Homework 2:
Shading, Raytracing, and Texture Mapping

Assigned: Tuesday, 9 November 1999
Due: Tuesday, 23 November 1999

DIRECTIONS

Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to talk over the problems with classmates, but please answer the questions on your own.

Name: ________________________________

Problem 1.

The Phong shading model can be summarized by the following equation:

$$I_{phong} = k_e + k_a I_a + \sum_i I_{l_i} \left(\frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \left[k_d \left(\mathbf{N} \cdot \mathbf{L}_i \right)_+ + k_s \left(\mathbf{V} \cdot \mathbf{R}_i \right)_+^{n_s} \right]$$

where the summation i is taken over all light sources.

The variables used in the Phong shading equation are summarized below:

I_{phong} a_0 a_1 a_2 d_i k_e k_a k_d k_s n_s I_a I_{li} $\mathbf{L}_i$ $\mathbf{R}_i$ $\mathbf{N}$ $\mathbf{V}$

(a) Which of the quantities above are affected if...(justify your answers)…

…the viewing direction changes?

…the position of the i^{th} light changes?

…the orientation of the surface changes?

Problem 1 - continued.

(b) Blinn and Newell have suggested that, when **V** and **L** are assumed to be constants, the computation of $\mathbf{V} \cdot \mathbf{R}$ can be simplified by associating with each light source a fictitious light source that will generate specular reflections. This second light source is located in a direction H halfway between **L** and **V**. The specular component is then computed from $(\mathbf{N} \cdot \mathbf{H})_+^{n_s}$ instead of from $(\mathbf{V} \cdot \mathbf{R})_+^{n_s}$.

Under what circumstances might **L** and **V** be assumed to be constant?

How does the new equation using **H** simplify shading equations?

(c) The ambient term in the Phong model is one way to guarantee that all visible surfaces receive some light. Another possibility is to use the "headlamp" method in which a point light source is positioned at the eye, but no ambient term is used.

Are these two methods equivalent? If so, explain why. If not, describe a scene in which the results would be clearly different.

Problem 1 - continued.

(d) Respond TRUE or FALSE to each of these statements and ***<u>explain your reasoning</u>***.

_____ The Phong model is a physical simulation of the behavior of real-world light.

_____ For polished metal, the specular component n_s would be large.

_____ A rough surface with many tiny microfacets is likely to have a large diffuse reflection coefficient.

(e) Describe the relationships between **N**, $\mathbf{L}_i$, and $\mathbf{R}_i$ that would result in a point shaded with the Phong model appearing maximally bright.

(f) The equation above is not the only hallmark to Mister Phong's fame. We also talked in class about the difference between two polygon shading interpolation methods, one called Phong and one called Gouraud. Describe a scene where the difference between Phong and Gouraud interpolated shading would be noticeable.

Problem 2.

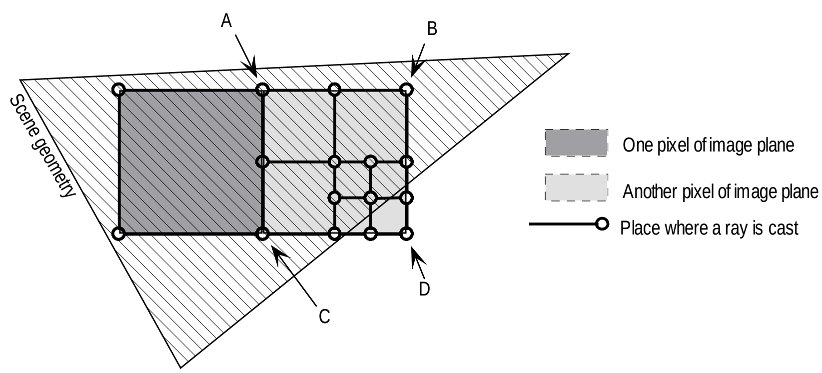
(a) The company you work for has just bought rights to a raytracing engine. Unfortunately, you don't have the source code, just a compiled library. You have been asked to determine how rays are terminated. So, you call the authors you find out even they don't remember for sure. All they can tell you is this: *The termination criteria for tracing rays is either (a) rays are traced to a maximum recursion depth of 5, or (b) rays are adaptively terminated based on their contribution to a pixel color.*

Describe a scene that can be used to determine which method is used. Be specific about all relevant aspects of the scene and what you would look for in the resulting image to determine which termination method is used.

Problem 2 - continued.

(b) One of the features included in the raytracing engine your company bought is a brand new algorithm for <u>antialiasing by adaptive supersampling</u>.

The normal implementation is to sample rays at the corner of every pixel, compare the colors of each sample, and if the difference between neighboring sample colors is too great, subdivide that region recursively and sample more times. (See the diagram below, or Foley, et al., 15.10.4)



However, in this new algorithm, we subdivide and supersample if neighboring rays *intersect different objects*. In other words, note the light-grey pixel above. Three of the four corner samples (a, b, and c) intersect the scene geometry. The fourth corner (d), misses the geometry completely. So we choose to supersample this pixel without ever comparing colors.

In what way(s) is this better than the traditional method? In what way(s) is it worse?

Problem 3 - Texture anti-aliasing

For each of the following types of texture maps, list the amount of time (in Big O notation) that it takes to return a value for a k by k square of texels that is getting mapped to a pixel p in screen space. For each type of texture map, also list the amount of storage that the preprocessed texture map takes up in big O notation. You may assume that the original image that represents the texture map is n pixels wide and n pixels high. Justify your answer.

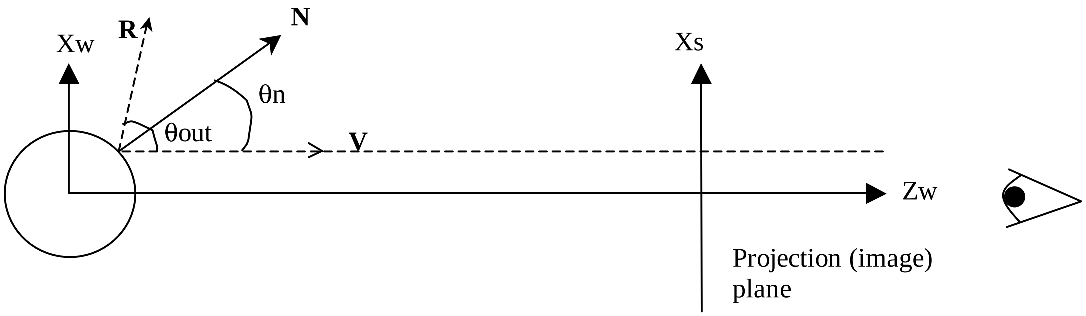
i. Brute Force

ii. Summed Area Tables

iii. Mip Maps

Problem 4 – Environment Mapping

One method of environment mapping (reflection mapping) involves using a 'gazing ball' to capture an image of the surroundings. The idea is to place a chrome sphere in an actual environment, take a photograph of the sphere, and warp the resulting image into an environment map. Let's examine this in two dimensions. Below is a diagram of our setup. Assume that the viewer is infinitely far away so that rays from the viewer are parallel. All angles are measured in <u>world</u> coordinates with respect to the Z-axis.



(a) For a ray of given Xs, determine the point of intersection (in world coordinates) with the two dimensional gazing ball. Assume that the gazing ball is a unit circle centered at the origin in world coordinates.

(b) Calculate the normal vector at the point of intersection as a function of Xs.

(c) Calculate θn (the angle between the normal and the Z-axis) as a function of Xs.

(d) Calculate θout (the angle between the reflected ray **R** and the Z-axis) as a function of Xs.

(e) Plot θout versus Xs. In what regions do small changes in image coordinates map to large changes in reflected angles?

(f) If we were to use θout to index into an environment map, what kind of errors can we expect when raytracing?