3,8,14,25] \oplus R_1[3,8,14,25] = K_1[26] \oplus 1$

- Round 3

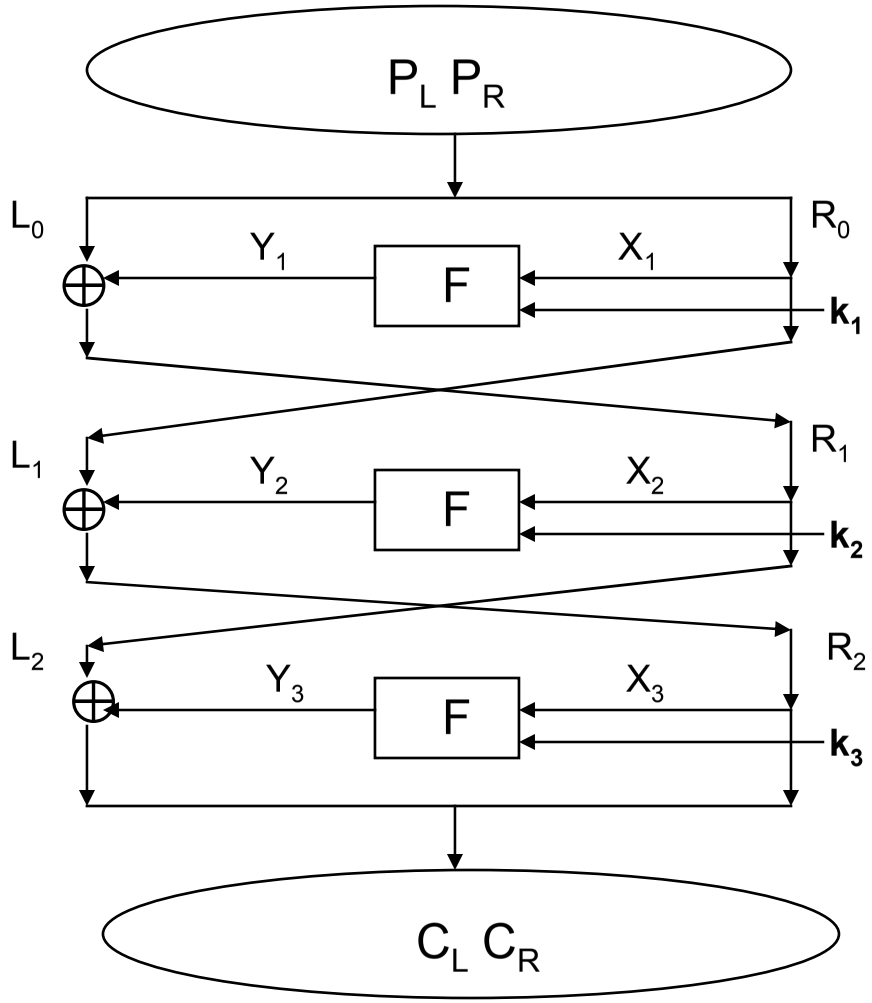
$X_3[17] \oplus Y_3[3,8,14,25] = K_3[26] \oplus 1$

$R_1[3,8,14,25] \oplus C_L[3,8,14,25] \oplus C_R[17] = K_3[26] \oplus 1$

- Adding the two get:

$P_R[17] \oplus P_L[3,8,14,25] \oplus C_L[3,8,14,25] \oplus C_R[17] = K_1[26] \oplus K_3[26]$

Thus holds with $p = (52/64)^2 + (12/64)^2 = .66$



Matsui's Per Round Constraints

Label	Equation	Pr
A	X[17] $\oplus$Y[3,8,14,25]=K[26]	12/64
B	X[1,2,4,5] $\oplus$Y[17]=K[2,3,5,6]	22/64
C	X[3] $\oplus$Y[17]=K[4]	30/64
D	X[17] $\oplus$Y[8,14,25]=K[26]	42/64
E	X[16,20] $\oplus$Y[8,14,25]=K[25,29]	16/64

Matsui: Linear Cryptanalysis Method for DES Cipher. Eurocrypt, 98.

Linear Cryptanalysis of 5 rounds

Pattern: BA-AB.

$P_L[17] \oplus R_1[17] \oplus P_R[1,2,4,5]=k_1[2,3,4,5]$

$L_1[3,8,14,25] \oplus R_2[3,8,14,25] \oplus R_1[17] = K_2[26]$

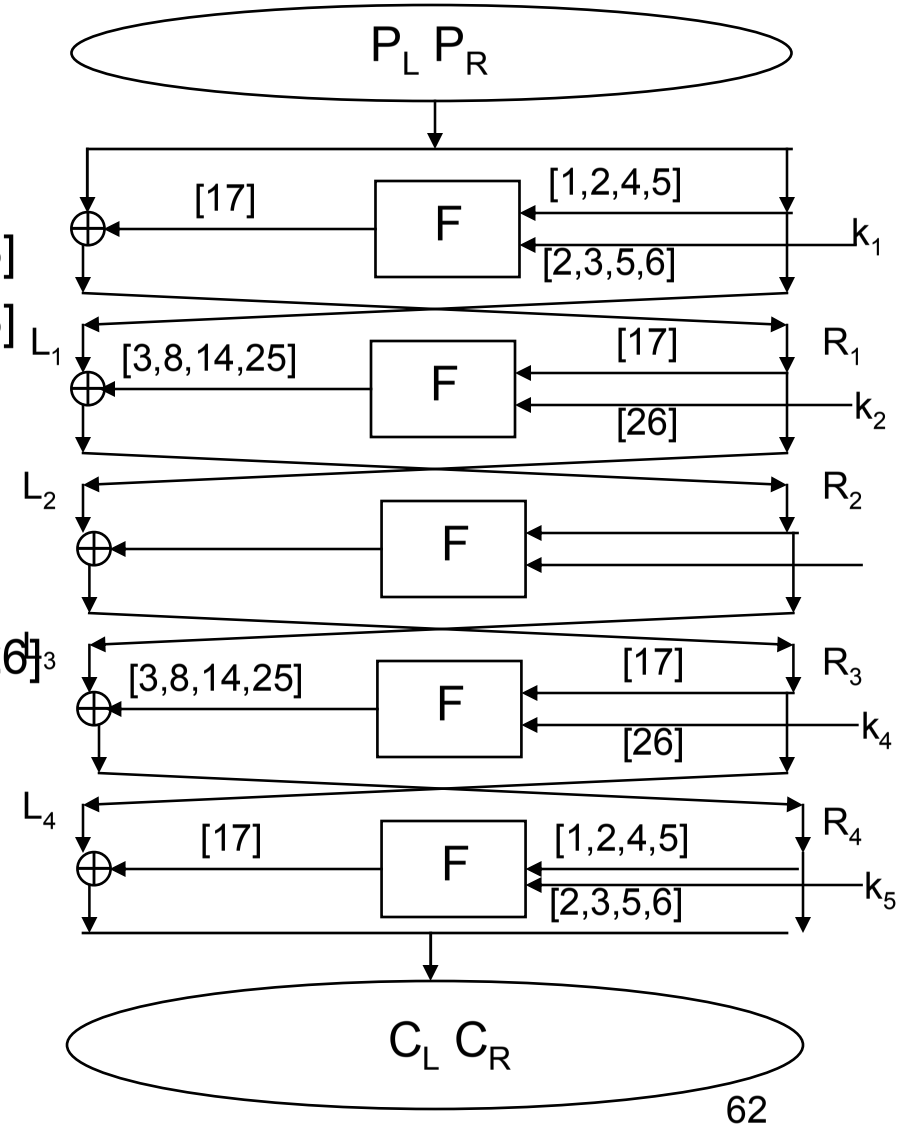
$R_2[3,8,14,25] \oplus C_R[3,8,14,25] \oplus R_3[17]= K_4[26]$

$R_3[17] \oplus C_L[17] \oplus C_R[1,2,4,5] = K_5[2,3,5,6]$

Adding yields

$P_L[17] \oplus P_R[1,2,4,5,3,8,14,25] \oplus C_L[17] \oplus C_R[1,2,4,5,3,8,14,25]=K_1[2,3,5,6] \oplus K_2[26] \oplus K_4[26] \oplus K_5[2,3,5,6]$

$P= p_A^2p_B^2+ (1-p_A)^2p_B^2+ p_A^2(1-p_B)^2+ (1-p_A)^2(1-p_B)^2+4p_Ap_B(1-p_A)(1-p_B)=0.519$



Piling Up Lemma

Let X_i $(1 \leqq i \leqq n)$ be independent random variables whose values are 0 with probability p_i. Then the probability that $X_1 \oplus X_2 \oplus ... \oplus X_n = 0$ is

$$½+2^{n-1} \prod_{[1,n]} (p_i-1/2)$$

Proof:

By induction on n. It's tautological for n=1.

Suppose $Pr[X_1 \oplus X_2 \oplus ... \oplus X_{n-1}= 0]= q= ½+2^{n-2} \prod_{[1,n-1]} (p_i-1/2)$. Then

$Pr[X_1 \oplus X_2 \oplus ... \oplus X_n= 0]= qp_n +(1-q)(1-p_n)=½+2^{n-1} \prod_{[1,n]} (p_i-1/2)$ as claimed.

Mathematics of biased voting

Central Limit Theorem. Let $X, X_1, \ldots, X_n$ be independent, identically distributed random variables and let $S_n = X_1 + X_2 + \ldots + X_n$. Let $\mu = E(X)$ and $\sigma^2 = E((X-\mu)^2)$. Finally set $T_n = (S_n - n\mu)/(\sigma\sqrt{n})$, $n(x) = 1/(\sqrt{2\pi})\exp(-x^2/2)$ and

$$N(a,b) = \int_{[a,b]} n(x)\,dx.$$

Then

$$Pr(a \leqq T_n \leqq b) = N(a,b).$$

n is called the Normal Distribution and is symmetric around x=0. $N(-\infty,0) = ½$.

N(-.5, .5)=.38, N(-.75,.75)= .55, N(-1,1)= .68, N(-2,2)=.9546, N(-3,3)= .9972

Application of CLT to LC

p= ½+ϵ, 1-p= ½-ϵ. Let L(k,P,E_k(P))=0 be an equation over GF(2) that holds with probability p. Let X_i be the outcome (1 if true, 0 if false) of an experiment picking P and testing whether L holds for the real k.

$E(X_i)= p$, $E((X_i-p)^2)= p(1-p)^2 + (1-p)(0-p)^2= p(1-p)$. Let T_n be as provided in the CLT.

Fixing n, what is the probability that more than half the X_i are 1 (i.e.- What is the probability that n random equations vote for the right key)?

This is just $Pr(T_n \geqq -\epsilon\sqrt{n}/\sqrt{(1/4 - \epsilon^2)})$. If $n=\delta^2\epsilon^{-2}$, this is just

$Pr(T_n \geqq -\delta/\sqrt{(1/4 - \epsilon^2)})$ or, if ϵ is small $Pr(T_n \geqq -2\delta)$.

Some numerical values: $\delta= .25, N(-.5, \infty) = .69, \delta= .5, N(-1, \infty) = .84,$

$\delta= 1, N(-2, \infty) = .98, \delta= 1.5, N(-3, \infty) = .999.$

15 Round Linear Approximation

Pattern: E-DCA-ACD-DCA-A. Note $L_i=R_{i-1}$, $L_i \oplus R_{i+1}=L_i \oplus L_{i+2}$.

1	$P_L[8,14,25]$	$\oplus$	$R_2[8,14,25]$	$\oplus$	$P_R[16,20]$	=	$K_1[23,25]$
3	$L_3[8,14,25]$	$\oplus$	$R_4[8,14,25]$	$\oplus$	$R_3[17]$	=	$K_3[26]$
4	$L_4[17]$	$\oplus$	$R_5[17]$	$\oplus$	$R_4[3]$	=	$K_4[4]$
5	$L_5[3,8,14,25]$	$\oplus$	$R_6[3,8,14,25]$	$\oplus$	$R_5[17]$	=	$K_5[26]$
7	$L_7[3,8,14,25]$	$\oplus$	$R_8[3,8,14,25]$	$\oplus$	$R_7[17]$	=	$K_7[26]$
8	$L_8[17]$	$\oplus$	$R_9[17]$	$\oplus$	$R_8[3]$	=	$K_8[4]$
9	$L_9[8,14,25]$	$\oplus$	$R_{10}[8,14,25]$	$\oplus$	$R_9[17]$	=	$K_9[26]$
11	$L_{11}[8,14,25]$	$\oplus$	$R_{12}[8,14,25]$	$\oplus$	$R_{11}[17]$	=	$K_{11}[26]$
12	$L_{12}[17]$	$\oplus$	$R_{13}[17]$	$\oplus$	$R_{12}[3]$	=	$K_{12}[4]$
13	$L_{13}[3,8,14,25]$	$\oplus$	$R_{14}[3,8,14,25]$	$\oplus$	$R_{13}[17]$	=	$K_{13}[26]$
15	$L_{15}[3,8,14,25]$	$\oplus$	$C_L[3,8,14,25]$	$\oplus$	$C_R[17]$	=	$K_{15}[26]$

15 Round Linear Approximation

Adding and canceling:

$$P_L[8,14,25] \oplus P_R[16,20] \oplus C_L[3,8,14,25] \oplus C_R[17] =$$
$$K_1[23,25] \oplus K_3[26] \oplus K_4[4] \oplus K_5[26] \oplus K_7[26] \oplus K_8[4]$$
$$\oplus K_9[26] \oplus K_{11}[26] \oplus K_{12}[4] \oplus K_{13}[26] \oplus K_{15}[26]$$

which holds (by Piling-up Lemma) with the indicated probability.

Matsui's Use of Constraints

Rounds	Equation	Pr	Equations Used
3	$P_L[3,8,14,25] \oplus P_R[17] \oplus C_L[3,8,14,25] \oplus C_R[17]=K_1[26] \oplus K_3[26]$	$\frac{1}{2}+1.56x2^{-3}$	A-A
5	$P_L[17] \oplus P_R[1,2,4,5,3,8,14,25] \oplus C_L[17] \oplus C_R[1,2,4,5,3,8,14,25]=K_1[2,3,5,6] \oplus K_2[26] \oplus K_4[26] \oplus K_5[2,3,5,6]$	$\frac{1}{2}+1.22x2^{-6}$	BA-AB
15	$P_L[8,14,25] \oplus P_R[16,20] \oplus C_L[3,8,14,25] \oplus C_R[17]=K_1[9,13] \oplus K_3[26] \oplus K_4[26] \oplus K_5[26] \oplus K_7[26] \oplus K_8[26] \oplus K_9[26] \oplus K_{11}[26] \oplus K_{12}[26] \oplus K_{13}[26] \oplus K_{15}[26]$	$\frac{1}{2}+1.19x2^{-22}$	E-DCA-ACD-DCA-A
16	$P_L[8,14,25] \oplus P_R[16,20] \oplus C_L[17] \oplus C_R[1,2,4,5,3,8,14,25]=K_1[9,13] \oplus K_3[26] \oplus K_4[26] \oplus K_5[26] \oplus K_7[26] \oplus K_8[26] \oplus K_9[26] \oplus K_{11}[26] \oplus K_{12}[26] \oplus K_{13}[26] \oplus K_{15}[26] \oplus K_{16}[2,3,5,6]$	$\frac{1}{2}-1.49x2^{-24}$	E-DCA-ACD-DCA-AB

Framework for differentials and linear approximation

- Differentials
  - $(\Delta X_i, \Delta Y_i) = \Pr[F_i(X_i \oplus \Delta X_i, K_i) \oplus F_i(X_i, K_i) = \Delta Y_i]$
  - $[p_1, p_2, \ldots, p_n] = \prod p_i$
  - $B_i = \text{Max}(\{\Delta X_i = \Delta X_{i-2} \oplus \Delta Y_{i-1}, 3 \leqq i \leqq n\})\ (\Delta X_1, \Delta Y_1)\ (\Delta X_2, \Delta Y_2) \ldots (\Delta X_n, Y_n)$
- Linear Approximations
  - $(\Gamma X_i, \Gamma Y_i) = \Pr[\Gamma X \cdot X_i \oplus \Gamma Y \cdot F_i(X_i, K_i) = 0] - .5$
  - $[p_1, p_2, \ldots, p_n] = 2^{t-1} \prod p_i$
  - $B_i = \text{Max}(\{\Gamma Y_i = \Gamma Y_{i-2} \oplus \Delta \Gamma X_{i-1}, 3 \leqq i \leqq n\})\ (\Gamma X_1, \Gamma Y_1)\ (\Gamma X_2, \Gamma Y_2) \ldots (\Gamma X_n, \Gamma Y_n)$

Matsui's algorithm for extending differentials and linear approximation

- Round -1
  Begin
    $p_1=\max_{\{\Delta Y\}}((\Delta X_1, \Delta Y))$
    If $[p_1, B_{n-1}] > AB_n$, call Round-2
  End
- Round-2
  Begin
    $p_2=\max_{\{\Delta X,\Delta Y\}}((\Delta X_2, \Delta Y_2))$
    If $[p_1, p_2, B_{n-2}] > AB_n$, call Round-3
  End
- Round –i ($3 \leqq i \leqq n$)
  Begin
    $\Delta X_i = \Delta X_{i-2} \oplus \Delta Y_{i-1}$
    $p_i=\max_{\{\Delta Y\}}((\Delta X_i, \Delta Y_i))$
    If $[p_1 , p_2, \ldots , p_i, B_{n-i}] > AB_n$, call Round-(i+1)
  End
- Round-n
  Begin
    $\Delta X_n = \Delta X_{n-2} \oplus \Delta Y_{n-1}$
    $p_n=\max_{\{\Delta X,\Delta Y\}}((\Delta X_n, \Delta Y))$
    If $[p_1 , p_2, \ldots , p_n] > AB_n$, then $AB_n = [p_1 , p_2, \ldots , p_n]$
  End

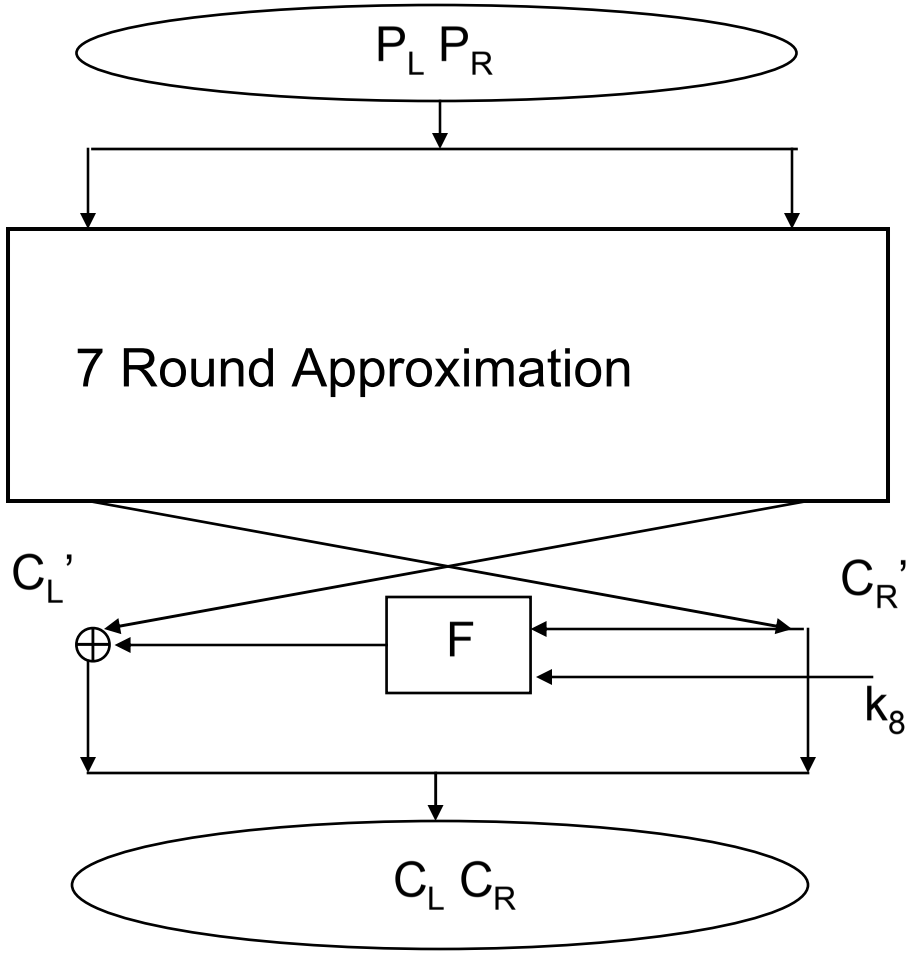
Linear Cryptanalysis of 8 rounds

7 Round approximation, p-½= 1.95 x 2^{-10}

$P_L[8,14,25] \oplus P_R[16,20] \oplus C_L[3,8,14,25] \oplus C_R[17] = K_1[9,13] \oplus K_3[26] \oplus K_4[4] \oplus K_5[26] \oplus K_7[26]$

$C_L = C_L' \oplus F(C_R', K_8)$

$C_R = C_L'$



Linear Cryptanalysis of full DES

Can be accomplished with ~2^{47} known plaintexts

- Using a slightly more sophisticated estimation 15 round approximation (with 2^{47} work factor)
  - For each 48 bit last round subkey, decrypt ciphertext backwards across last round for all sample ciphertexts
  - Increment count for all subkeys whose linear expression holds true to the penultimate round
  - This is done for the first and last round yielding 7 key bits each (total: 14)

Linear Cryptanalysis of full DES

Can be accomplished with ~2^{43} known plaintexts, using a more sophisticated estimation 14 round approximation

- For each 48 bit last round subkey, decrypt ciphertext backwards across last round for all sample ciphertexts
- Increment count for all subkeys whose linear expression holds true to the penultimate round
- This is done for the first and last round yielding 13 key bits each (total: 26)

- Here they are:

$P_R[8,14,25] \oplus C_L[3,8,14,25] \oplus C_R[17] = K_1[26] \oplus K_3[4] \oplus K_4[26] \oplus K_6[26] \oplus K_7[4] \oplus K_8[26] \oplus K_{10}[26] \oplus K_{11}[4] \oplus K_{12}[26] \oplus K_{14}[26]$ with probability $\frac{1}{2} - 1.19 \times 2^{-21}$

$C_R[8,14,25] \oplus P_L[3,8,14,25] \oplus P_R[17] = K_{13}[26] \oplus K_{12}[24] \oplus K_{11}[26] \oplus K_9[26] \oplus K_8[24] \oplus K_7[26] \oplus K_5[26] \oplus K_4[4] \oplus K_3[26] \oplus K_1[26]$ with probability $\frac{1}{2} - 1.19 \times 2^{-21}$

DesX

- DesX fixes two critical problems with DES:
  - Key size is too small
  - Practical Linear and Differential attacks rely on access to first and last rounds of iterated cipher
- DesX replaces the 56 bit DES key, k with a 184 bit key (k_i,k,k_o)
- $DESX(k_i,k,k_o;x)= k_o \oplus DES_k(x \oplus k_i)$
- Xoring key bits at input and output of cipher is called "whitening" and is used extensively by AES candidates
- DESX was proposed by Rivest and doesn't adversely affect the encryption/decryption speed

Brute Force Known-Plaintext Attacks on DES

- There are 2^{56} = about 7.2 x10^{16} possible keys
  - about 72 thousand trillion
- Diffie and Hellman - 1976
  - a $20 M machine with 1-million processors, each capable of testing 1 M DES keys/sec, could search 10^{12} keys/sec and all keys in a day.
- Michael Wiener - 1993
  - $1 M machine with 57,600 search chips, each testing 50 M keys/s, could break in 3.5 hours on average.
- EFF
  - 7/13/98, $10K RSA prize awarded to EFF which broke DES with $210K machine.

DES Attacks

- Differential cryptanalysis: This attack requires the encryption of 2^{47} chosen plaintexts
- Linear cryptanalysis. By means of this method, a DES key can be recovered by the analysis of 2^{43} known plaintexts.
- First Aid: use 3-DES or DES-X: $DESX(k_1,k_2,k_3,x)= k_2 \oplus DES(k_1,x\oplus k_3)$
  - This “whitens” input and output.

Other attacks on DES

- Multiple Linear Approximations, Knudsen & Robshaw
- Related Keys, Biham
  - Introduce controlled diffusion producing predictable effect that depends on related keys. Also applies to hash functions.
  - Applies to LOKI and Lucifer but not DES (because of irregular key selection schedule)
- Non-Linear Approximations, Knudsen and Robshaw

  Hard to knit constraints together, some help on first and last rounds
- Davies Attack, improved by Biham and Biryukov

  Output on adjacent S-boxes share keys and parity of non-shared keys have non-uniform distributions. Marginally better than exhaustive search
- Boomerang Attack, Slide attack, Wagner.

Triple-DES

Part of FIPS 46-3 standard

Encrypt: $C = E(K_3, D(K_2, E(K_1, P)))$

Decrypt: $P = D(K_3, E(K_2, D(K_1, C))$

3-key: K_1, K_2, K_3 all different

get 168-bit key

2-key: $K_1 = K_3$ (Two key version is subject to a time-memory trade-off attack)

get 112-bit key

better than double encryption

Post DES Design Criteria

- Use inverses in GF(2^k). (k=8 is popular) – Mars, Rijndael
- S-Boxes should be surjective
- If S-Boxes define permutations (π),
  - $DP_{max}(\pi) = \max_{\{a \neq 0, b\}} Pr(\pi(x \oplus a) \oplus \pi(x) = b) < 10/256$
  - $LP_{max}(\pi) = (2Pr(x \cdot a = \pi(a) \cdot b) - 1)^2 < 1/16$
  - Few Fixed points (3 for 256)

DES Summary

- DES was a great cipher.
- They should have stuck with a 64-bit key.
  - More secure
  - Better as a Hash primitive
- To make DES better.
  - *Increase key size*!
  - Pre and post whitening and non-key based mixing
  - Calculate differentials and dependencies and make them small enough so not enough plain/cipher is available for linear and differential attacks.

Homework 6 – Problem 1

S-box 4 is observed to have the indicated output xor when presented with the indicated inputs

```
In1: 0x22, In2: 0x16,  Output xor: 0x0c
In1: 0x12, In2: 0x0c , Output xor: 0x05
```

Perform a differential cryptanalysis and produce the possible candidate key(s). You may find the tables provided in “DC.txt” helpful.

Homework 6, problem 2 (omit)

Consider the 2 round iterative differential characteristic for DES
0x1960000000000→0x19600000000000, p=1/234

Suppose for the following questions we can always find chosen plaintext with S/N ratio high enough to require only 10 “right pairs” for a successful differential cryptanalysis (“DC”).

a. On average, how many chosen plain ciphertext pairs are required for a successful DC on two rounds?

b. On average, how many chosen plain ciphertext pairs are required for a successful DC on ten rounds?

c. After how many rounds is DC impossible because there cannot possibly be enough plain ciphertext pairs to succeed?

Homework 6 – Problem 3

A certain cipher X with 6 bit key k_1, k_2, k_3, k_4, k_5, k_6 has 4 linear constraints.

Given the corresponding plaintext, ciphertext pairs and substituting the equations become:

$0 = k_1 \oplus k_3 \oplus k_4$

$0 = k_4 \oplus k_5$

$0 = k_1 \oplus k_2$

$1 = k_1 \oplus k_6$

Guessing k_1 and k_3 calculate k_2, k_4, k_5, k_6. How many encryptions are needed to discover the correct key with exhaustive search in the worst case?

How many are needed with these constraints?

Homework 6, problem 4

(A) Suppose the cipher X has a linear constraint (Equation 1) that holds with probability *p*=.75 where the input to X is plaintext bits $i_1||i_2||\ldots||i_6$; the output is the ciphertext bits $o_1||o_2||\ldots||o_6$ under key bits $k_1||k_2||\ldots||k_6$. The constants $a_1, a_2, \ldots, a_6, b_1, b_2, \ldots, b_6, c_1, c_2, \ldots, c_6$, d are all known.

Equation 1: $a_1i_1 \oplus a_2i_2 \oplus a_3i_3 \oplus a_4i_4 \oplus a_5i_5 \oplus a_6i_6 \oplus$
$b_1o_1 \oplus b_2o_2 \oplus b_3o_3 \oplus b_4o_4 \oplus b_5o_5 \oplus b_6o_6 =$
$c_1k_1 \oplus c_2k_2 \oplus c_3k_3 \oplus c_4k_4 \oplus c_5k_5 \oplus c_6k_6 \oplus d$

Finally, suppose upon substituting values from 3 plaintext/ciphertext pairs the left hand side of equation 1 has values 1,1,0, respectively.

What are the odds that $c_1k_1 \oplus c_2k_2 \oplus c_3k_3 \oplus c_4k_4 \oplus c_5k_5 \oplus c_6k_6 \oplus d=1$ rather than 0?

(B) Suppose the same setup as in A but 3 out of 4 plaintext/ciphertext pairs "vote" that $c_1k_1 \oplus c_2k_2 \oplus c_3k_3 \oplus c_4k_4 \oplus c_5k_5 \oplus c_6k_6 \oplus d=1$.

What are the odds that $c_1k_1 \oplus c_2k_2 \oplus c_3k_3 \oplus c_4k_4 \oplus c_5k_5 \oplus c_6k_6 \oplus d=1$ rather than 0?

Homework 6, problem 4

(C) Constructing a multi-round constraint

Suppose X is a four round iterative cipher with plaintext input, P and ciphertext output C where each round has 6 bit input I and 6 bit output O and per round keys $K^{(1)}$, $K^{(2)}$,…$K^{(6)}$. Using Matsui's notation suppose the contraints:

$I[1,2] \oplus O[3,4] = K^{(1)}[1,3]$ R1

$I[3,4] \oplus O[1,5] = K^{(2)}[4,6]$ R2

$I[1,5] \oplus O[1,6] = K^{(3)}[1,5]$ R3

$I[1,6] \oplus O[2,5] = K^{(4)}[2]$ R4

hold with probabilities $p_1 = .8$, $p_2 = .9$, $p_3 = .8$, $p_4 = .9$, respectively.

What is the probability that

$P[1,2] \oplus C[2,5] = K^{(1)}[1,3] \oplus K^{(2)}[4,6] \oplus K^{(3)}[1,5] \oplus K^{(4)}[2]$?

Homework 6, problem 4

(D) Suppose X is a multi round iterative cipher with 40 bit plaintext input, P, and ciphertext output, C, and 40 bit key. Suppose, using Matsui's notation, that the following four linearly independent constraints:

i. $P[a_1^{(1)}, a_2^{(1)},\ldots, a_{40}^{(1)}] \oplus C[b_1^{(1)}, b_2^{(1)},\ldots, b_{40}^{(1)}] = K[c_1^{(1)}, c_2^{(1)},\ldots, c_{40}^{(1)}]$

ii. $P[a_1^{(2)}, a_2^{(2)},\ldots, a_{40}^{(2)}] \oplus C[b_1^{(2)}, b_2^{(2)},\ldots, b_{40}^{(2)}] = K[c_1^{(2)}, c_2^{(2)},\ldots, c_{40}^{(2)}]$

iii. $P[a_1^{(3)}, a_2^{(3)},\ldots, a_{40}^{(3)}] \oplus C[b_1^{(3)}, b_2^{(3)},\ldots, b_{40}^{(3)}] = K[c_1^{(3)}, c_2^{(3)},\ldots, c_{40}^{(3)}]$

iv. $P[a_1^{(4)}, a_2^{(4)},\ldots, a_{40}^{(4)}] \oplus C[b_1^{(4)}, b_2^{(4)},\ldots, b_{40}^{(4)}] = K[c_1^{(4)}, c_2^{(4)},\ldots, c_{40}^{(4)}]$

hold with probabilities $p_1 = .75$, $p_2 = .7$, $p_3 = .8$, $p_4 = .9$, respectively.

Suppose that on 10 plaintext/ciphertext pairs the LHS of i, ii, iii and iv "vote" that the RHS of the equations are 0 with tallies (2,8,2,8)

What is the probabilities that each of the most popular choices for the resulting constraints is correct? What is the probability that all 4 are correct? If all 4 are correct, and assuming X takes 1 microsecond/encrypt, what is the time to break X by exhaustive search (assuming a serial processor)? How about by applying the 4 constraints and searching for the remaining key bits (assuming a serial processor)?

PS: Key search is a "trivially parallelizable" operation.

Homework 6, problem 4

(E) In the lecture we noted that there was a linear attack that worked on 16 round DES with 2^{43} plaintext/ciphertext pairs where the basic constraint held with probability $p = \frac{1}{2} + \epsilon$ where $\epsilon = 1.19 \times 2^{-21}$ is the "bias". Using this fact, estimate for what p, there are not enough corresponding plain/cipher texts to enable applying the Linear cryptanalysis to reduce the search keyspace.

End Paper

- Done