2. Color

Reading

Suggested:

- Watt (2nd ed.), Chapter 14.

Further reading:

- Brian Wandell. *Foundations of Vision. Chapter 4.* Sinauer Associates, Sunderland, MA, 1995.
- Gerald S. Wasserman. *Color Vision: An Historical Introduction.* John Wiley & Sons, New York, 1978

Outline

Color matching

The CIE XYZ color space

Color spaces for computer graphics

What is color?

The eyes and brain turn an incoming emission spectrum into a discrete set of values.

The signal sent to our brain is somehow interpreted as *color*.

Color science asks some basic questions:
- When are two colors alike?
- How many pigments or primaries does it take to match another color?

One more question: why should we care?

Newton's Experiments

Newton was the first to perform a scientific experiment on color in 1666.

Newton's experimental setup (Wandell 4.1)
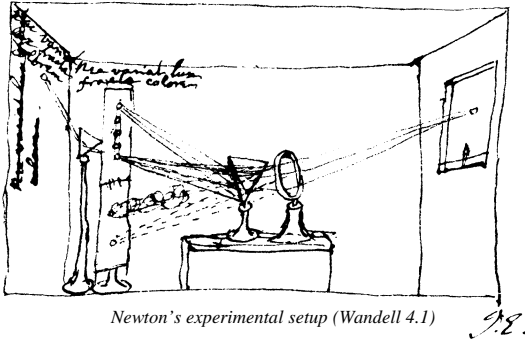
He built a simple colorimeter:

- Hole in a shutter
- Prism to disperse white light into a spectrum
- Comb-shaped aperture to manipulate the spectrum
- Converging lens to recombine the spectrum

Newton's Experiments, cont'd

Newton defined two types of light:

- Simple: Light that cannot be furthered dispersed by a prism (now called monochromatic)
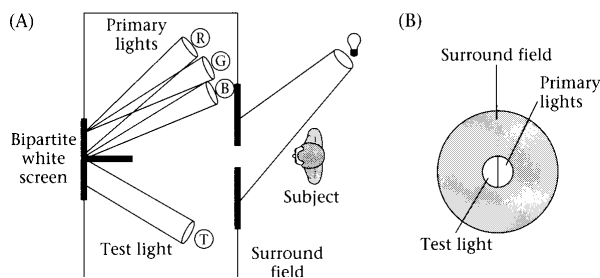- Compound: Light that can be dispersed.

He called the colors of simple lights primaries.

[This term means many things today.]

Newton's Experiments, cont'd

To study the appearance of colors, Newton recombined primaries to create new colors.

A modern day version of this kind of experiment looks like:

The color matching experiment (Wandell, 4.10)

Rods and "color matching"

A rod responds to a spectrum through its spectral sensitivity function, $r(\lambda)$. The response to a test light, $t(\lambda)$, is simply:

$$R = \int r(\lambda)t(\lambda)d\lambda$$

For convenience, we can also write this as:

$$R = \sum_i r(\lambda_i)t(\lambda_i)$$

If we consider only the visible wavelengths, then we can think of the r and t samples as defining vectors, leading to a simple matrix equation:

$$[R] = \begin{bmatrix} & \mathbf{r^T} & \end{bmatrix}\begin{bmatrix} \mathbf{t} \end{bmatrix}$$

What does this tell us about rod color discrimination?

Cones and color matching

Color is perceived through the responses of the cones to light.

The response of each cone can be written simply as:

$$L_t = \int l(\lambda)t(\lambda)d\lambda$$

$$M_t = \int m(\lambda)t(\lambda)d\lambda$$

$$S_t = \int s(\lambda)t(\lambda)d\lambda$$

We can also use matrix notation, which will prove useful in a moment:

$$\begin{bmatrix} L_t \\ M_t \\ S_t \end{bmatrix} = \begin{bmatrix} & \mathbf{l^T} & \\ & \mathbf{m^T} & \\ & \mathbf{s^T} & \end{bmatrix} \begin{bmatrix} \\ \mathbf{t} \\ \\ \end{bmatrix}$$

How many primaries do you think we'll need to match t?

Color matching

Let's assume that we need 3 primaries to perform the color matching experiment.

Consider three primaries, $\mathbf{p_1}$, $\mathbf{p_2}$, $\mathbf{p_3}$, with three emissive power knobs, e_1, e_2, e_3.

The three knobs create spectra of the form:

How do we set the knobs to match the test spectrum, $\mathbf{t}$?

Color matching, cont'd

First, we compute the response to the primaries:

$$\begin{bmatrix} L_p \\ M_p \\ S_p \end{bmatrix} = \begin{bmatrix} & \mathbf{l^T} & \\ & \mathbf{m^T} & \\ & \mathbf{s^T} & \end{bmatrix} \begin{bmatrix} \\ e_1\mathbf{p_1} + e_2\mathbf{p_2} + e_3\mathbf{p_3} \\ \\ \end{bmatrix}$$

$$= \begin{bmatrix} & \mathbf{l^T} & \\ & \mathbf{m^T} & \\ & \mathbf{s^T} & \end{bmatrix} \begin{bmatrix} \mathbf{p_1} & \mathbf{p_2} & \mathbf{p_3} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{l \cdot p_1} & \mathbf{l \cdot p_2} & \mathbf{l \cdot p_3} \\ \mathbf{m \cdot p_1} & \mathbf{m \cdot p_2} & \mathbf{m \cdot p_3} \\ \mathbf{s \cdot p_1} & \mathbf{s \cdot p_2} & \mathbf{s \cdot p_3} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Color matching, cont'd

In order for the primaries to match the test, we require the cone responses to be identical:

$$\begin{bmatrix} L_t \\ M_t \\ S_t \end{bmatrix} = \begin{bmatrix} & \mathbf{l^T} & \\ & \mathbf{m^T} & \\ & \mathbf{s^T} & \end{bmatrix} \begin{bmatrix} \\ \mathbf{t} \\ \\ \end{bmatrix} = \begin{bmatrix} L_p \\ M_p \\ S_p \end{bmatrix}$$

This gives us:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \mathbf{l \cdot p_1} & \mathbf{l \cdot p_2} & \mathbf{l \cdot p_3} \\ \mathbf{m \cdot p_1} & \mathbf{m \cdot p_2} & \mathbf{m \cdot p_3} \\ \mathbf{s \cdot p_1} & \mathbf{s \cdot p_2} & \mathbf{s \cdot p_3} \end{bmatrix}^{-1} \begin{bmatrix} & \mathbf{l^T} & \\ & \mathbf{m^T} & \\ & \mathbf{s^T} & \end{bmatrix} \begin{bmatrix} \\ \mathbf{t} \\ \\ \end{bmatrix}$$

And finally:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} & \overline{\mathbf{p}}_1^{T} & \\ & \overline{\mathbf{p}}_2^{T} & \\ & \overline{\mathbf{p}}_3^{T} & \end{bmatrix} \begin{bmatrix} \\ \mathbf{t} \\ \\ \end{bmatrix}$$

Color matching, cont'd

Key observations:

1. Three primaries are "sufficient" for color matching.

2. We can compute the knob settings using three vectors (functions). These are called the **color matching functions**.

3. Color matching functions are linear transforms of the cone responses.

4. All sets of color matching functions are linear transforms of each other.

5. The resulting knob settings can take on negative values.

Negative light

What does it mean to use a negative amount of a primary?

Consider:

$$
\begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix}
\begin{bmatrix} \mathbf{t} \end{bmatrix} =
\begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix}
\begin{bmatrix} 0.5\mathbf{p_1} - 0.3\mathbf{p}_2 + 0.4\mathbf{p}_3 \end{bmatrix}
$$

To make e_2 behave like a "real" (i.e., positive valued) knob, we have to move it over to the other side:

$$
\begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix}
\begin{bmatrix} \mathbf{t} + 0.3\mathbf{p}_2 \end{bmatrix} =
\begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix}
\begin{bmatrix} 0.5\mathbf{p_1} + 0.4\mathbf{p}_3 \end{bmatrix}
$$

So, if we are allowed to move a primary to the other side, we will be able to match *any* color.
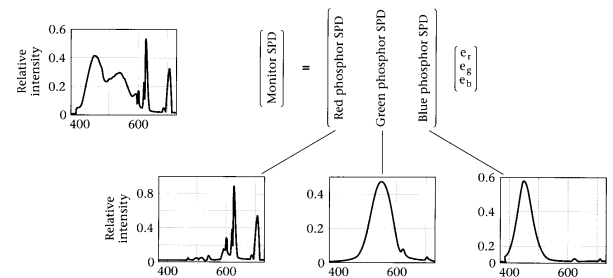
Example: Wright's experiments

In the late 20's, Wright found that the colors of all wavelengths could be reproduced with combinations of 3 primaries at 460, 530, and 650nm:
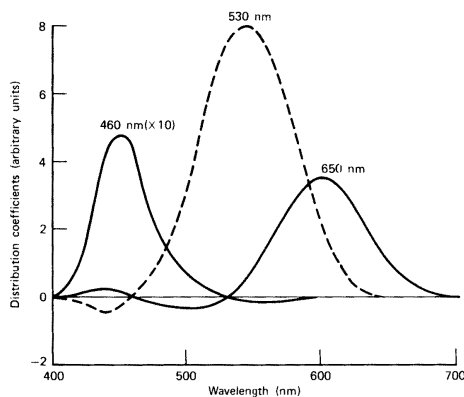
Color matching functions for primaries at 460, 530 and 650 nm (Wasserman 3.3)

These functions are color-matching functions for the given primaries.
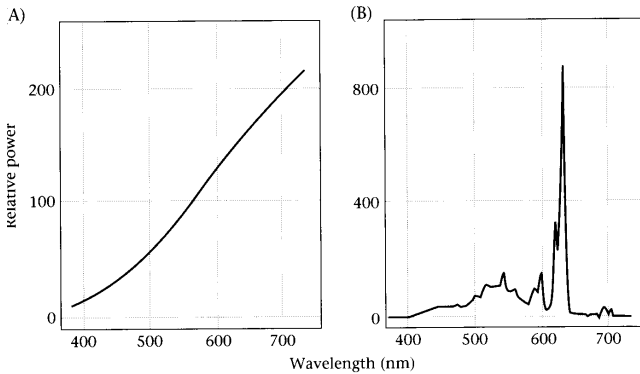
Choosing Primaries

Emission spectra for RGB monitor phosphors (Wandell B.3)

Primaries don't have to be monochromatic. You can still derive color matching functions.

Emission Spectrum is not Color

Recall how much averaging the eye does. Light is infinite dimensional!
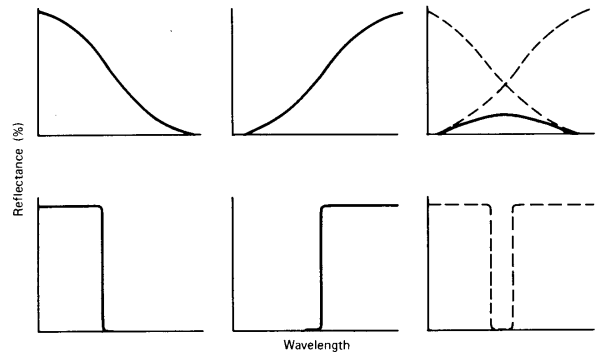
Different light sources can evoke exactly the same colors. Such lights are called **metamers**.

A dim tungsten bulb and an RGB monitor set up to emit a metameric spectrum (Wandell 4.11)

Coloured Surfaces

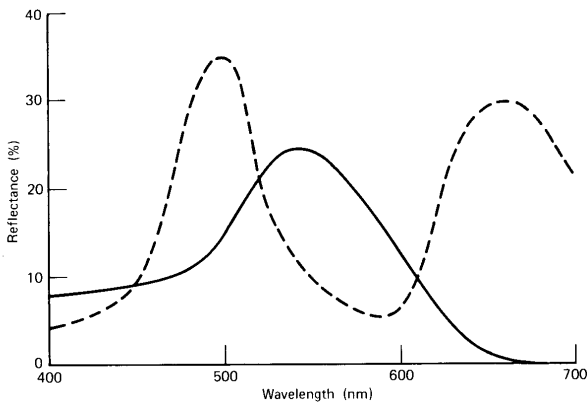So far, we've discussed the colors of lights. How do *surfaces* acquire color?

Subtractive colour mixing (Wasserman 2.2)

A surface's **reflectance** is its tendency to reflect incoming light across the spectrum.

Reflectance is combined **subtractively** with incoming light. (Actually, the process is multiplicative.)
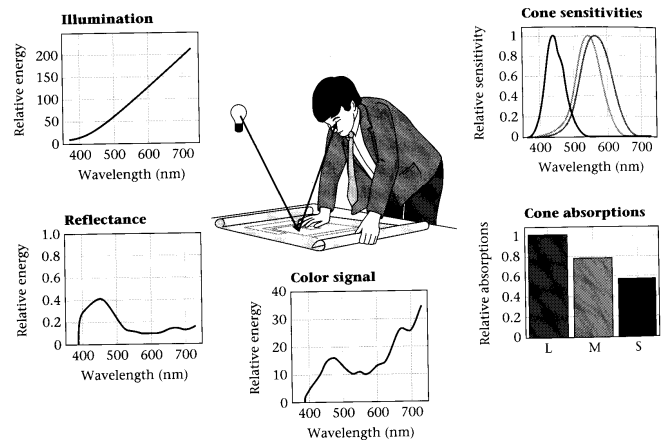
Subtractive Metamers

Surfaces that are metamers under only some lighting conditions (Wasserman 3.9)

Reflectance adds a whole new dimension of complexity to color perception.

The solid curve appears green indoors and out. The dashed curve looks green outdoors, but brown under incandescent light.
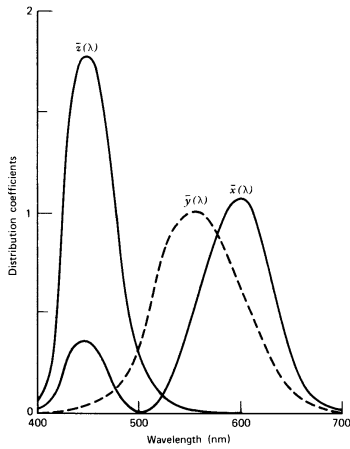
Illustration of Color Appearance

How light and reflectance become cone responses (Wandell, 9.2)

The CIE *XYZ* System

A standard created in 1931 by CIE, defined in terms of three color matching functions.

The XYZ colour matching functions (Wasserman 3.8)

What are the primaries?

Why were these color matching functions chosen?

CIE Coordinates

Given an emission spectrum, we can use the CIE matching functions to obtain the *X, Y* and *Z* coordinates.

$$X = \int \overline{x}(\lambda)t(\lambda)d\lambda$$

$$Y = \int \overline{y}(\lambda)t(\lambda)d\lambda$$
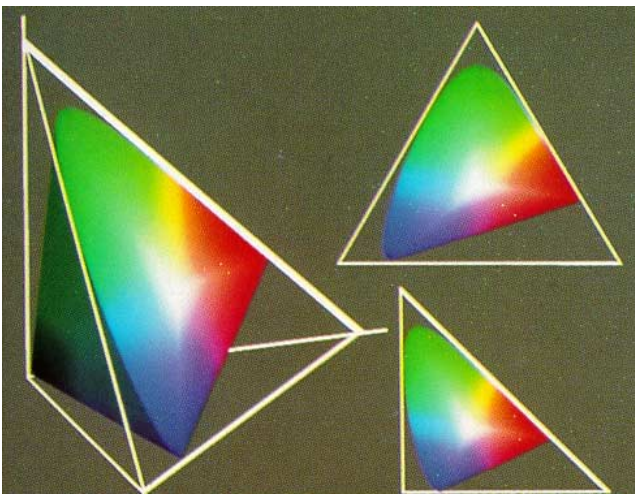
$$Z = \int \overline{z}(\lambda)t(\lambda)d\lambda$$

Then we can compute *chromaticity coordinates*. This gives a brightness independent notion of color.
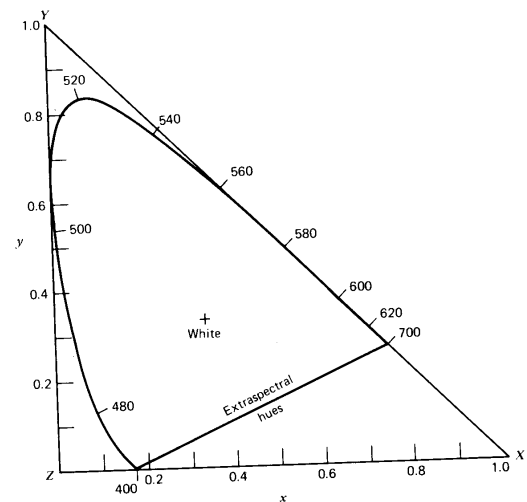
$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

The CIE Colour Blob

Different views of the CIE color space (Foley II.1)

The CIE Chromaticity Diagram

The chromaticity diagram (Wasserman 3.7)

A projection of the plane *X*+*Y*+*Z*=1.

Each point is a chromaticity value, which depends on **dominant wavelength**, or **hue**, and **excitation purity**, or **saturation**.

More About Chromaticity

Dominant wavelengths go around the perimeter of the chromaticity blob.

◆ A color's dominant wavelength is where a line from white through that colour intersects the perimeter.

◆ Some colours, called *nonspectral* color's, don't have a dominant wavelength.

Excitation purity is measured in terms of a color's position on the line to its dominant wavelength.

Complementary colors lie on opposite sides of white, and can be mixed to get white.
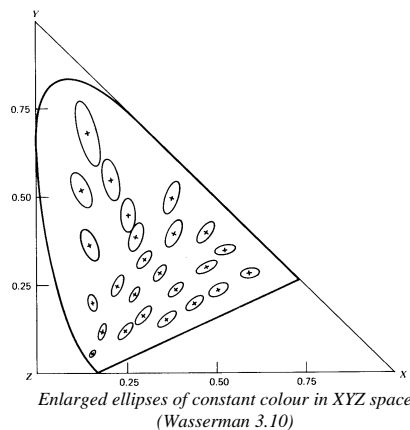
Gamuts

Not every output device can reproduce every color. A device's range of reproducible colors is called its **gamut**.
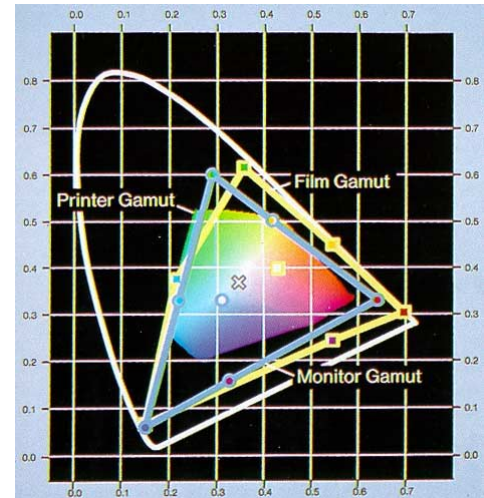
Gamuts of a few common output devices in CIE space (Foley, II.2)

Perceptual (Non-)uniformity

The *XYZ* color space is not perceptually uniform!

Enlarged ellipses of constant colour in XYZ space (Wasserman 3.10)

Some modified spaces attempt to fix this:

• $L^*u^*v^*$
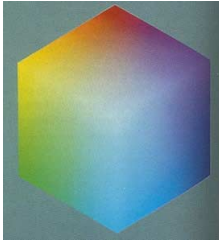• $L^*a^*b^*$

Colour Spaces for Computer Graphics

In practice, there's a set of more commonly-used colour spaces in computer graphics:

• RGB for display
• CMY (or CMYK) for hardcopy
• HSV for user selection
• YIQ for television broadcast

RGB

Perhaps the most familiar colour space, and the most convenient for display on a CRT.
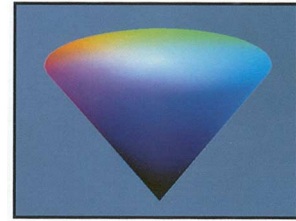
What does the RGB colour space look like?

HSV

More natural for user interaction, corresponds to the artistic concepts of tint, shade and tone.
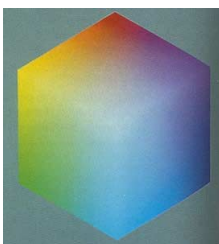
The HSV space looks like a cone:

CMY

A subtractive colour space used for printing.
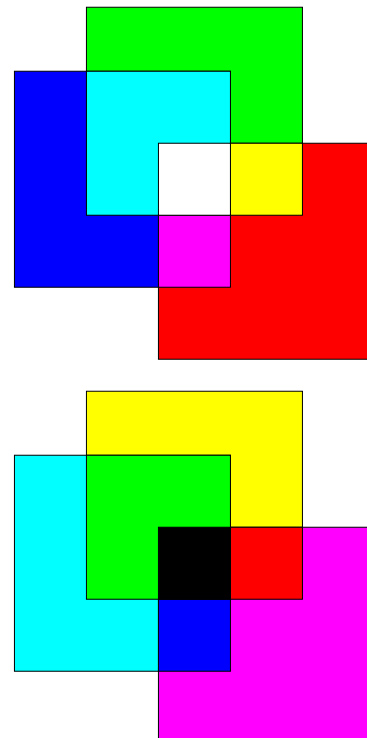
Involves three subtractive primaries:

- Cyan - subtracts red
- Magenta - subtracts green
- Yellow - subtracts blue

Mixing two pigments subtracts their opposites from white.

CMYK adds blacK ink rather than using equal amounts of all three.

RGB vs. CMY

YIQ

Used in TV broadcasting, YIQ exploits useful properties of the visual system.

- Y - luminance (taken from CIE)
- I - major axis of remaining colour space
- Q - remaining axis

YIQ is broadcast with relative bandwidth ratios 8:3:1

- We're best as distinguishing changes in luminance.
- Small objects can be compressed into a single colour dimension.

Why do we devote a channel to luminance?

Summary

Here's what you should take home from this lecture:

- All the **boldfaced terms**.
- How the color matching experiment works
- The relationship between color matching and functions cone responses
- The difference between emissive and reflective color
- The CIE XYZ color standard and how to interpret the chromaticity diagram
- The color spaces used in computer graphics