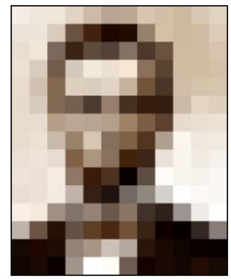
Filtering

CSE P 576

Larry Zitnick (larryz@microsoft.com)

What do computers see?

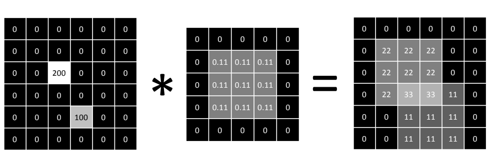


Images can be viewed as a 2D function.

Image filtering

Linear filtering = Applying a local function to the image using a sum of weighted neighboring pixels.

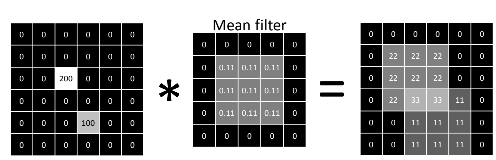
Such as blurring:



Input image Kernel Output image

Image filtering

$$g(x,y) = \sum_{x'} \sum_{y'} f(x+x', y+y') h(x', y')$$



Input image f Filter h Output image g

Image filtering

- Linear filters can have arbitrary weights.
- Typically they sum to 0 or 1, but not always.
- Weights may be positive or negative.
- Many filters aren't linear (median filter.)

0	0	0	0	0
0	0	0	0	0
0	0	0	1	0
0	0	0	0	0
0	0	0	0	0

What does this filter do?

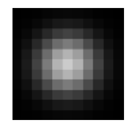
Gaussian filter

$$\mathcal{G}_\sigma(x,y) = \frac{1}{Z} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

Compute empirically

Input image *f*

$*$

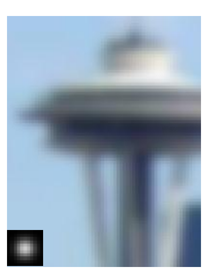


Filter *h*

$=$

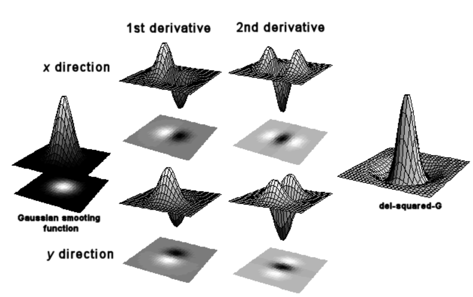
Output image *g*

Gaussian vs. mean filters



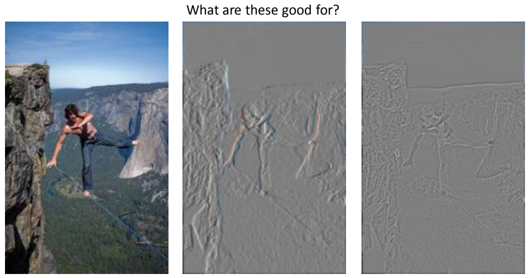
What does real blur look like?

First and second derivatives

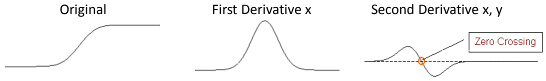


First and second derivatives

What are these good for?

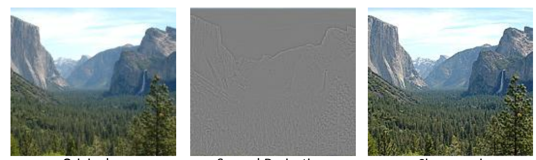


Original First Derivative x Second Derivative x, y



Subtracting filters

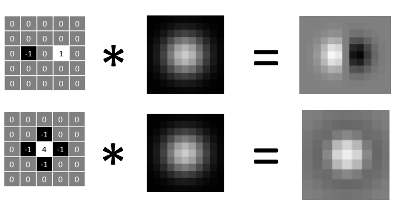
$$Sharpen(x,y) = f(x,y) - \alpha(f * \nabla^2 \mathcal{G}_\sigma(x,y))$$



Original Second Derivative Sharpened

Combining filters

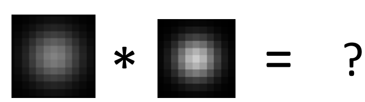
$$f * g * g' = f * h \text{ for some } h$$



It's also true: $f * (g * h) = (f * g) * h$

$$f * g = g * f$$

Combining Gaussian filters



$$f * \mathcal{G}_\sigma * \mathcal{G}_{\sigma'} = f * \mathcal{G}_{\sigma''}$$

$$\sigma'' = \sqrt{\sigma^2 + \sigma'^2}$$

More blur than either individually (but less than $\sigma'' = \sigma + \sigma'$)

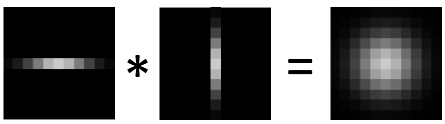
Separable filters

$$\mathcal{G}_\sigma = \mathcal{G}_\sigma^x * \mathcal{G}_\sigma^y$$

$$\mathcal{G}_\sigma^x(x,y) = \frac{1}{Z} e^{\frac{-(x^2)}{2\sigma^2}}$$

$$\mathcal{G}_\sigma^y(x,y) = \frac{1}{Z} e^{\frac{-(y^2)}{2\sigma^2}}$$

Compute Gaussian in horizontal direction, followed by the vertical direction. **Much faster!**



Not all filters are separable.

Freeman and Adelson, 1991

Sums of rectangular regions

How do we compute the sum of the pixels in the red box?

After some pre-computation, this can be done in constant time for any box.

This "trick" is commonly used for computing Haar wavelets (a fundemental building block of many object recognition approaches.)

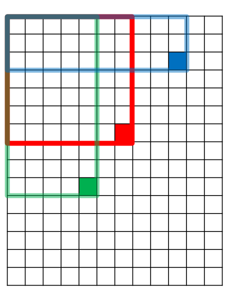
243	239	240	225	206	185	188	218	211	206	216	225
242	239	218	110	67	31	34	152	213	206	208	221
243	242	123	58	94	82	132	77	108	208	208	215
235	217	115	212	243	236	247	139	91	209	208	211
233	208	131	222	219	226	196	114	74	208	213	214
232	217	131	116	77	150	69	56	52	201	228	223
232	232	182	186	184	179	159	123	93	232	235	235
232	236	201	154	216	133	129	81	175	252	241	240
235	238	230	128	172	138	65	63	234	249	241	245
237	236	247	143	59	78	10	94	255	248	247	251
234	237	245	193	55	33	115	144	213	255	253	251
248	245	161	128	149	109	138	65	47	156	239	255
190	107	39	102	94	73	114	58	17	7	51	137
23	32	33	148	168	203	179	43	27	17	12	8
17	26	12	160	255	255	109	22	26	19	35	24

Sums of rectangular regions

The trick is to compute an "integral image." Every pixel is the sum of its neighbors to the upper left.

Sequentially compute using:

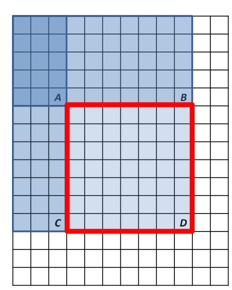
$$I(x,y) = I(x-1,y) + I(x,y-1) - I(x-1,y-1)$$



Sums of rectangular regions

Solution is found using:

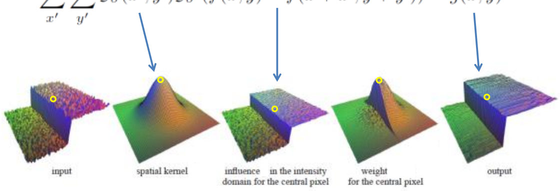
A + D – B - C



Spatially varying filters

Some filters vary spatially.

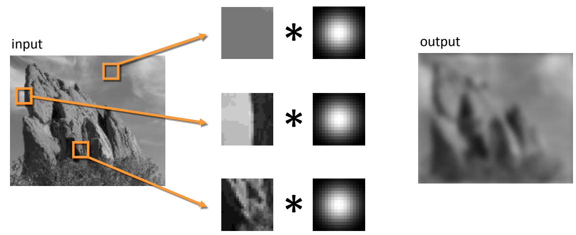
$$\sum_{x'}\sum_{y'} \mathcal{G}_{\sigma}(x',y')\mathcal{G}_{\sigma'}(f(x,y)-f(x+x',y+y'))=g(x,y)$$



Durand, 02

Useful for deblurring.

Constant blur

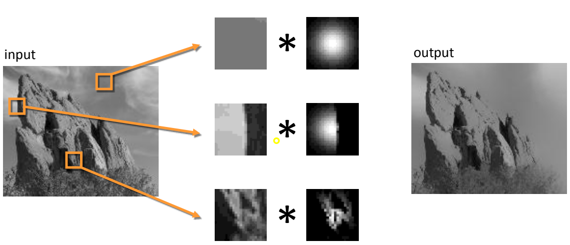


Same Gaussian kernel everywhere.

Slides courtesy of Sylvian Paris

Bilateral filter

Maintains edges when blurring!



The kernel shape depends on the image content.

Slides courtesy of Sylvian Paris

Borders

What to do about image borders:

black fixed periodic reflected

Sampling

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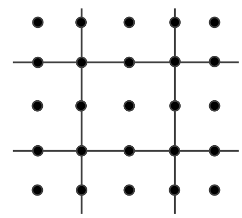
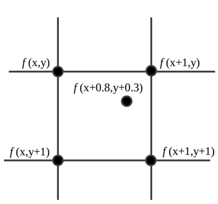
Up-sampling

How do we compute the values of pixels at fractional positions?



Up-sampling

How do we compute the values of pixels at fractional positions?

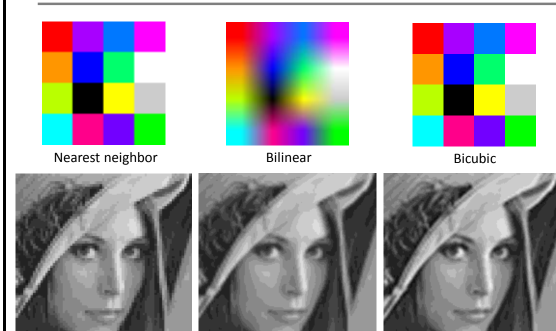


Bilinear sampling:

$$f(x + a, y + b) = (1 - a)(1 - b)\, f(x, y) + a(1 - b)\, f(x + 1, y) + (1 - a)b\, f(x, y + 1) + ab\, f(x + 1, y + 1)$$

Bicubic sampling fits a higher order function using a larger area of support.

Up-sampling

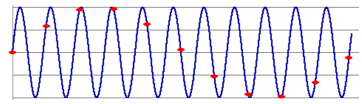


Down-sampling

If you do it incorrectly your images could look like this:

Check out Moire patterns on the web.

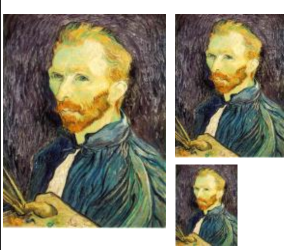
Down-sampling



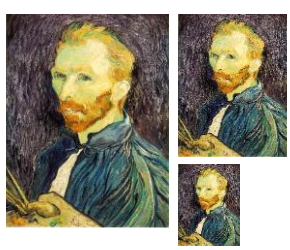
- **Aliasing** can arise when you sample a continuous signal or image
  - occurs when your sampling rate is not high enough to capture the amount of detail in your image
  - Can give you the wrong signal/image—an *alias*
  - formally, the image contains structure at different scales
    - called "frequencies" in the Fourier domain
  - the sampling rate must be high enough to capture the highest frequency in the image

Solution

Filter before sampling, i.e. blur the image first.



With blur



Without blur