Stereo

CSE P 576

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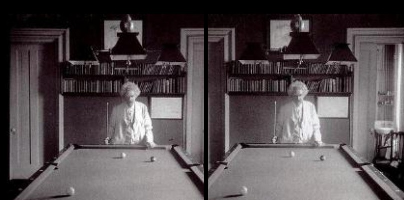
Many slides courtesy of Steve Seitz

Thomas Edison



Mark Twain, 1908

Why do we perceive depth?

What do humans use as depth cues?

Motion

Convergence
When watching an object close to us, our eyes point slightly inward. This difference in the direction of the eyes is called convergence. This depth cue is effective only on short distances (less than 10 meters).

Binocular Parallax
As our eyes see the world from slightly different locations, the images sensed by the eyes are slightly different. This difference in the sensed images is called binocular parallax. Human visual system is very sensitive to these differences, and binocular parallax is the most important depth cue for medium viewing distances. The sense of depth can be achieved using binocular parallax even if all other depth cues are removed.

Monocular Movement Parallax
If we close one of our eyes, we can perceive depth by moving our head. This happens because human visual system can extract depth information in two similar images sensed after each other, in the same way it can combine two images from different eyes.

Focus

Accommodation
Accommodation is the tension of the muscle that changes the focal length of the lens of eye. Thus it brings into focus objects at different distances. This depth cue is quite weak, and it is effective only at short viewing distances (less than 2 meters) and with other cues.

Marko Teittinen http://www.hitl.washington.edu/scivw/EVE/III.A.1.c.DepthCues.html

What do humans use as depth cues?

Image cues

Retinal Image Size
When the real size of the object is known, our brain compares the sensed size of the object to this real size, and thus acquires information about the distance of the object.

Linear Perspective
When looking down a straight level road we see the parallel sides of the road meet in the horizon. This effect is often visible in photos and it is an important depth cue. It is called linear perspective.

Texture Gradient
The closer we are to an object the more detail we can see of its surface texture. So objects with smooth textures are usually interpreted being farther away. This is especially true if the surface texture spans all the distance from near to far.

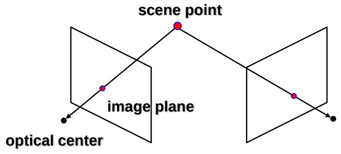
Overlapping
When objects block each other out of our sight, we know that the object that blocks the other one is closer to us. The object whose outline pattern looks more continuous is felt to lie closer.

Aerial Haze
The mountains in the horizon look always slightly bluish or hazy. The reason for this are small water and dust particles in the air between the eye and the mountains. The farther the mountains, the hazier they look.

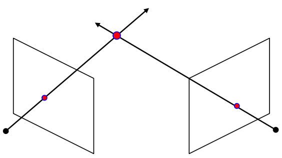
Shades and Shadows
When we know the location of a light source and see objects casting shadows on other objects, we learn that the object shadowing the other is closer to the light source. As most illumination comes downward we tend to resolve ambiguities using this information. The three dimensional looking computer user interfaces are a nice example on this. Also, bright objects seem to be closer to the observer than dark ones.

Marko Teittinen http://www.hitl.washington.edu/scivw/EVE/III.A.1.c.DepthCues.html

Stereo



Stereo



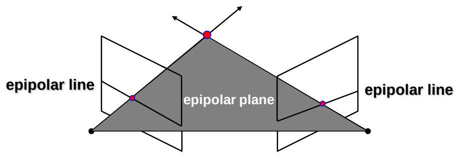
Basic Principle: Triangulation

- Gives reconstruction as intersection of two rays
- Requires
  - camera pose (calibration)
  - ***point correspondence***

Stereo correspondence

Determine Pixel Correspondence

- Pairs of points that correspond to same scene point

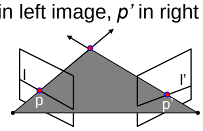


Epipolar Constraint

- Reduces correspondence problem to 1D search along *conjugate epipolar lines*
- Java demo: http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

Fundamental matrix

Let *p* be a point in left image, *p'* in right image



Epipolar relation

- *p* maps to epipolar line *l'*
- *p'* maps to epipolar line *l*

Epipolar mapping described by a 3x3 matrix *F*

$$\begin{aligned} l' &= Fp \\ l &= p'F \end{aligned}$$

It follows that

$$p'Fp = 0$$

Fundamental matrix

This matrix F is called

- the "Essential Matrix"
  - when image intrinsic parameters are known
- the "Fundamental Matrix"
  - more generally (uncalibrated case)

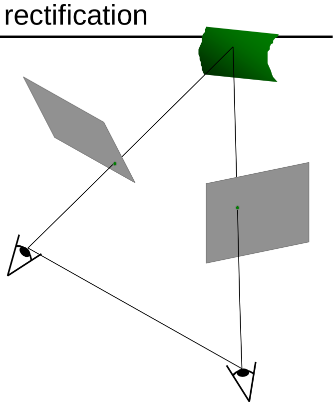
Can solve for F from point correspondences

- Each (p, p') pair gives one linear equation in entries of F

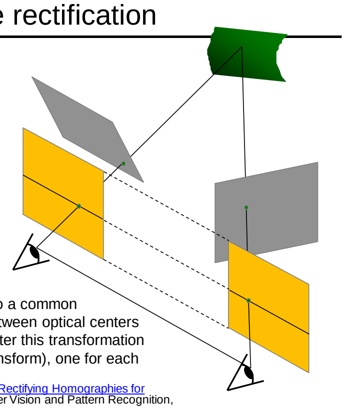
$$p'Fp = 0$$

- 8 points give enough to solve for F (8-point algorithm)
- see Marc Pollefey's notes for a nice tutorial

Stereo image rectification

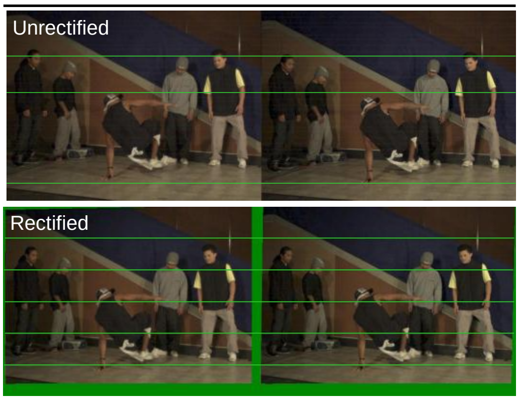


Stereo image rectification

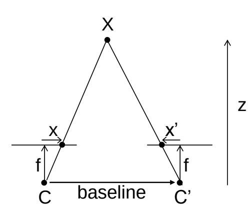


- reproject image planes onto a common plane parallel to the line between optical centers
- pixel motion is horizontal after this transformation
- two homographies (3x3 transform), one for each input image reprojection
  - C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). IEEE Conf. Computer Vision and Pattern Recognition, 1999.

Example



Depth from disparity



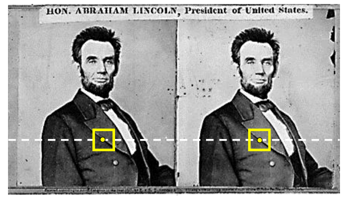
$$disparity = x - x' = \frac{baseline * f}{z}$$

Stereo matching algorithms

Match Pixels in Conjugate Epipolar Lines

- Assume brightness constancy
- This is a tough problem
- Numerous approaches
  - A good survey and evaluation: http://www.middlebury.edu/stereo/

Your basic stereo algorithm



For each epipolar line

For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match ***windows***

- This should look familar...

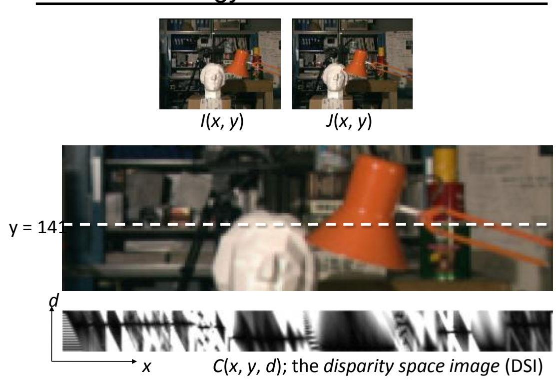
Stereo as energy minimization

- Find disparities *d* that minimize an energy function $E(d)$
- Simple pixel / window matching

$$E(d) = \sum_{(x,y)\in I} C(x, y, d(x, y))$$

$C(x, y, d(x, y))$ = SSD distance between windows *I*(*x*, *y*) and *J*(*x*, *y* + *d*(*x*,*y*))

Stereo as energy minimization



C(*x*, *y*, *d*); the *disparity space image* (DSI)

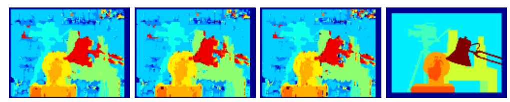
Stereo as energy minimization

Simple pixel / window matching: choose the minimum of each column in the DSI independently:

$$d(x,y) = \arg\min_{d'} \; C(x,y,d')$$

Matching windows

Similarity Measure	Formula
Sum of Absolute Differences (SAD)	$\sum_{(i,j)\in W} \lvert I_1(i,j) - I_2(x+i, y+j)\rvert$
Sum of Squared Differences (SSD)	$\sum_{(i,j)\in W} \left(I_1(i,j) - I_2(x+i,y+j)\right)^2$
Zero-mean SAD	$\sum_{(i,j)\in W} \lvert I_1(i,j) - \bar{I}_1(i,j) - I_2(x+i,y+j) + \bar{I}_2(x+i,y+j)\rvert$
Locally scaled SAD	$\sum_{(i,j)\in W} \lvert I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i,y+j)} I_2(x+i,y+j)\rvert$
Normalized Cross Correlation (NCC)	$\frac{\sum_{(i,j)\in W} I_1(i,j).I_2(x+i,y+j)}{\sqrt[2]{\sum_{(i,j)\in W} I_1^2(i,j).\sum_{(i,j)\in W} I_2^2(x+i,y+j)}}$

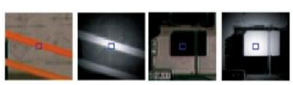


SAD SSD NCC Ground truth

http://siddhantahuja.wordpress.com/category/stereo-vision/

More window techniques

Bilateral filtering



Adaptive weighting

K.-J. Yoon and I.-S. Kweon. Adaptive support-weight approach for correspondence search. PAMI 28(4):650-656, 2006

Yoon and Kweon

Ground truth

Window size

W = 3 W = 20

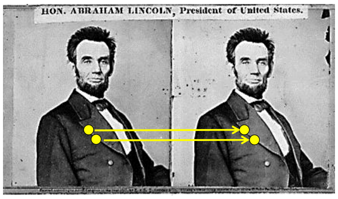
Effect of window size

- Smaller window
  - +
  - –
- Larger window
  - +
  - –

Better results with *adaptive window*

- T. Kanade and M. Okutomi, *A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment*,, Proc. International Conference on Robotics and Automation, 1991.
- D. Scharstein and R. Szeliski. Stereo matching with nonlinear diffusion. International Journal of Computer Vision, 28(2):155-174, July 1998

Stereo as energy minimization



What defines a good stereo correspondence?

1. Match quality
   - Want each pixel to find a good match in the other image
2. Smoothness
   - If two pixels are adjacent, they should (usually) move about the same amount

Stereo as energy minimization

Better objective function

$$E(d) = \underbrace{E_d(d)}_{\text{match cost}} + \lambda \underbrace{E_s(d)}_{\text{smoothness cost}}$$

Want each pixel to find a good match in the other image

Adjacent pixels should (usually) move about the same amount

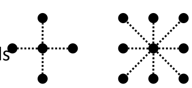
Stereo as energy minimization

$$E(d) = E_d(d) + \lambda E_s(d)$$

match cost: $$E_d(d) = \sum_{(x,y)\in I} C(x, y, d(x, y))$$

smoothness cost: $$E_s(d) = \sum_{(p,q)\in \mathcal{E}} V(d_p, d_q)$$

$\mathcal{E}$: set of neighboring pixels



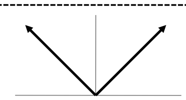
4-connected neighborhood 8-connected neighborhood

Smoothness cost

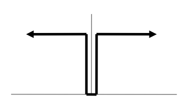
$$E_s(d) = \sum_{(p,q)\in \mathcal{E}} V(d_p, d_q)$$

$$V(d_p, d_q) = |d_p - d_q|$$

L_1 distance



$$V(d_p, d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$



“Potts model”

Dynamic programming

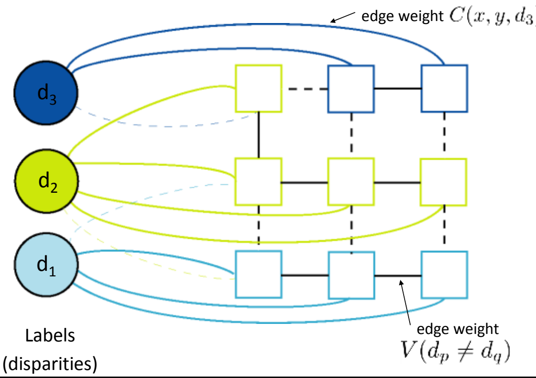
$$E(d) = E_d(d) + \lambda E_s(d)$$

Can minimize this independently per scanline using dynamic programming (DP)

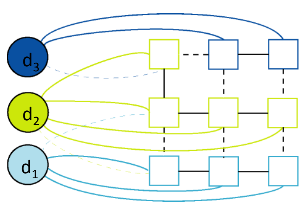
$D(x,y,d)$: minimum cost of solution such that *d*(*x*,*y*) = *d*

$$D(x,y,d) = C(x,y,d) + \min_{d'} \{D(x-1,y,d') + \lambda |d - d'|\}$$

Energy minimization via graph cuts



Energy minimization via graph cuts



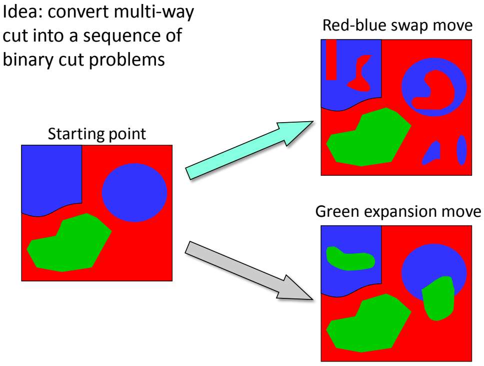
- Graph Cut
  - Delete enough edges so that
    - each pixel is connected to exactly one label node
  - Cost of a cut: sum of deleted edge weights
  - Finding min cost cut equivalent to finding global minimum of energy function

Computing a multiway cut

- With 2 labels: classical min-cut problem
  - Solvable by standard flow algorithms
    - polynomial time in theory, nearly linear in practice
  - More than 2 terminals: NP-hard
    [Dahlhaus *et al.*, STOC '92]
- Efficient approximation algorithms exist
  - Boykov, Veksler and Zabih, Fast Approximate Energy Minimization via Graph Cuts, ICCV 1999.
  - Within a factor of 2 of optimal
  - Computes local minimum in a strong sense
    - even very large moves will not improve the energy

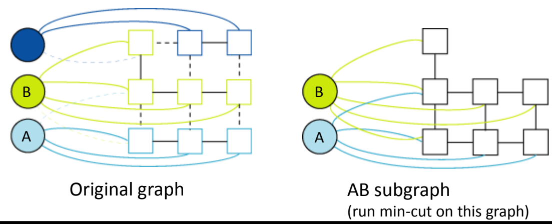
Move examples

Idea: convert multi-way cut into a sequence of binary cut problems



The swap move algorithm

1. Start with an arbitrary labeling
2. Cycle through every label pair *(A,B)* in some order
   2.1 Find the lowest *E* labeling within a single *AB*-swap
   2.2 Go there if it's lower *E* than the current labeling
3. If *E* did not decrease in the cycle, we're done
   Otherwise, go to step 2



Alpha-expansion

Similar to swap move algorithm, except it's one label vs. all others.

Other energy functions

Can optimize other functions (exactly or approximately) with graph cuts

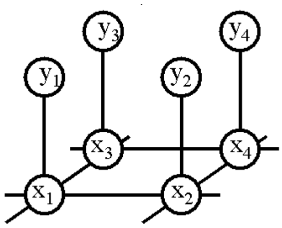
$$V(d_p, d_q) = (d_p - d_q)^2$$
$$V(d_p, d_q) = |d_p - d_q|$$

But many functions are much harder…

Markov Random Fields

Allows rich probabilistic models for images.

But built in a local, modular way. Learn local relationships, get global effects



MRF slides courtesy of Antonio Torralba

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Network joint probability

$$P(x,y) = \frac{1}{Z} \prod_{i,j} \Psi(x_i, x_j) \prod_i \Phi(x_i, y_i)$$

disparity

images

Disparity-disparity compatibility function

neighboring disparity nodes

Images-disparity compatibility function

local observations

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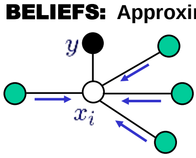
Belief Propagation

BELIEFS: Approximate posterior marginal distributions

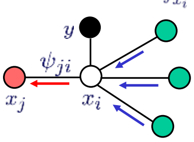


$$\widehat{p}(x_i \mid y) \propto \psi_i(x_i, y) \prod_{k \in \Gamma(i)} m_{ki}(x_i)$$

$\Gamma(i) \longrightarrow$ ***neighborhood* of node *i***

MESSAGES: Approximate sufficient statistics

$$m_{ij}(x_j) \propto \int_{x_i} \psi_{j,i}(x_j, x_i) \psi_i(x_i, y) \prod_{k \in \Gamma(i) \setminus j} m_{ki}(x_i)\, dx_i$$

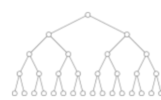


I. Belief Update (Message Product)

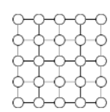
II. Message Propagation (Convolution)

Justifications for BP

- **Gives *exact* marginals for trees**
  - → ***Optimal estimates***
  - → ***Confidence measures***



- For general graphs, *loopy BP* has excellent empirical performance in many applications
- Recent theory provides some guarantees:
  - Statisical physics: *variational method* *(Yedidia, Freeman, & Weiss)*
  - BP as reparameterization: *error bounds* *(Wainwright, Jaakkola, & Willsky)*
  - Many others…



References on BP and GBP

J. Pearl, 1985
- classic

Y. Weiss, NIPS 1998
- Inspires application of BP to vision

W. Freeman et al learning low-level vision, IJCV 1999
- Applications in super-resolution, motion, shading/paint discrimination

H. Shum et al, ECCV 2002
- Application to stereo

M. Wainwright, T. Jaakkola, A. Willsky
- Reparameterization version

J. Yedidia, AAAI 2000
- The clearest place to read about BP and GBP.

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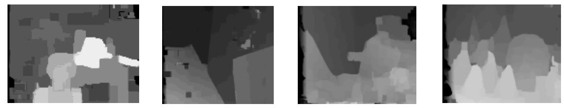
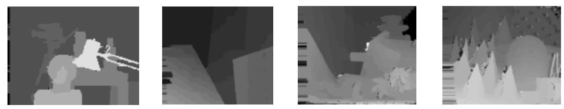
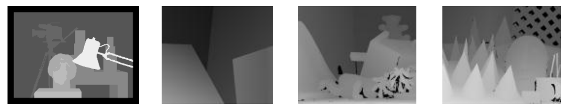
MRF results



SSD

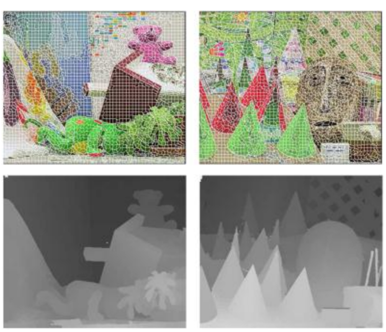


V. Kolmogorov and R. Zabih. Computing visual correspondence with occlusions using graph cuts. ICCV 2001.



Ground truth

Segmentation approaches



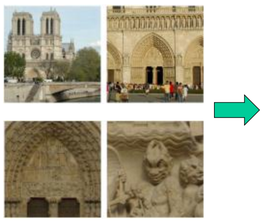
L. Zitnick, S.B. Kang, M. Uyttendaele, S. Winder, and R. Szeliski. High-quality video view interpolation using a layered representation. SIGGRAPH 2004.

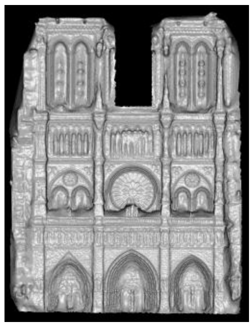
Real-time stereo

Nomad robot searches for meteorites in Antartica
http://www.frc.ri.cmu.edu/projects/meteorobot/index.html

Used for robot navigation (and other tasks)
- Several software-based real-time stereo techniques have been developed (most based on simple discrete search)

Using more than two images





[Multi-View Stereo for Community Photo Collections](#)
M. Goesele, N. Snavely, B. Curless, H. Hoppe, S. Seitz
Proceedings of [ICCV 2007](#).

Why does stereo fail?

Fronto-Parallel Surfaces: Depth is constant within the region of local support

Why does stereo fail?

Monotonic Ordering - Points along an epipolar scanline appear in the same order in both stereo images
Occlusion – All points are visible in each image

Why does stereo fail?

Image Brightness Constancy: Assuming Lambertian surfaces, the brightness of corresponding points in stereo images are the same.

Why does stereo fail?

Match Uniqueness: For every point in one stereo image, there is at most one corresponding point in the other image.

Stereo reconstruction pipeline

Steps

- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

What will cause errors?

- Camera calibration errors
- Poor image resolution
- Occlusions
- Violations of brightness constancy (specular reflections)
- Large motions
- Low-contrast image regions