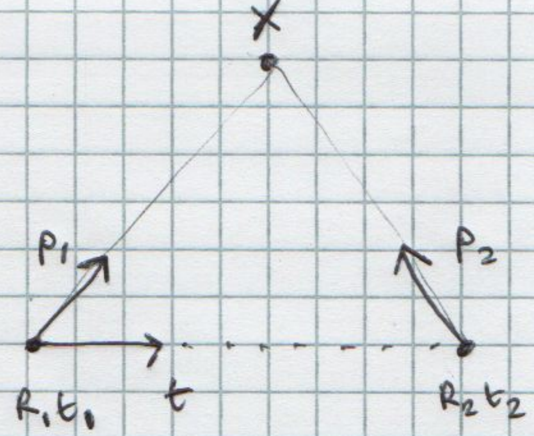
4.1



$$X = R_1 X_{c_1} + t_1$$
$$= R_2 X_{c_2} + t_2$$

$$\tilde{u}_1 = s\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = K_1 \begin{bmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} \end{bmatrix} = K \underline{X}_{c_1}$$

$$\tilde{u}_2 = K_2 \underline{X}_{c_2}$$

$P_2^T (t \times P_1) = 0$ camera rays + translation are coplanar

$$P_1 = R_1 X_{c_1} = R_1 K_1^{-1} \tilde{u}_1$$
$$P_2 = R_2 X_{c_2} = R_2 K_2^{-1} \tilde{u}_2$$
$$t = t_2 - t_1$$

$$\therefore \quad \tilde{u}_2 \underbrace{K_2^{-T} R_2^T (t_2 - t_1) \times R_1 K_1^{-1}}_{3\times 3 \text{ matrix } F} \tilde{u}_1 = 0$$

$$\tilde{u}_2 F \tilde{u}_1 = 0 \qquad [u_2 \; v_2 \; 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = 0$$

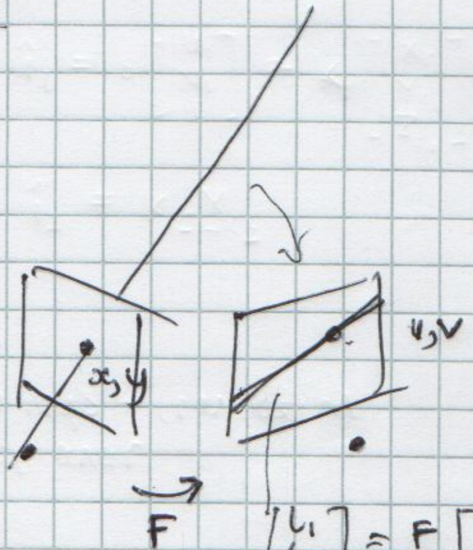
4.2

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} & & \\ & \underset{3\times 3}{F} & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$au + bv + c = 0$$

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = F \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = F \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

4.3

$$\check{u}_2^T \underset{\parallel}{F} \check{u}_1 = 0$$

$$F = \hat{u}_2^T K_2^{-T} R_2 (t_2 - t_1)^{\times} R_1 K_1^{-1} \check{u}_1$$

$$X_{c_1} = K_1^{-1} u_1 \quad X_{c_2} = K_2^{-1} u_2$$

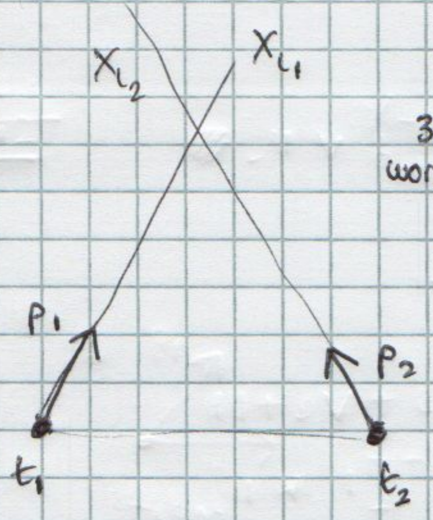
let $t_2 - t_1 = t \quad R_2 = I \quad R_1 = R$

$$X_{c_2}^T \underbrace{t \times R}_{E} X_{c_1} = 0$$

E essential matrix

this can be solved for t, R

4.4



3D world

$$X_{L_1} = t_1 + \lambda_1 p_1$$

$$X_{L_2} = t_2 + \lambda_2 p_2$$

$$\lambda_1^* \lambda_2^* = \arg\min_{\lambda_1 \lambda_2} \left| X_{L_1} - X_{L_2} \right|^2$$

$$= \arg\min_{\lambda_1 \lambda_2} \left| \underbrace{\begin{bmatrix} \vdots & \\ p_1 & -p_2 \\ \vdots & \end{bmatrix}}_{3\times 2} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + t_1 - t_2 \right|^2$$

of form $\min_{\underline{\lambda}} |A\underline{\lambda} + b|^2 \rightarrow$ linear least squares

---

or minimise image dist $\tilde{u}_1 = K_1 X_{c_1} = K_1 R_1^T (X - t_1)$

proj of X

$$X^* = \min_X d(\tilde{u}_1, m_1) + d(\tilde{u}_2, m_2)$$

measure (feature)

$$= |p(K_1 R_1^T (\underline{x} - t_1)) - m_1|^2 + |p(K_2 R_2^T (x - t_2) - m_2|^2$$

where $p\begin{pmatrix} x \\ y \\ s \end{pmatrix} = \begin{pmatrix} x/s \\ y/s \end{pmatrix}$ non-linear least squares.

4.5

$$r^k = \begin{pmatrix} u_2 \\ v_2 \end{pmatrix}^k - p\left(H_2 H_1^{-1} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}^k \right)$$

k^{th} residual

$$p\begin{pmatrix} x \\ y \\ s \end{pmatrix} = \begin{pmatrix} x/s \\ y/s \end{pmatrix}$$ projective → pixel

$$e = \underbrace{\sum_i \sum_j}_{\text{image pairs}} \sum_k |r_{ij}^k|^2$$

$\uparrow$ matches

$$\{H_1 H_2 \cdots H_n\}^* = \arg\min_{H_1 H_2 \cdots H_n} e$$

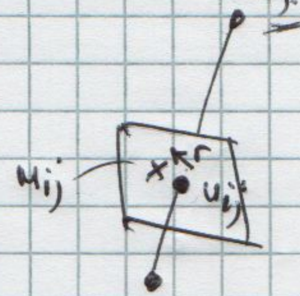
non linear in H

Levenberg Marquardt.

non linear LS.

$\frac{\partial e}{\partial h_{11}}$ etc.

4.6



$$r_{ij} = u_{ij} - \underline{\mu_{ij}}$$

i image / camera
j 3D point index

$$\underline{u_{ij}} = p\Big(\overbrace{K_i\,(\underbrace{R_i\,\underline{X_j} + t_i}_{X_{c_i}})}^{\tilde{u}_{ij}}\Big)$$

$$p\begin{pmatrix} x \\ y \\ s \end{pmatrix} = \begin{pmatrix} x/s \\ y/s \end{pmatrix}$$

$$e = \sum_{i \in \text{images}} \; \sum_{j \in \text{points}} \left| r_{ij}(R_i, t_i, X_j) \right|^2$$

min e wrt $R_1 R_2 R_3 \cdots \; t_1 t_2 t_3 \cdots \; X_1 X_2 X_3 \cdots X_n$

- non-linear LS solvers CERES