Hierarchical Modeling
Reading

Required:

- Angel, sections 10.1 – 10.6, 10.8

Optional:

- *OpenGL Programming Guide*, chapter 3
Symbols and instances

Most graphics APIs support a few geometric primitives:
- spheres
- cubes
- cylinders

These symbols are instanced using an instance transformation.

Q: What is the matrix for the instance transformation above?

\[ M = TRS \]
3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:

- Base rotates about its vertical axis by $\theta$
- Upper arm rotates in its $xy$-plane by $\phi$
- Lower arm rotates in its $xy$-plane by $\psi$

(Note that the angles are set to zero in the figure; i.e., the parts are shown in their “default” positions.)

Q: What matrix do we use to transform the base?

Q: What matrix for the upper arm?

Q: What matrix for the lower arm?

\[
\begin{align*}
    R_y(\theta) & \quad T(0, h_1, 0) & \quad R_2(\phi) & \quad T(0, h_2, 0) & \quad R_2(\psi) \\
    \text{base} & \quad \text{upper arm} & \quad \text{lower arm} \\
\end{align*}
\]
3D Example: A robot arm

An alternative interpretation is that we are taking the original coordinate frames...

... and translating and rotating them into place:
Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```c
Matrix M_model;

main()
{
    ...
    robot_arm();
    ...
}

robot_arm()
{
    M_model = R_y(theta);
    base();
    M_model = R_y(theta)*T(0, h1, 0)*R_z(phi);
    upper_arm();
    M_model = R_y(theta)*T(0, h1, 0)*R_z(phi)
               *T(0, h2, 0)*R_z(psi);
    lower_arm();
}

Do the matrix computations seem wasteful?
Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it in place by concatenating matrices on the right:

Matrix M_model;

main()
{
    ... 
    M_model = Identity();
    robot_arm();
    ... 
}

robot_arm()
{
    M_model *= R_y(theta);
    base();
    M_model *= T(0, h1, 0)*R_z(phi);
    upper_arm();
    M_model *= T(0, h2, 0)*R_z(psi);
    lower_arm();
}
Robot arm implementation, OpenGL

OpenGL maintains a global state matrix called the **model-view matrix**, which is updated by concatenating matrices on the **right**.

```c
main()
{
    ...
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
    robot_arm();
    ...
}

robot_arm()
{
    glRotatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi, 0.0, 0.0, 1.0 );
    lower_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi, 0.0, 0.0, 1.0 );
    upper_arm();
}
```
Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:

- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

How might we draw the tree for the robot arm?
A complex example: human figure

Q: What's the most sensible way to traverse this tree?

depth first search / stack
figure()
{
    torso();
    glPushMatrix();
    glTranslate( ... );
    glRotate( ... );
    head();
    glPopMatrix();
    glPushMatrix();
    glTranslate( ... );
    glRotate( ... );
    left_upper_arm();
    glPushMatrix();
    glTranslate( ... );
    glRotate( ... );
    left_lower_arm();
    glPopMatrix();
    glPopMatrix();
    glPopMatrix();
    ...
Animation

The above examples are called articulated models:

- rigid parts
- connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.
Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- Each joint specified at various **key frames** (not necessarily the same as other joints)
- System does interpolation or **in-betweening**

Doing this well requires:

- A way of smoothly interpolating key frames: **splines**
- A good interactive system
- A lot of skill on the part of the animator
Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- many different objects
- lights
- camera position

This is called a **scene tree** or **scene graph**.
Summary

Here’s what you should take home from this lecture:

- All the **boldfaced terms**.
- How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- How the notion of a model tree or DAG can be extended to entire scenes.
- How OpenGL transformations can be used in hierarchical modeling.
- How keyframe animation works.