Hidden Surface Determination

Reading

- Foley *et al*, Chapter 15

Optional

- I. E. Sutherland, R. F. Sproull, and R. A. Schumacker, A characterization of ten hidden surface algorithms, *ACM Computing Surveys* 6(1): 1-55, March 1974.

The Quest for 3D

- Construct a 3D hierarchical geometric model
- Define a virtual camera
- Map points in 3D space to points in an image

- produce a wireframe drawing in 2D from a 3D object

- Of course, there’s more work to be done…

Introduction

- Not every part of every 3D object is visible to a particular viewer. We need an algorithm to determine what parts of each object should get drawn.
- Known as "hidden surface elimination" or "visible surface determination".
- Hidden surface elimination algorithms can be categorized in three major ways:
  - Object space vs. image space
  - Object order vs. image order
  - Sort first vs. sort last

Object Space Algorithms

- Operate on geometric primitives
  - For each object in the scene, compute the part of it which isn't obscured by any other object, then draw.
  - Must perform tests at high precision
  - Resulting information is resolution-independent
- Complexity
  - Must compare every pair of objects, so O(n^2) for *n* objects
  - For an *m* x *m* display, have to fill in colors for m^2 pixels.
  - Overall complexity can be $O(k_{obj}\, n^2 + k_{disp}\, m^2)$.
  - Best for scenes with few polygons or resolution-independent output
- Implementation
  - Difficult to implement!
  - Must carefully control numerical error

Image Space Algorithms

- Operate on pixels
  - For each pixel in the scene, find the object closest to the COP which intersects the projector through that pixel, then draw.
  - Perform tests at device resolution, result works only for that resolution
- Complexity
  - Naïve approach checks all *n* objects at every pixel.Then, $O(\overset{n\,m^2}{n\ m^2})$.
  - Better approaches check only the objects that *could* be visible at each pixel. Let's say, on average, *d* objects are visible at each pixel (a.k.a., depth complexity). Then, $m^2 d$ $O(d\ m^2)$.
- Implementation
  - Usually very simple!
  - Used a lot in practice.

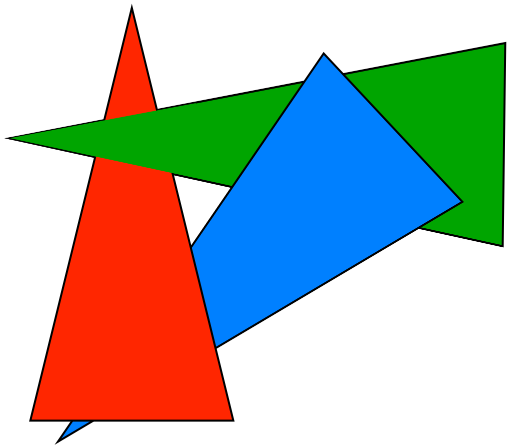
Object Order vs. Image Order

- Object order
  - Consider each object only once - draw its pixels and move on to the next object
  - Might draw the same pixel multiple times

- Image order
  - Consider each pixel only once - draw part of an object and move on to the next pixel
  - Might compute relationships between objects multiple times

Sort First vs. Sort Last

- Sort first
  - Find some depth-based ordering of the objects relative to the camera, then draw from back to front
  - Build an ordered data structure to avoid duplicating work
- Sort last
  - Sort implicitly as more information becomes available

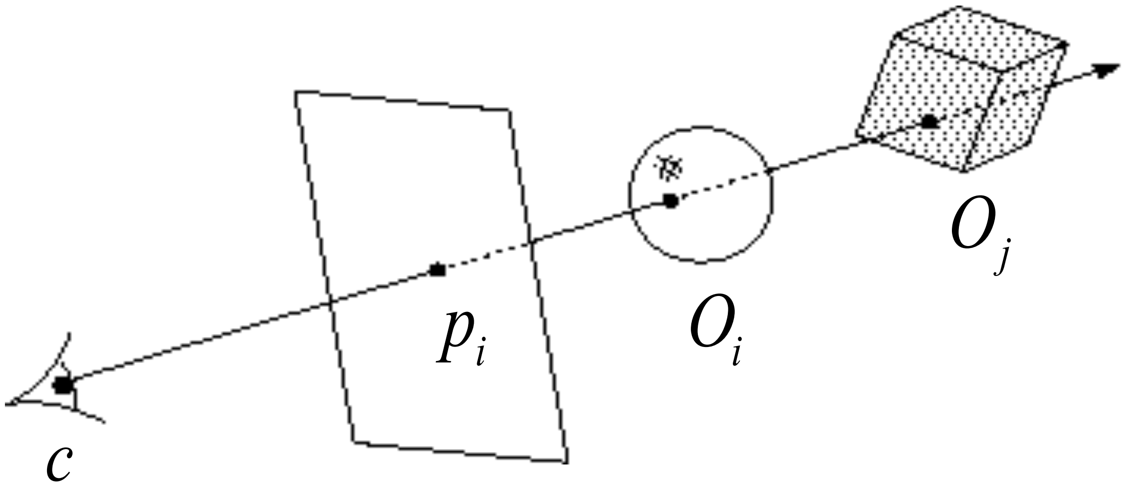


Important Algorithms

- Ray casting
- Z-buffer
- Binary space partitioning
- Back face culling

Ray Casting

1. Partition the projection plane into pixels to match screen resolution:
2. For each pixel p_i, construct ray from COP through PP at that pixel and into scene
3. Intersect the ray with every object in the scene
4. Color the pixel according to the object with the closest intersection



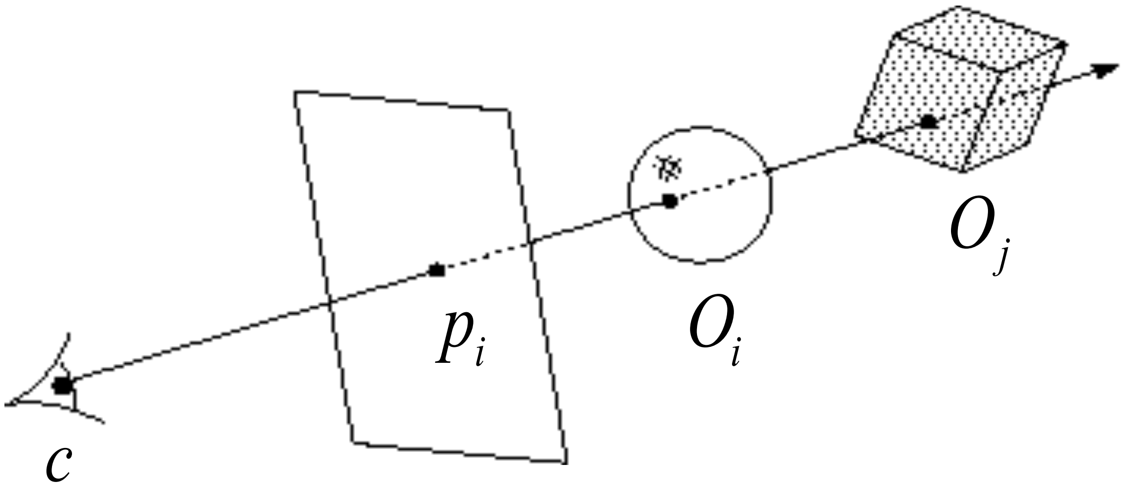
Ray Casting, cont.

- Parameterize each ray:

  $\mathbf{r}(t) = \mathbf{c} + t\,(\boldsymbol{P}_{ij} - \mathbf{c})$

- Each object O_i returns $t_i > 1$ such that first intersection with O_i occurs at $\mathbf{r}(t_i)$.

Q: Given the set $\{t_i\}$ what is the first intersection point?



Aside: Definitions

- An algorithm exhibits *coherence* if it uses knowledge about the continuity of the objects on which it operates
- An *online* algorithm is one that doesn’t need all the data to be present when it starts running
  - Example: insertion sort

Ray Casting Analysis

- Easy to implement?
- Hardware implementation?
- Pre-processing required?
- Incremental drawing calculations (uses coherence)?
- On-line (doesn’t need all objects before drawing begins)?
- Memory intensive?
- Handles transparency and refraction?
- Polygon-based?
- Extra work for moving objects?
- Extra work for moving viewer?
- Efficient shading?
- Handles cycles and self-intersections?

Z-buffer

- Idea: along with a pixel's red, green and blue values, maintain some notion of its *depth*
  - An additional channel in memory, like alpha
  - Called the depth buffer or Z-buffer

```
void draw_mode_setup( void ) {
  …
  GlEnable( GL_DEPTH_TEST );
  …
}
```

- When the time comes to draw a pixel, compare its depth with the depth of what's already in the framebuffer. Replace only if it's closer
- Very widely used
- History
  - Originally described as "brute-force image space algorithm"
  - Written off as impractical algorithm for huge memories
  - Today, done easily in hardware

Z-buffer Implementation

```
for each pixel $p_i$
{
        Z-buffer[ $p_i$ ] = FAR
        Fb[ $p_i$ ] = BACKGROUND_COLOR
}

for each polygon P
{
        for each pixel $p_i$ in the projection of P
        {
                Compute depth z and shade s of P at $p_i$
                if z < Z-buffer[ $p_i$ ]
                {
                        Z-buffer[ $p_i$ ] = z
                        Fb[ $p_i$ ] = s
                }
        }
}
```

Visibility tricks for Z-buffers

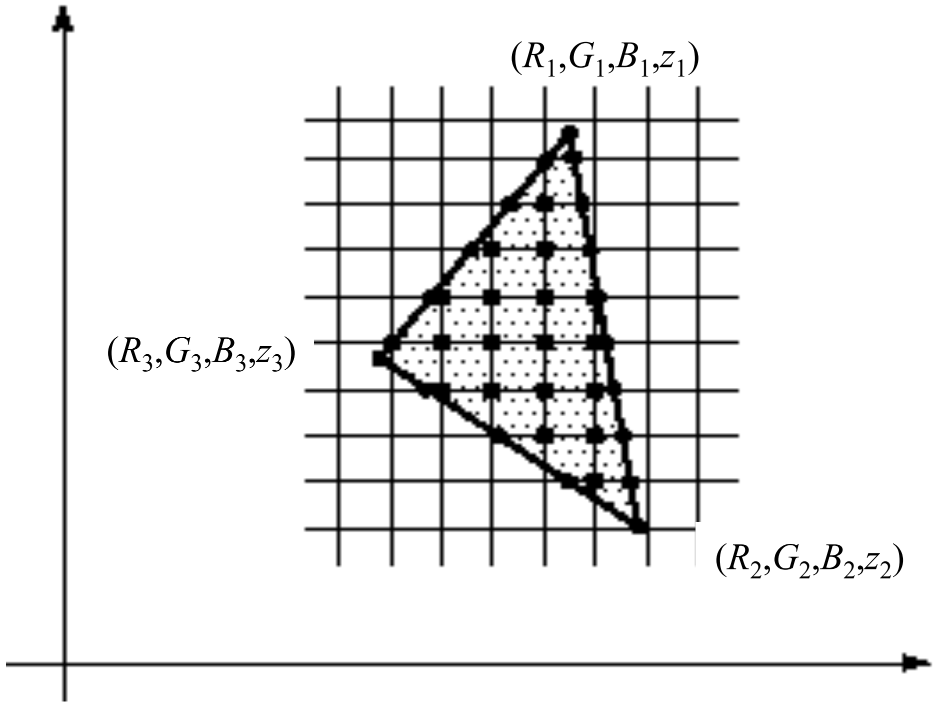
Z-buffering is ***the*** algorithm of choice for hardware rendering

What is the complexity of the Z-buffer algorithm?

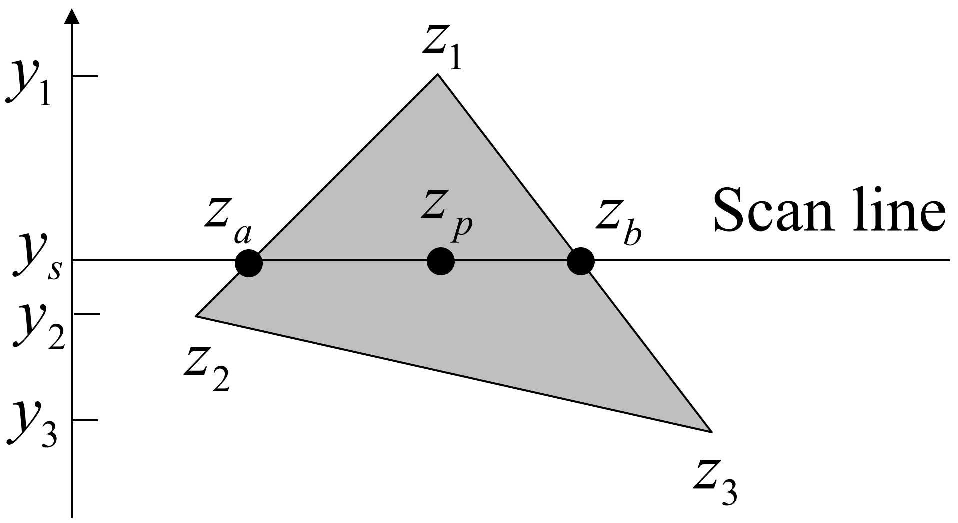
What can we do to decrease the constants?

Z-buffer Tricks

- The shade of a triangle can be computed incrementally from the shades of its vertices
- Can do the same with depth



Z value interpolation

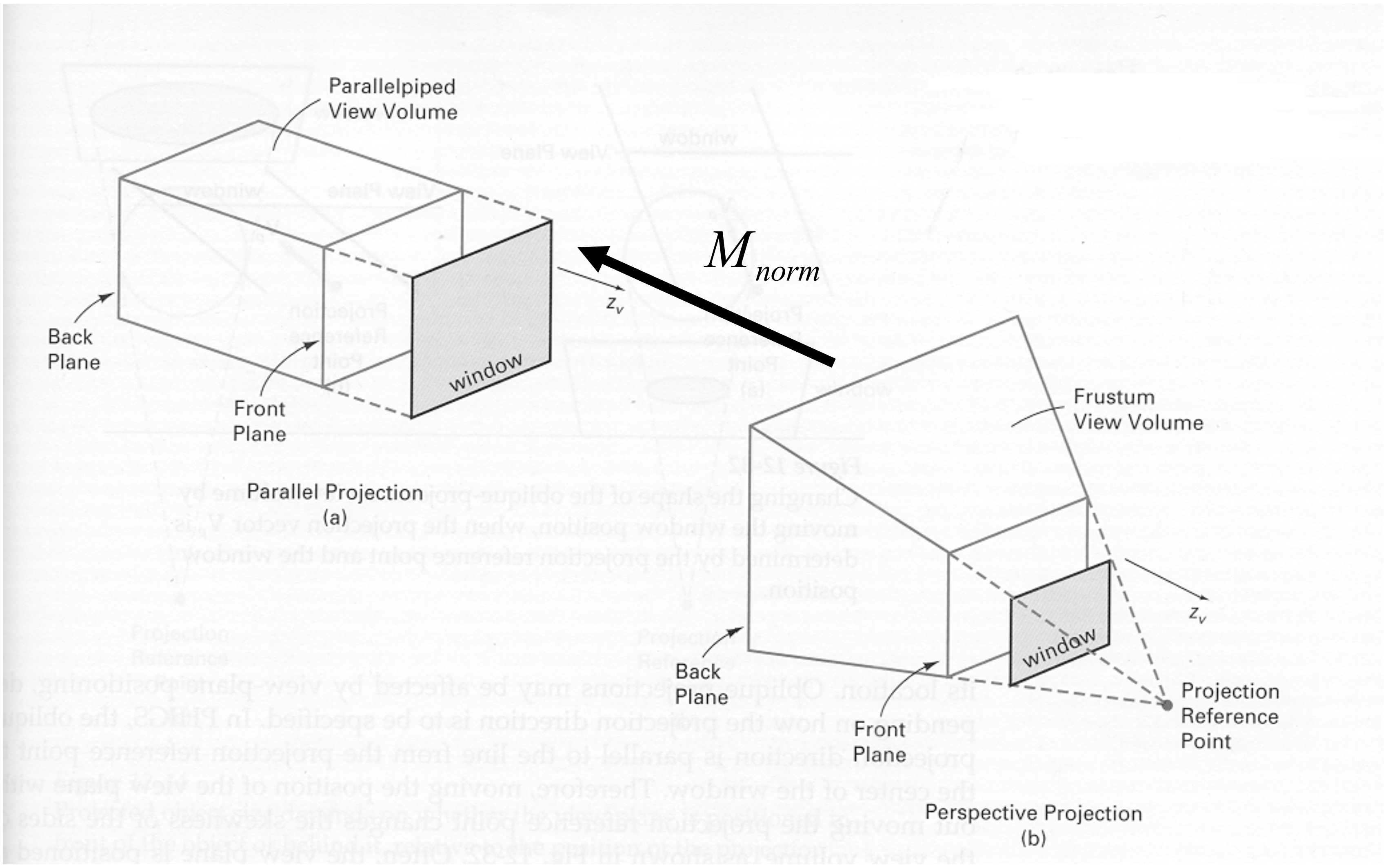


$$z_a = z_1 - (z_1 - z_2)\frac{y_1 - y_s}{y_1 - y_2}$$

$$z_b = z_1 - (z_1 - z_3)\frac{y_1 - y_s}{y_1 - y_3}$$

$$z_p = z_b - (z_b - z_a)\frac{x_b - x_p}{x_b - x_a}$$

Depth Preserving Conversion to Parallel Projection



Computing Z

- Use 3x4 projective transformation

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- And keep around z (e.g. z′=z)
- To make sure that z bits are unifromly distributed between far and near clipping planes

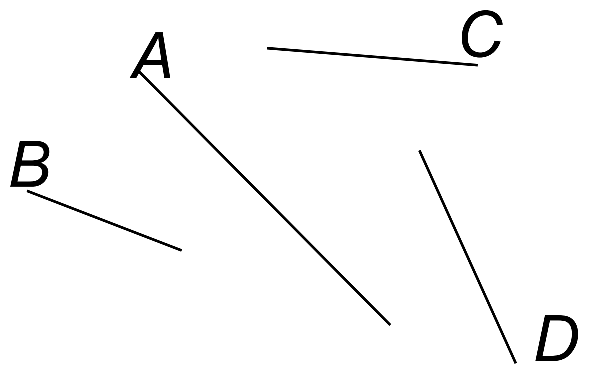
$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{f+n}{d(f-n)} & \frac{2fn}{d(f-n)} \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} =$$

Z-buffer Analysis

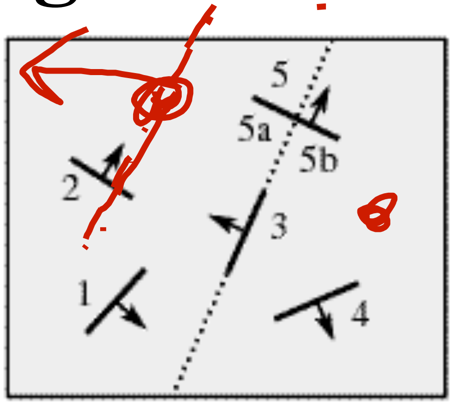
- Easy to implement?
- Hardware implementation?
- Pre-processing required?
- Incremental drawing calculations (uses coherence)?
- On-line (doesn’t need all objects before drawing begins)?
- Memory intensive?
- Handles transparency and refraction?
- Polygon-based?
- Extra work for moving objects?
- Extra work for moving viewer?
- Efficient shading?
- Handles cycles and self-intersections?

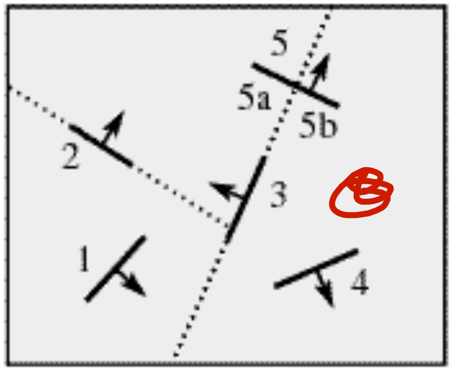
Binary Space Partitioning

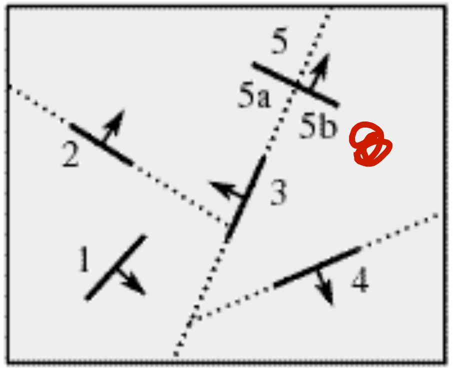
- **Goal**: build a structure that captures some relative depth information between objects. Use it to draw objects in the right order from any viewpoint.
  - Called the binary space partitioning tree, or BSP tree
- **Key observation**: The polygons in the scene are painted in the correct order if for each polygon *P*,
  - Polygons on the far side of *P* are painted first
  - *P* is painted next
  - Polygons in front of *P* are painted last

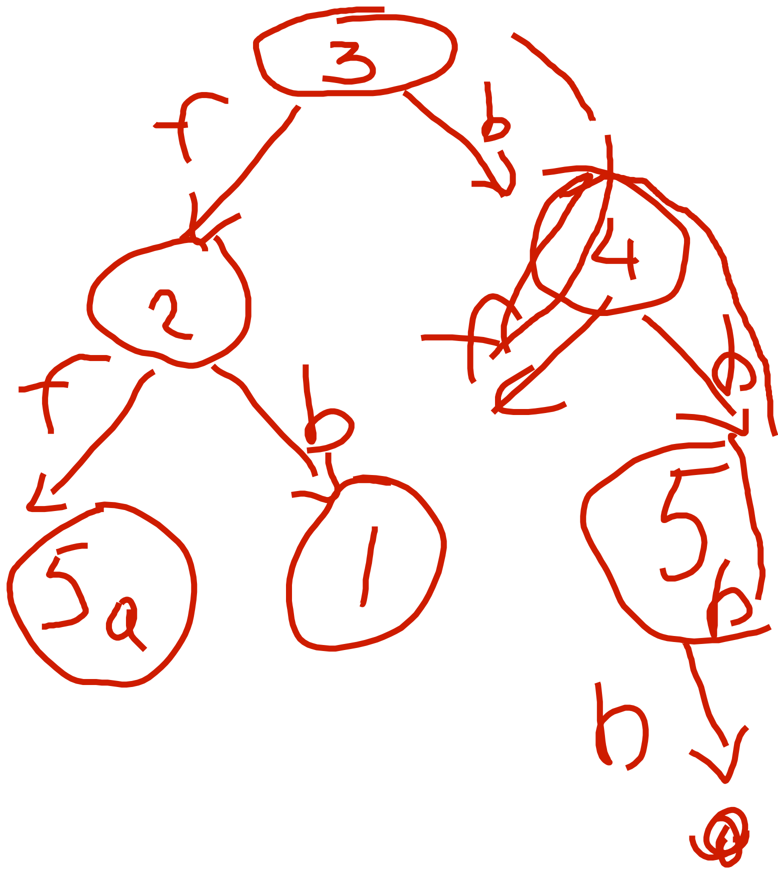


Building a BSP Tree (in 2D)

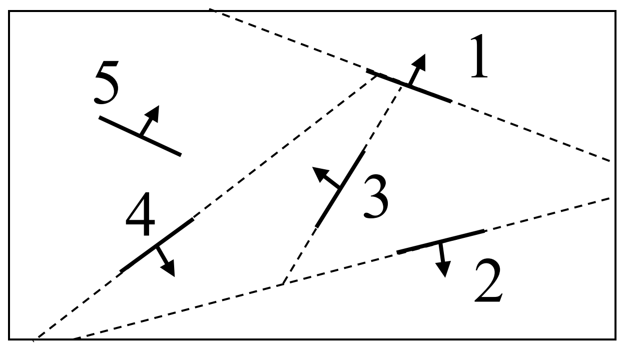


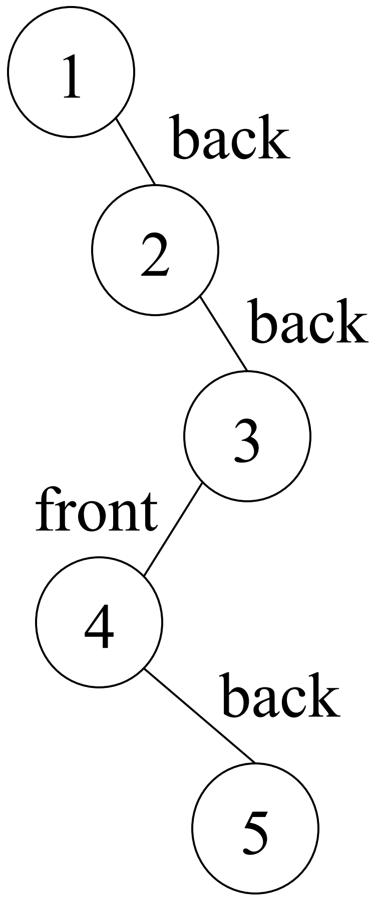






Alternate BSP Tree





BSP Tree Construction

```
BSPtree makeBSP( L: list of polygons )
{
        if L is empty
        {
                return the empty tree
        }

        Choose a polygon P from L to serve as root
        Split all polygons in L according to P
        return new TreeNode(
                P,
                makeBSP( polygons on negative side of P ),
                makeBSP( polygons on positive side of P ))
}
```

- Splitting polygons is expensive! It helps to choose P wisely at each step.
  - Example: choose five candidates, keep the one that splits the fewest polygons

BSP Tree Display

```
showBSP( v: Viewer, T: BSPtree )
{
        if T is empty then return

        P := root of T
        if viewer is in front of P
        {
                showBSP( back subtree of T )
                draw P
                showBSP( front subtree of T )
        } else {
                showBSP( front subtree of T )
                draw P
                showBSP( back subtree of T )
        }
}
```

BSP Tree Applications

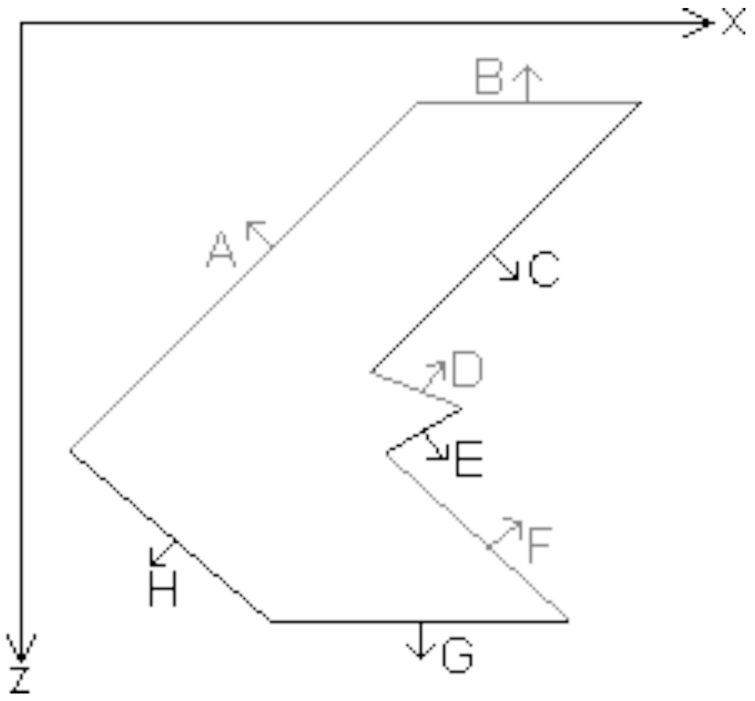
- Hidden surface removal
- Ray casting speedup
- Collision detection
- Robot motion planning

BSP Analysis

- Easy to implement?
- Hardware implementation?
- Pre-processing required?
- Incremental drawing calculations (uses coherence)?
- On-line (doesn’t need all objects before drawing begins)?
- Memory intensive?
- Handles transparency and refraction?
- Polygon-based?
- Extra work for moving objects?
- Extra work for moving viewer?
- Efficient shading?
- Handles cycles and self-intersections?

Back Face Culling

- Can be used in conjunction with polygon-based algorithms
- Often, we don't want to draw polygons that face away from the viewer. So test for this and eliminate (cull) back-facing polygons before drawing
- How can we test for this?



Summary

- Classification of hidden surface algorithms
- Understanding of Z-buffer
- Familiarity with BSP trees and back face culling