Distribution Ray Tracing
Reading

Required:

♦ Shirley, section 10.11

Further reading:

♦ Watt, sections 10.4-10.5
♦ A. Glassner. An Introduction to Ray Tracing. Academic Press, 1989. [In the lab.]
Pixel anti-aliasing

No anti-aliasing

\[ \frac{1}{A_{\text{pixel}}} \int l(x) dx \]

Pixel anti-aliasing
BRDF, revisited

The reflection model on the previous slide assumes that inter-reflection behaves in a mirror-like fashion.

Recall that we could view light reflection in terms of the general **Bi-directional Reflectance Distribution Function (BRDF):**

\[ f_r(\omega_{in}, \omega_{out}) \]

Sometimes this is written as:

\[ f_r(\omega_{in} \rightarrow \omega_{out}) \]

Which we could visualize for a given \( \omega_{in} \):

This is like a ray of light coming in at direction \( \omega_{in} \) and scattering into directions \( \omega_{out} \).
Surface reflection equation

BRDF’s exhibit reciprocity:

\[ f_r(\omega_{in} \rightarrow \omega_{out}) = f_r(\omega_{out} \rightarrow \omega_{in}) \]

This, combined with the idea of tracing rays from the viewer into the scene, means that we can turn things around:

Now, we can think of the BRDF as weighting light coming in from all directions \( \omega_{in} \) and summing their effect into \( \omega_{out} \).

This idea gives rise to the surface reflection equation:

\[ l(\omega_{out}) = \int_H l(\omega_{in}) f_r(\omega_{in} \rightarrow \omega_{out})(\omega_{in} \cdot \mathbf{N}) \, d\omega_{in} \]

Where we are integrating over all incoming directions from the hemisphere \( H \) above the surface point.
Simulating gloss and translucency

The mirror-like form of reflection, when used to approximate glossy surfaces, introduces a kind of aliasing, because we are undersampling reflection (and refraction).

For example:

Distributing rays over reflection directions gives:
Reflection anti-aliasing
Pixel and reflection anti-aliasing

\[ \frac{1}{A_{\text{pixel}}} \int l(x)dx \]

\[ \int_{\omega_{\text{in}}} I(\omega_{\text{in}}) f_r(\omega_{\text{in}} \rightarrow \omega_{\text{out}})(\omega_{\text{in}} \cdot \mathbf{N}) d\omega_{\text{in}} \]
Full anti-aliasing... lots of nested integrals!

Computing these integrals is prohibitively expensive, especially after following the rays recursively.

We'll look at ways to approximate high-dimensional integrals...
Glossy reflection revisited

Let's return to the glossy reflection model, and modify it – for purposes of illustration – as follows:

We can visualize the span of rays we want to integrate over, within a pixel:
Whitted ray tracing

Returning to the reflection example, Whitted ray tracing replaces the glossy reflection with mirror reflection:

Thus, we render with anti-aliasing as follows:
Monte Carlo path tracing

Let’s return to our original (simplified) glossy reflection model:

An alternative way to follow rays is by making random decisions along the way – a.k.a., Monte Carlo path tracing. If we distribute rays uniformly over pixels and reflection directions, we get:
Importance sampling

The problem is that lots of samples are “wasted.” Using again the glossy reflection model:

Let’s now randomly choose rays, but according to a probability that favors more important reflection directions, i.e., use importance sampling:
Stratified sampling

We still have a problem that rays may be clumped together. Let’s simplify the reflection model some and split it into zones:

Now let’s restrict our randomness to within these zones, i.e. use **stratified sampling**:
Stratified sampling of a 2D pixel

Here we see random (pure Monte Carlo) vs. stratified sampling over a 2D pixel (here 16 rays/pixel):

![Random sampling pattern](image)

![Stratified sampling pattern](image)

The stratified pattern on the right is also sometimes called a **jittered** sampling pattern.

One interesting side effect of these stochastic sampling patterns is that they actually injects noise into the solution (slightly grainier images). This noise tends to be less objectionable than aliasing artifacts.
Distribution ray tracing

These ideas can be combined to give a particular method called **distribution ray tracing** [Cook84]:

- uses non-uniform (jittered) samples.
- replaces aliasing artifacts with noise.
- provides additional effects by distributing rays to sample:
  - Reflections and refractions
  - Light source area
  - Camera lens area
  - Time

[This approach was originally called “distributed ray tracing,” but we will call it distribution ray tracing (as in probability distributions) so as not to confuse it with a parallel computing approach.]
DRT pseudocode

`TraceImage()` looks basically the same, except now each pixel records the average color of jittered sub-pixel rays.

```plaintext
function traceimage (scene):
    for each pixel (i, j) in image do
        l(i, j) ← 0
        for each sub-pixel id in (i, j) do
            s ← pixelToWorld(jitter(i, j, id))
            p ← COP
            d ← (s - p).normalize()
            l(i, j) ← l(i, j) + traceRay(scene, p, d, id)
        end for
        l(i, j) ← l(i, j)/numSubPixels
    end for
end function
```

A typical choice is numSubPixels = 5*5.
DRT pseudocode (cont’d)

Now consider \textit{traceRay()}, modified to handle (only) opaque glossy surfaces:

\begin{verbatim}
function traceRay(scene, \textbf{p}, \textbf{d}, \textit{id}):
    (\textbf{q}, \textbf{N}, \textit{material}) ← intersect(scene, \textbf{p}, \textbf{d})
    \textit{l} ← shade(…)
    \textbf{R} ← jitteredReflectDirection(material, \textbf{N}, -\textbf{d}, \textit{id})
    \textit{l} ← \textit{l} + material.\textit{k}_r * traceRay(scene, \textbf{q}, \textbf{R}, \textit{id})
    return \textit{l}
\end{verbatim}

end function

Pre-sampling glossy reflections
Soft shadows

Distributing rays over light source area gives:
The pinhole camera, revisited

Recall the pinhole camera:

![Pinhole camera diagram]

We can equivalently turn this around by following rays from the viewer:

![Pinhole camera diagram]
The pinhole camera, revisited

Given this flipped version:

Pinhole camera

how can we simulate a pinhole camera more accurately?
Lenses

Pinhole cameras in the real world require small apertures to keep the image in focus.

Lenses focus a bundle of rays to one point => can have larger aperture.

For a “thin” lens, we can approximately calculate where an object point will be in focus using the the Gaussian lens formula:

\[
\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}
\]

where \( f \) is the **focal length** of the lens.
Depth of field

Lenses do have some limitations. The most noticeable is the fact that points that are not in the object plane will appear out of focus.

The **depth of field** is a measure of how far from the object plane points can be before appearing “too blurry.”

![Diagram of depth of field](http://www.cambridgeincolour.com/tutorials/depth-of-field.htm)
Simulating depth of field

Distributing rays over a finite aperture gives:
Chaining the ray id’s

In general, you can trace rays through a scene and keep track of their id’s to handle all of these effects:
DRT to simulate motion blur.

Distributing rays over time gives:
Summary

What to take home from this lecture:

1. The limitations of Whitted ray tracing.
2. How distribution ray tracing works and what effects it can simulate.