Ray Tracing

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Reading

Required:
- Shirley, section 10.1-10.7 (online handout)
- Triangle intersection (online handout)

Further reading:
- Shirley errata on syllabus page, needed if you work from his book instead of the handout, which has already been corrected.

Geometric optics

Modern theories of light treat it as both a wave and a particle.

We will take a combined and somewhat simpler view of light – the view of geometric optics.

Here are the rules of geometric optics:

- Light is a flow of photons with wavelengths. We'll call these flows “light rays.”
- Light rays travel in straight lines in free space.
- Light rays do not interfere with each other as they cross.
- Light rays obey the laws of reflection and refraction.
- Light rays travel from the light sources to the eye, but the physics is invariant under path reversal (reciprocity).

Eye vs. light ray tracing

Where does light begin?

At the light: light ray tracing (a.k.a. forward ray tracing or photon tracing)

At the eye: eye ray tracing (a.k.a. backward ray tracing)

We will generally follow rays from the eye into the scene.
Precursors to ray tracing

Local illumination
- Cast one eye ray, then shade according to light

Appel (1968)
- Cast one eye ray to light

Whitted ray-tracing algorithm

In 1980, Turner Whitted introduced ray tracing to the graphics community.
- Combines eye ray + rays to light
- Recursively traces rays

Algorithm:
1. For each pixel, trace a primary ray in direction $\mathbf{V}$ to the first visible surface.
2. For each intersection, trace secondary rays
   - Shadow rays in directions $\mathbf{L}_i$ to light sources
   - Reflected ray in direction $\mathbf{R}$
   - Refracted ray or transmitted ray in direction $\mathbf{T}$.

Whitted algorithm (cont'd)

Let's look at this in stages:

Primary rays
- $\mathbf{L}_i$ from light sources
- $\mathbf{L}_e$ from eye

Shadow rays
- $\mathbf{R}$ from reflected ray
- $\mathbf{T}$ from refracted ray

Reflection rays
- $\mathbf{R}$ from reflected ray
- $\mathbf{T}$ from refracted ray

Reflected rays
- $\mathbf{R}$ from reflected ray

Ray casting and local illumination

Now let's actually build the ray tracer in stages. We'll start with ray casting and local illumination:
Direct illumination

A ray is defined by an origin $P$ and a unit direction $d$, and is parameterized by $r > 0$:

$$r(t) = P + td$$

Let $I(P, d)$ be the intensity seen along a ray. Then:

$$I(P, d) = I_{\text{direct}}$$

where

- $I_{\text{direct}}$ is computed from the Blinn-Phong model.

Shading in "Trace"

The Trace project uses a version of the Blinn-Phong shading equation we derived in class, with two modifications:

- Distance attenuation is clamped to be at most 1:
  $$I_{\text{att}} = \min\left\{ \frac{1}{d^2 + d_{\text{min}}^2} \right\}$$

- Shadow attenuation is included.

Here's what it should look like:

$$I = I_{\text{att}} + \sum f_j I_{\text{shadow}} g_j E_j \cdot [N_j (N_j \cdot I_j) + b_j (N_j \cdot H_j)]$$

This is the shading equation to use in the Trace project!

Ray-tracing pseudocode

We build a ray-traced image by casting rays through each of the pixels.

```plaintext
function traceImage(scene):
    for each pixel $(i, j)$ in image
        $A = pixelToWorld(i, j)$
        $P = A + \epsilon d$
        $d = (A + 0.5) / \epsilon - P$
        $I(i, j) = traceRay(scene, P, d)$
    end for
end function

function traceRay(scene, P, d):
    $N, mtrl \leftarrow scene.intersect(P, d)$
    Q = ray $(P, d)$ evaluated at $t = 1$
    $I = shader(mtrl, N, d, Q, scene)$
    return $I$
end function
```

Shading pseudocode

Next, we need to calculate the color returned by the shading function.

```plaintext
function shade(mtrl, scene, Q, N, d):
    $I = mtrl.k_d + mtrl.k_s \cdot I_{\text{shadow}}$
    for each light source $Light$ do:
        $atten = Light > distance\_attenuation( Q )$
        $L = Light > get\_direction( Q )$
        $I = I + atten \cdot (I_{\text{diffuse term}} + I_{\text{specular term}})$
    end for
    return $I$
end function
```
Ray casting with shadows

Now we'll add shadows by casting shadow rays:

Shading with shadows

To include shadows, we need to modify the shade function:

```
function shade(material, scene, Q, N, d)
    I ← mtransfer + mtransfer * I_d
    for each light source Light do:
       atten ← Light -> distance attenuation(0) +
               Light -> shadow attenuation(scene, Q)
        L ← Light -> getDirection(Q)
        I ← I + atten * (diffuse term + specular term)
    end for
    return I
end function
```

Shadow attenuation

Computing a shadow can be as simple as checking to see if a ray makes it to the light source.

For a point light source:

```
function PointLight:shadow attenuation(scene, P)
    d ← getDirection(P)
    (L: N, m) ← scene.intersect(P, d)
    Compute light
    if (L < Llight) then:
        atten ← (0, 0, 0)
    else
        atten ← (1, 1, 1)
    end if
    return atten
end function
```

Recursive ray tracing with reflection

Now we'll add reflection:

Note: we will later handle color-filtered shadowing, so this function needs to return a color value.

For a directional light, \( \theta_{\text{light}} = \alpha \).
Shading with reflection

Let \( I(\rho, d) \) be the intensity seen along ray. Then:

\[ I(\rho, d) = I_{\text{direct}} + I_{\text{reflected}} \]

where:

- \( I_{\text{direct}} \) is computed from the Blinn-Phong model, plus shadow attenuation
- \( I_{\text{reflected}} = k_r I(O, R) \)

Typically, we set \( k_r = k_r \). (\( k_r \) is a color value.)

Ray-tracing pseudocode, revisited

```plaintext
function traceRay(scene, P, d):
    Q, N, mtr ← scene.intersect(P, d)
    Q ← ray(P, d) evaluated at t
    I ← shade(scene, mtr, Q, N, d)
    R ← reflectedDirection(-d, N)
    I ← I + mtr.kr * traceRay(scene, Q, R)
    return I
end function
```

Terminating recursion

Q. How do you bottom out of recursive ray tracing?

Possibilities:

1. \( \text{depth} = \text{depth}_{\text{max}} \) ⇒ return (stop recursion)
2. \( I = I_1 + k_{r_1} (I_2 + k_{r_2} (I_3 + k_{r_3} ( \ldots ))) \)
3. \( T T K_{r_i} < \text{thresh} \) Early ray termination

Reflection

Law of reflection:

\[ d = d' \]

R is co-planar with \( d \) and \( N \).
Whitted ray tracing

Finally, we'll add refraction, giving us the Whitted ray tracing model.

Shading with reflection and refraction

Let \( I(\mathbf{n}, \mathbf{d}) \) be the intensity seen along a ray. Then:

\[
I(\mathbf{n}, \mathbf{d}) = I_{\text{direct}} + I_{\text{refracted}} + I_{\text{transmit}}
\]

where

- \( I_{\text{direct}} \) is computed from the Blinn-Phong model plus shadow attenuation
- \( I_{\text{refracted}} = I_{\mathbf{Q}}(Q, R) \)
- \( I_{\text{transmit}} = I_{\mathbf{Q}}(Q, T) \)

Typically, we set \( k_r = k_t \) and \( k_d = 1 - k_r \) (or \( 0, 0, 0 \)); if opaque, where \( k_r \) is a color value.

(Generally, \( k_r \) and \( k_t \) are determined by “Fresnel reflection,” which depends on angle of incidence and changes the polarization of the light. This is discussed in Shirley’s textbook and can be implemented for extra credit.)

Refraction

Snell's law of refraction:

\[
\eta_1 \sin \theta_1 = \eta_2 \sin \theta_2
\]

where \( \eta_1, \eta_2 \) are indices of refraction.

In all cases, \( R \) and \( T \) are coplanar with \( d \) and \( N \).

The index of refraction is material dependent.

It can also vary with wavelength, an effect called dispersion that explains the colorful light rainbows from prisms. (We'll generally assume no dispersion.)

Total Internal Reflection

The equation for the angle of refraction can be computed from Snell's law:

\[
\frac{\eta_1}{\eta_2} \sin \theta_1 = \sin \theta_2 \quad \Rightarrow \quad \frac{\eta_1}{\eta_2} \sin \theta_1 = \sin \theta_2 = \sin^{-1}
\]

What happens when \( \eta_1 > \eta_2 \)?

When \( \theta_1 \) is exactly 90°, we say that \( \theta_1 \) has achieved the “critical angle” \( \theta_c \).

For \( \theta_1 > \theta_c \), no rays are transmitted, and only reflection occurs, a phenomenon known as “total internal reflection” or TIR.
Shirley handout

Shirley uses different symbols. Here is the translation between them:

\[ r = R \]
\[ t = T \]
\[ a = d \]
\[ b = d = c \]
\[ n = \eta \]
\[ n_s = \eta_s \]

Also, Shirley has two important errors that have already been corrected in the handout.

But if you’re consulting the original text, be sure to refer to the errata posted on the syllabus and on the project page for corrections.

Ray-tracing pseudocode, revisited

```
function traceRay(scene, P, d):
    t, N, mtrl[0] ← scene.intersect(P, d)
    I ← ray(P, d) evaluated at t
    I ← shade(scene, mtrl, f, N, d]
    R ← reflectDirection(N, d)
    if ray is entering object then
        n_i ← index_of_air
        n_t ← mtrl.index
    else
        n_i ← mtrl.index
        n_t ← index_of_air
    if \( \bar{n} \cdot \bar{N} \cdot \bar{N} \) then
        T ← refractDirection(N, d)
    I ← I + mtrl.\( c_s \cdot \) traceRay(scene, O, T)
end if
return I
end function
```

Q: How do we decide if a ray is entering the object?

Terminating recursion, incl. refraction

Q: How do you bottom out of recursive ray tracing?

\[ T \cdot K_r \cdot k_3 < \text{thresh} \Rightarrow \text{terminate} \]

Shadow attenuation (cont’d)

Q: What if there are transparent objects along a path to the light source?

We’ll take the view that the color is really only at the surface, like a glass object with a colored transparency coating on it. In this case, we multiply in the transparency constant, \( K_p \), every time an object is entered or exited, possibly more than once for the same object.
Shadow attenuation (cont’d)

Another model would be to treat the glass as solidly colored in the interior. Shirley’s textbook describes the resulting volumetric attenuation based on Beer’s Law, which you can implement for extra credit.

Photon mapping

Combine light ray tracing (photon tracing) and eye ray tracing:

...to get photon mapping.

Normals and shading when inside

When a ray is inside an object and intersects the object’s surface on the way out, the normal will be pointing away from the ray (i.e., the normal always points to the outside by default).

You must negate the normal before doing any of the shading, reflection, and refraction that follows.

Finally, when shading a point inside of an object, apply \( k_a \) to the ambient component, since that "ambient light" had to pass through the object to get there in the first place.

Intersecting rays with spheres

Now we’ve done everything except figure out what that "spheres intersect?" function does.

 Mostly, it calls each object to find out the value at which the ray intersects the object. Let’s start with intersecting spheres...

Given:

1. The coordinates of a point along a ray passing through \( P \) in the direction \( d \):

\[
\begin{align*}
X &= X_0 + t d_x \\
Y &= Y_0 + t d_y \\
Z &= Z_0 + t d_z
\end{align*}
\]

2. A unit sphere \( S \) centered at the origin defined by the equation:

\[
x^2 + y^2 + z^2 = 1
\]

Find: the point \( t \) at which the ray intersects \( S \).
Intersecting rays with spheres

Solution by substitution:

$$x^2 + y^2 + z^2 - 1 = 0$$

$$(p_x + td_x)^2 + (p_y + td_y)^2 + (p_z + td_z)^2 - 1 = 0$$

$$at^2 + bt + c = 0$$

where

$$a = d_x^2 + d_y^2 + d_z^2$$
$$b = 2(p_x d_x + p_y d_y + p_z d_z)$$
$$c = p_x^2 + p_y^2 + p_z^2 - 1$$

Q: What are the solutions of the quadratic equation in $t$ and what do they mean?

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

Q: What is the normal to the sphere at a point $(p_x, p_y, p_z)$ on the sphere?

$$N = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Ray-plane intersection

We can write the equation of a plane as:

$$ax + by + cz = k$$

The coefficients $a$, $b$, and $c$ form a vector that is normal to the plane $n = (a \ b \ c)^T$. Thus, we can rewrite the plane equation as:

We can solve for the intersection parameter (and thus the point):

$$r(t) = p + td$$

$$t \cdot \hat{n} = k$$

$$\hat{n} \cdot p + t \hat{n} \cdot d = k$$

$$\hat{n} \cdot d = 0 \Rightarrow \text{no}$$

Ray-triangle intersection

To intersect with a triangle, we first solve for the equation of its supporting plane.

How might we compute the (un-normalized) normal?

$$\hat{n} = (B - A) \times (C - A)$$

Given this normal, how would we compute $K$?

$$\hat{n} \cdot A = \hat{n} \cdot B = \hat{n} \cdot C = k$$

Using these coefficients, we can solve for $Q$. Now we need to decide if $Q$ is inside or outside of the triangle.

3D inside-outside test

One way to do this “inside-outside test,” is to see if $Q$ lies on the left side of each edge as we move counterclockwise around the triangle.

How might we use cross products to do this?

$$\left[(B-A)\times(Q-A)\right] \cdot N \geq 0 \Rightarrow \text{all true } \Rightarrow Q \text{ inside } \triangle ABC$$

$$\left[(C-B)\times(Q-B)\right] \cdot N \geq 0 \Rightarrow Q \text{ inside } \triangle ABC$$

$$\left[(A-C)\times(Q-C)\right] \cdot N \geq 0$$

$$\Rightarrow Q \text{ inside } \triangle ABC$$
2D inside-outside test

Without loss of generality, we can perform this same test after projecting down a dimension:

If Q is inside of ABC, then Q is inside of A'B'C'.

Why is this projection desirable? Faster.

Which axes should you "project away"? Project away the one that is largest.

Barycentric coordinates

As we'll see in a moment, it is often useful to represent Q as an affine combination of A, B, and C:

\[ Q = \alpha A + \beta B + \gamma C \]

where:

\[ \alpha + \beta + \gamma = 1 \]

We call \( \alpha, \beta, \) and \( \gamma \) the barycentric coordinates of \( Q \) with respect to \( A, B, \) and \( C. \)

\[ Q = \beta (B-A) + \gamma (C-A) + A = \beta B - \beta A + \gamma C - \gamma A + A = (1-\beta-\gamma)A + \beta B + \gamma C \]

\[ \alpha + \beta + \gamma = 1 \]

Computing barycentric coordinates

Given a point \( Q \) that is inside of triangle \( ABC, \) we can solve for \( Q \)'s barycentric coordinates in a simple way:

\[ \alpha = \frac{\text{Area}(QBC)}{\text{Area}(ABC)} \]

\[ \beta = \frac{\text{Area}(QAC)}{\text{Area}(ABC)} \]

\[ \gamma = \frac{\text{Area}(QAB)}{\text{Area}(ABC)} \]

How can cross products help here?

\[ \text{Area}(ABC) = \frac{1}{2} \left| (B-A) \times (C-A) \right| \]

In the end, these calculations can be performed in the 2D projection as well.

Interpolating vertex properties

The barycentric coordinates can also be used to interpolate vertex properties such as:

- material properties
- texture coordinates
- normals

For example:

\[ k_2(Q) = \alpha k_2(A) + \beta k_2(B) + \gamma k_2(C) \]

Interpolating normals, known as Phong interpolation, gives triangle meshes a smooth shading appearance. (Note: don't forget to normalize interpolated normals.)

\[ \hat{N} = \frac{N}{\|N\|} \]
Epsilons

Due to finite precision arithmetic, we do not always get the exact intersection at a surface.

Q: What kinds of problems might this cause?

Q: How might we resolve this?

$ t > RAY\_EPSILON$

t be considered an $N$

Intersecting with xformed geometry

In general, objects will be placed using transformations. What if the object being intersected were transformed by a matrix $M$?

Apply $M^{-1}$ to the ray first and intersect in object (local) $P$ coordinates.

$M^{-1}(Q = P + t_0d)$

$Q' = M^{-1}(P + t_0 d)$

$Q' = M^{-1}P + M^{-1}t_0d$

$Q' = M^{-1}P + t_0 M^{-1}d$

$Q' = P' + t_0 d'$

Intersecting with xformed geometry

The intersected normal is in object (local) coordinates. How do we transform it to world coordinates?

$$a \cdot b = a^T b$$

$$A^T A = I$$

$$(A^T A)^T = I^T$$

$$A^T (A^T) = I$$

$$A^T (A^T)^t = I$$

$$A^T = A^{-1}$$

$$M = \begin{bmatrix} U & a \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} U \\ 0 \end{bmatrix} \begin{bmatrix} t \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$N^T t_1 = 0$$

$$(N)^T U t_1 = 0$$

$$v^T t_1 = 0$$

$$v = U^{-1}N$$

$$(v)^T U v = 0$$

$$(v)^T = (N)^T U^{-1} v$$

$$t_1 = R - Q$$

$$M t_1 = M(R - Q) = MR - MQ = R - Q \times t_1$$

In the limit, $mt_1 = t_1$ and $Mt_1 = t_2$.

--- drop the happy coord for all vectors

$$t_1 = U t_1$$

$$t_2 = U t_2$$

$$N^T t_1 = 0$$

$$v^T t_1 = 0$$

$$v^T N = 0$$

$$v = U^{-1} N$$

$$v \sim N$$
Summary

What to take home from this lecture:

- The meanings of all the boldfaced terms.
- Enough to implement basic recursive ray tracing.
- How reflection and transmission directions are computed.
- How ray-object intersection tests are performed on spheres, planes, and triangles.
- How barycentric coordinates within triangles are computed.
- How ray epsilons are used.