2.1

$$I_{corr}(x,y) = \underbrace{\int_t \int_s}_{\text{range of kernel}} I(x+s, y+t)\, k(s,t)\, ds\, dt$$

1D $$I_{corr}(x) = \int_s I(x+s)\, k(s)\, ds$$

I = 9 5 2 1 3 4 6 2 4

k = 1 2 1

↓

21 10 7 ...

2.2

$$I_{corr}(x,y) = \int_t \int_s I(x+s, y+t)\, k(s,t)\, ds\, dt$$

let $s = -a$, $t = -b$

$$I_{corr} = \int_b \int_a I(x-a, y-b)\, k(-a,-b)\, |J|\, da\, db = \cancel{I_{conv}}$$

I_{corr} = corr(I,k) ≠ conv(I,k')

$k'(a,b) = k(-a,-b)$

flipped in x,y == rotate 180°

2.3 $\frac{1}{4}[1\ 2\ 1] * [72\ \ 88\ \ 62\ \ 52\ \ 37]$ (0 … 0)

$$= \frac{1}{4}\begin{bmatrix} 1&2&1&0&0&0&0 \\ 0&1&2&1&0&0&0 \\ 0&0&1&2&1&0&0 \\ 0&0&0&1&2&1&0 \\ 0&0&0&0&1&2&1 \end{bmatrix} \begin{bmatrix} 0 \\ 72 \\ 88 \\ 62 \\ 52 \\ 37 \\ 0 \end{bmatrix}$$

linear,

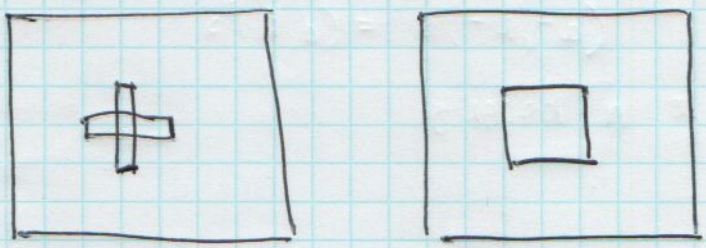
associative $a * (b * c) = (a * b) * c$

$$I' = \int_s \int_t I(x-s, y-t)\, k(s,t)\, ds\, dt \qquad a = x-s,\ b = y-t$$

$$= \int_a \int_b I(a,b)\, k(x-a, y-b)\, db\, da$$

$I * k = k * I$ → commutative.

2.4



$2n$ */+ $\qquad$ n^2 */+

$$I(x,y) * g(x,y) = I(x,y) * g(x) * g(y)$$

$$g(x)g(y) = g(x) * g(y)$$

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$$SSD = \sum_{R} |I(x+\Delta x) - I(x)|^2$$

region of patch 16×16 ?

$(|a|^2 = a^T a)$

$$I(x+\Delta x) = I(x) + \nabla I^T \Delta x + \dots$$

$$SSD \approx \sum_{R} |\nabla I^T \Delta x|^2 = \Delta x^T \left(\sum_{R} \nabla I \nabla I^T \right) \Delta x = \Delta x^T H \Delta x$$

$$H = \sum_{R} \nabla I \nabla I^T \qquad \nabla I = \begin{pmatrix} I_x \\ I_y \end{pmatrix} \begin{matrix} \leftarrow \text{grd in } x \\ \leftarrow \text{grd in } y \end{matrix}$$

$$= \sum_{R} \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

$$I(x,y) * (1,-1) * g_{\sigma_d}(x,y) = I_x$$

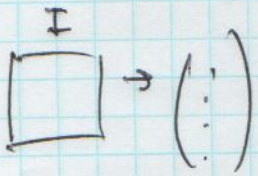
$$f_{Harris} = \det(H) - k\,Tr(H)^2 \qquad \text{req both } \lambda_1, \lambda_2 \text{ large}$$

derivative scale σ_d used for $I_x I_y$

integration scale σ_I used for summation/smoothing

$$H = \sum_{R} (\dots) \longrightarrow g_{\sigma_I}(x,y) * \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

2.6 Normalised correlation $\frac{I^T J}{|I||J|}$



$$SSD = |I - J|^2 = (I-J)^T(I-J)$$
$$= |I|^2 + |J|^2 - 2I^T J$$

if $|I| = |J| = 1$

then $SSD = 2 - 2\,CORR$