4. Image processing

Reading

Jain, Kasturi, Schunck, *Machine Vision*. McGraw-Hill, 1995. Chapter 4 and Sections 5.1-5.4.

What is an image?

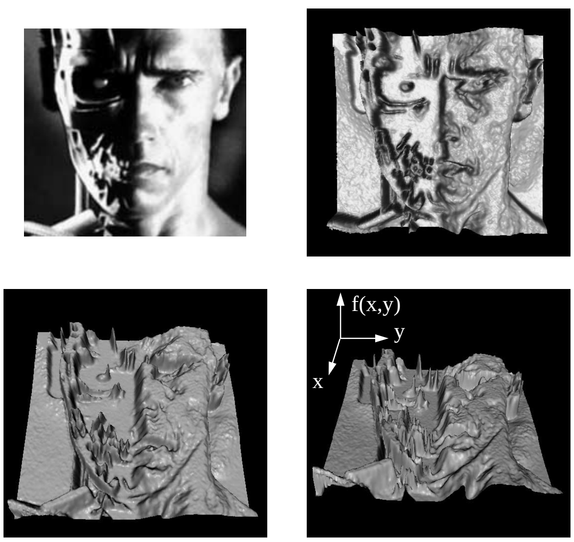
We can think of an **image** as a function, *f*, from R^2 to R:

- $f(x, y)$ gives the intensity of a channel at position (x, y)
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - f: $[a,b]\times[c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

Images as functions



What is a digital image?

In computer graphics, we usually operate on **digital** (**discrete**) images:

- **Sample** the space on a regular grid
- **Quantize** each sample (round to nearest integer)

If our samples are Δ apart, we can write this as:

$f[i, j] = \text{Quantize}\{ f(i\,\Delta, j\,\Delta) \}$

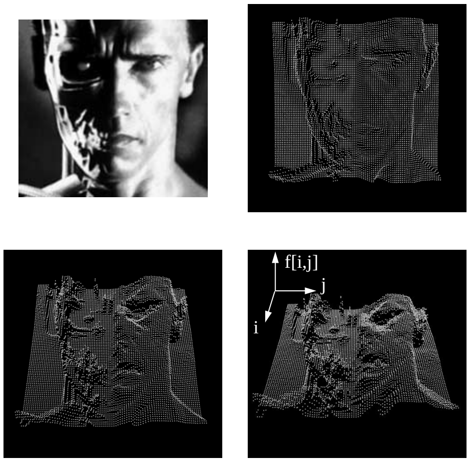


Image processing

An **image processing** operation typically defines a new image g in terms of an existing image f.

The simplest operations are those that transform each pixel in isolation. These pixel-to-pixel operations can be written:

$$g(x, y) = t(f(x, y))$$

Example: threshold, RGB ➔ grayscale

Note: a typical choice for mapping to grayscale is to apply the YIQ television matrix and keep the Y.

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Pixel movement

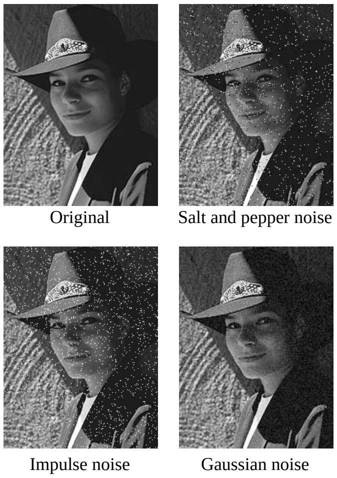
Some operations preserve intensities, but move pixels around in the image

$$g(x, y) = f(\tilde{x}(x, y), \tilde{y}(x, y))$$

Examples: many amusing warps of images

Noise

Image processing is also useful for noise reduction and edge enhancement. We will focus on these applications for the remainder of the lecture…



Common types of noise:

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Noise reduction

How can we "smooth" away noise?

Is there a more abstract way to represent this sort of operation? *Of course there is!*

Convolution

One of the most common methods for filtering an image is called **convolution**.

In 1D, convolution is defined as:

$$
\begin{aligned}
g(x) &= f(x) * h(x) \\
&= \int_{-\infty}^{\infty} f(x')h(x-x')dx' \\
&= \int_{-\infty}^{\infty} f(x')\tilde{h}(x'-x)dx'
\end{aligned}
$$

where $\tilde{h}(x) = h(-x)$.

Example:

Discrete convolution

For a digital signal, we define **discrete convolution** as:

$$
\begin{aligned}
g[i] &= f[i] * h[i] \\
&= \sum_j f[j]h[j-i] \\
&= \sum_j f[j]\tilde{h}[i-j]
\end{aligned}
$$

where $\tilde{h}[i] = h[-i]$.

Example

Convolution in 2D

In two dimensions, convolution becomes:

$$
\begin{aligned}
g(x,y) &= f(x,y) * h(x,y) \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')h(x-x',y-y')dx'dy' \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')h(x'-x,y'-y)dx'dy'
\end{aligned}
$$

where $\tilde{h}(x,y) = h(-x,-y)$.

Similarly, discrete convolution in 2D becomes:

$$
\begin{aligned}
g[i,j] &= f[i,j] * h[i,j] \\
&= \sum_k \sum_l f[k,l]h[k-i,l-j] \\
&= \sum_k \sum_l f[k,l]h[i-k,j-l]
\end{aligned}
$$

where $\tilde{h}[i,j] = h[-i,-j]$.

Convolution representation

Since f and g are defined over finite regions, we can write them out in two-dimensional arrays:

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

X .2	X 0	X .2
X 0	X .2	X 0
X .2	X 0	X .2

Note: *This is not matrix multiplication*!

Mean filters

How can we represent our noise-reducing averaging filter as a convolution diagram?

Effect of mean filters

Gaussian filters

Gaussian filters weigh pixels based on their distance from the center of the convolution filter. In particular:

$$h[i, j] = \frac{e^{-(i^2+j^2)/(2\sigma^2)}}{C}$$

This does a decent job of blurring noise while preserving features of the image.

What is the constant C? What should we set it to?

Effect of Gaussian filters

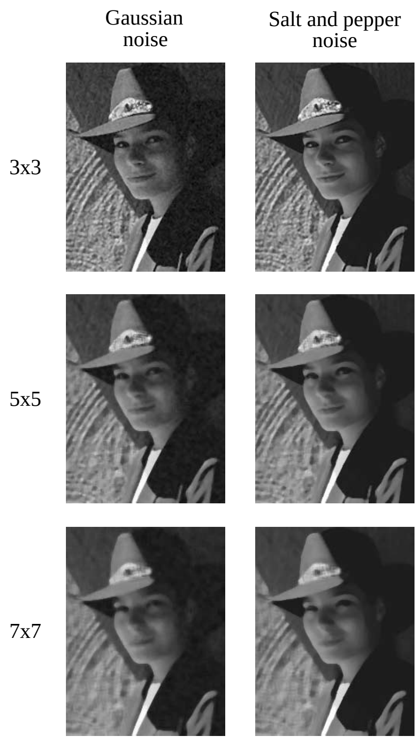
Median filters

A **Median Filter** operates over an mxm region by selecting the median intensity in the region.

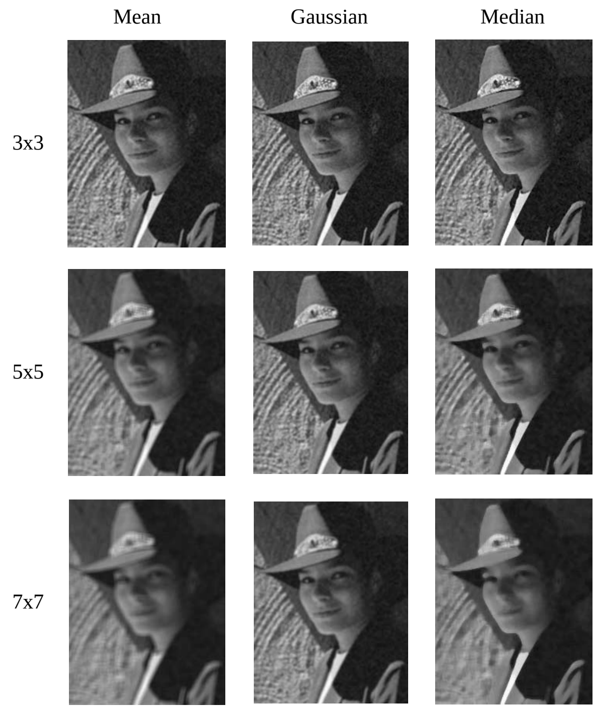
What advantage does a median filter have over a mean filter?

Is a median filter a kind of convolution?

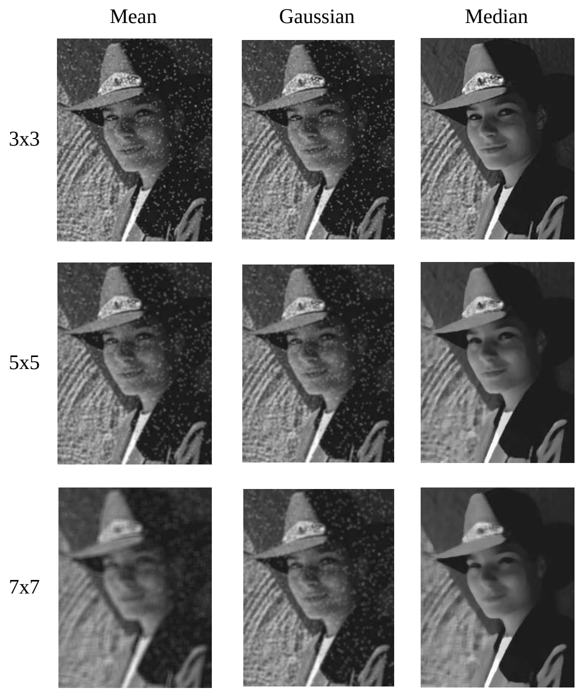
Effect of median filters



Comparison: Gaussian noise



Comparison: salt and pepper noise



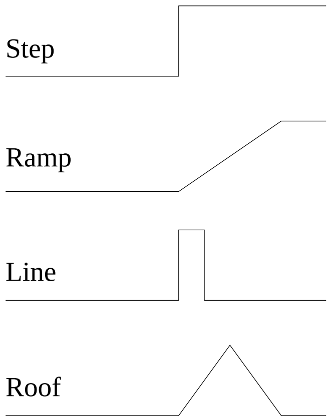
Edge detection

One of the most important uses of image processing is **edge detection:**

- Really easy for humans
- Really difficult for computers

- Fundamental in computer vision
- Important in many graphics applications

What is an edge?



Q: How might you detect an edge in 1D?

Gradients

The **gradient** is the 2D equivalent of the derivative:

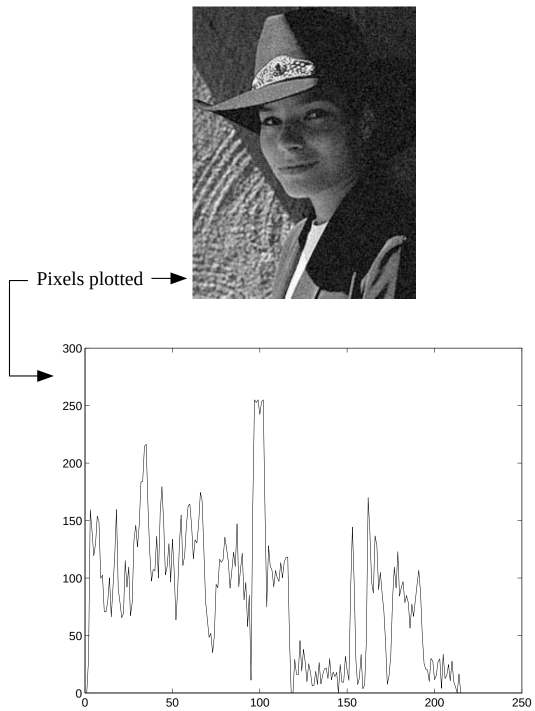
$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

Properties of the gradient

- It's a vector
- Points in the direction of maximum increase of f
- Magnitude is rate of increase

How can we approximate the gradient in a discrete image?

Less than ideal edges



Steps in edge detection

Edge detection algorithms typically proceed in three or four steps:

- **Filtering**: cut down on noise
- **Enhancement**: amplify the difference between edges and non-edges
- **Detection**: use a threshold operation
- **Localization** (optional): estimate geometry of edges beyond pixels

Edge enhancement

A popular gradient magnitude computation is the **Sobel operator**:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

We can then compute the magnitude of the vector (s_x, s_y).

Results of Sobel edge detection


Original


Smoothed


Sx + 128

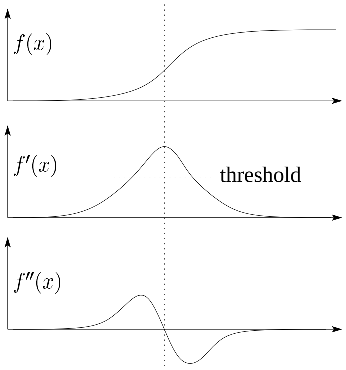

Sy + 128


Magnitude


Threshold = 64


Threshold = 128

Second derivative operators



The Sobel operator can produce thick edges. Ideally, we're looking for infinitely thin boundaries.

An alternative approach is to look for local extrema in the first derivative: places where the change in the gradient is highest.

Q: A peak in the first derivative corresponds to what in the second derivative?

Localization with the Laplacian

An equivalent measure of the second derivative in 2D is the **Laplacian**:

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Using the same arguments we used to compute the gradient filters, we can derive a Laplacian filter to be:

$$\Delta^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Zero crossings of this filter correspond to positions of maximum gradient. These zero crossings can be used to localize edges.

Summary

What you should take away from this lecture:

- The meanings of all the boldfaced terms.
- A richer understanding of the terms "image" and "image processing"
- How noise reduction is done
- How convolution filtering works
- The effect of mean, Gaussian, and median filters
- What an image gradient is and how it can be computed
- How edge detection is done