Computer Graphics *Instructor: Zoran Popovic*

CSE P 557, Autumn 2003

Homework #1

Display Devices, Image Processing, Affine Transformations, Hierarchical Modeling

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Assigned: Tuesday, October 21, 2003
Due: Tuesday, November 4, 2003, **at the beginning of class**

Points: 63 + 10 extra

Directions: Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to discuss the problems with classmates, but please ***answer the questions on your own*.**

Name:____

Problem 1: Short Answer (10 points)

Provide a short answer (typically one or two sentences) to each of the following questions. In each case, you must clearly justify your answer.

1. (2 points) Color lookup tables are useful even when used with a true color frame buffer. Give an example of what the lookup table might be used for.

2. (2 points) Which blurring filter will do a better job removing salt and pepper noise: median or mean? Why?

3. (2 points) Why do we use a 4x4 matrix for transformations in 3-space?

4. (2 points) Name one application where you would use a parallel projection instead of a perspective projection. Explain why.

5. (2 points) Explain what happens when you push and pop the transformation matrix in OpenGL.

Problem 2: Display Devices (9 points)

In order to allow more time for transmitting and displaying each pixel, the American broadcast television standard (NTSC) uses an "interlaced" type of refresh, in which each video frame is broken into two "fields," each containing one-half of the picture. The two fields are "interlaced" in the sense that each field contains every other scan line: all odd-numbered scan lines are displayed in the first field, and all even-numbered scan lines are displayed in the second.

The purpose of an interlaced scan is to place some new information in all areas of the screen at a high enough rate to avoid flicker, while allowing the hardware more time for accessing and displaying each pixel.

1. (1 point) If the video controller displays each field in $1/60^{th}$ of a second, what is the overall frame rate for displaying the entire screen?

2. (2 points each) An interlaced refresh works well as long as adjacent scan lines display similar information. In which parts, if any, of the following images would you expect to see flicker on an interlaced display (and briefly mention why):

- A single-pixel-wide horizontal white line on a black background?

- A single pixel wide vertical white line on a black background?

- A checkerboard of black and white, where each black or white square is a single pixel?

3. (2 points) On interlaced displays, there is a very noticeable artifact when objects on the screen are moving fast. What is this artifact, and how will it appear if the video is of a white box moving horizontally on a black background?

Problem 3: Filters (12 points)

There is an image $I(x,y)$ and two kernel filters $G_1(i,j)$ and $G_2(i,j)$.

$$G_1 = \begin{cases} (\pm 1,0) & := 1/4 \\ (0,0) & := 1/2 \\ \text{otherwise} & ,\ 0 \end{cases} \qquad G_2 = \begin{cases} (0,\pm 1) & := 1/4 \\ (0,0) & := 1/2 \\ \text{otherwise} & ,\ 0 \end{cases}$$

1. (1 points) Fill in the corresponding 3x3 filter masks for G_1 and G_2.

$\mathbf{G_1}$

$\mathbf{G_2}$

2. (2 points) If we applied filter $\mathbf{G_1}$ to an image $\mathbf{I}$ as $\mathbf{G_1} \times \mathbf{I}$, describe in words the resulting image transformation.

3. (2 points) Apply filter G_1 to filter G_2 using convolution. Assume zero values extending from the edges of filter G_2. If you were to use the result as a filter, what kind of filter would it be?

$G_2 \times G_1$

4. (3 points) Convolution is an associative operation. What can we say about the result of applying both filters G_1 and G_2 to an image, one after the other, using convolution? Explain.

5. (3 points) Create a 3x3 filter G_3 such that $I \times G_3 \approx I - \nabla^2 I$. Assume that the Laplacian filter is $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

G_3

6. (1 point) What sort of use might we have for filter G_3? **Hint: Try it in Photoshop.**

Problem 4: Perspective Projections (6 points)

Recall the matrix for perspective projection $P(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix}$

1. (4 points) The perspective projection matrix takes a set of points p_i in 3-space and maps them to points on the viewing plane $p_i' = P(d)p_i$. If we first apply a uniform scale by a factor s to all points in 3-space, before performing the projection indicated by this matrix $p_i' = P(d)S(s)p_i$, how will the projected points be different from the case where no scale is applied? Show your reasoning mathematically and explain the result geometrically (in words and/or with a sketch). ***Hint: it may be useful to think about what this looks like in two dimensions instead of three.***

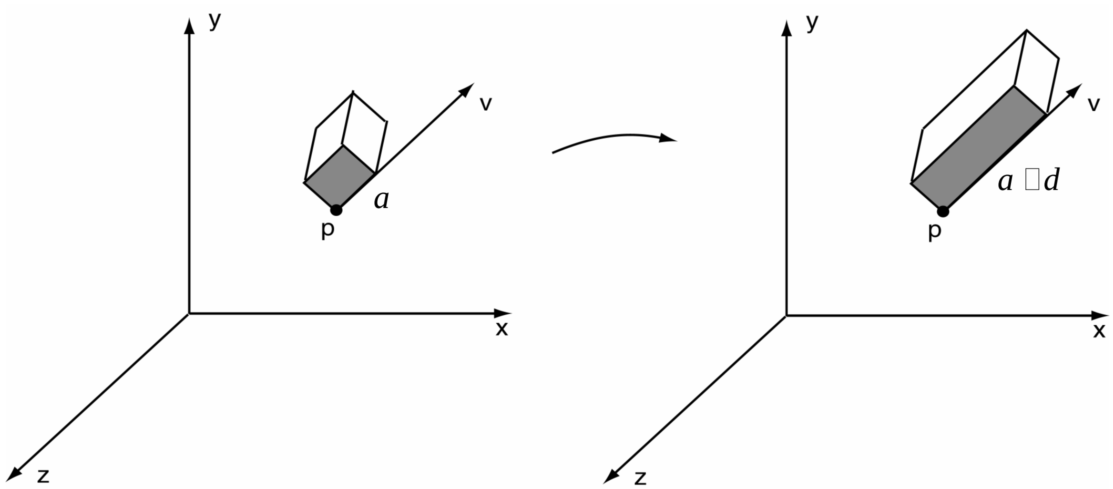
2. (2 points) Using the perspective projection matrix above, we observed in class that any set of parallel lines will converge and intersect at a vanishing point. In fact, some parallel lines will ***not*** intersect, i.e., they will remain parallel, even after undergoing this perspective projection. Which lines are these? Geometrically speaking, what do all sets of these lines have in common?

Problem 5: 3D Affine transformations (6 points)

The basic scaling matrix discussed in lecture scales only with respect to the x, y, and/or z axes. Using the basic translation, scaling, and rotation matrices, specify how to build a transformation matrix that scales along any ray in 3D space. This new transformation is described by the ray origin $\mathbf{p} = (x_0, y_0, z_0)$ and a ***unit*** direction vector $\mathbf{v} = (x_1, y_1, z_1)$, and the amount of *s*.

You can use any of the following standard matrices (from lecture) as building blocks: Euler angle rotations $R_x(\theta)$, $R_y(\theta)$, $R_z(\theta)$, scales $S(s_x,s_y,s_z)$, and translations $T(t_x,t_y,t_z)$. You don't need to compute exact formulas for the rotation angles, but you must describe how to compute each of the rotation angles using words and drawings. You don't need to write out the entries of the 4x4 matrices, it is sufficient to use the symbols given above.

For clarity, a diagram has been provided, showing a box with an edge of length *a* being scaled by a factor *d* with respect to a given ray. Your answer should work for any ray, not just the case shown in the picture.



Problem 6: Hierarchies (14 points)

Suppose you want to model the sun, Jupiter, Jupiter's moon Io, and the satellite Galileo using these transformations:

$R(q)$ rotate q degrees clockwise
$T(x, y)$ translate by x and y

The following constrains define the arrangement of the model:

- Jupiter orbits the sun at a distance d from the center.
- Io and Galileo both orbit Jupiter at distance 10 from its center.
- The same side of Io should always be facing Jupiter no matter what position it is at in its orbit as shown in Figure 2 (think about how the same features on the moon are always visible from earth).
- The same side of Galileo should always point parallel to the axis between the center of the sun and the center of Jupiter as shown in Figure 2 (so that it always has solar power).

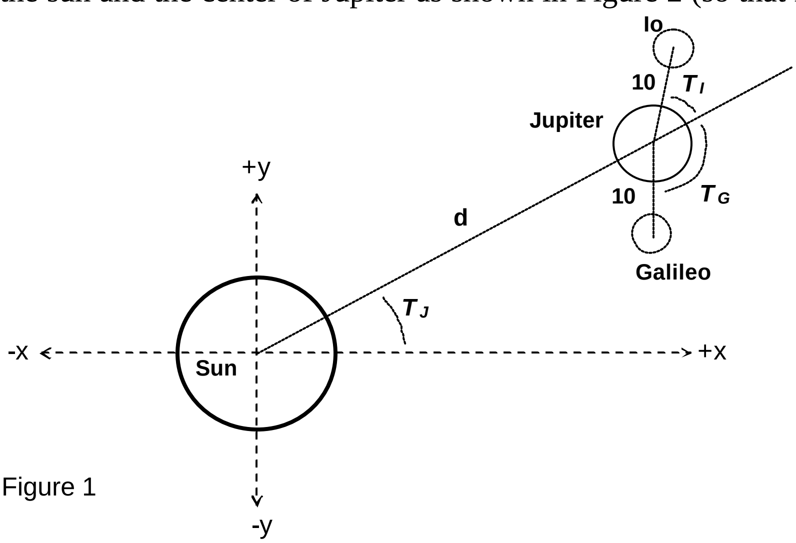


Figure 1

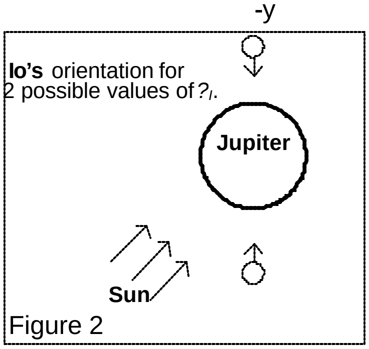


Figure 2

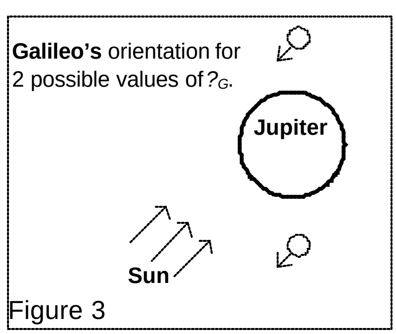


Figure 3

Arrows in Figure 2 and Figure 3 indicate the orientation of Io and Galileo respectively.

1. (10 points) Construct a transformation tree, rooted at the sun, that describes the position of the sun, Jupiter, Io, and Galileo for a given set of parameters d, $?_I$, $?_J$ and $?_G$. Along each of the edges of the tree, write expressions for the transformations that are applied along that edge, using the notation given above (you do not need to write out the matrices). Remember that order is important!

2. (2 points) Write out the full transformation expression for Io. Order the parts of the transformation so that if **T** is your complete transformation and p is a point that we want to transform then **T***p will transform the point p correctly.

3. (2 points) Write out the full transformation expression for Galileo. Order the parts of the transformation so that if **T** is your complete transformation and p is a point that we want to transform then **T***p will transform the point p correctly.

Problem 7: Inverse Kinematics (6 points)

In the hierarchies we have studied in class so far, we have dealt only with forward kinematics; that is, if we want to determine a particular position of a limb, we specify joint angles for all the joints such that the limb is positioned in the way we want.

In certain cases, however, it would be useful to instead specify coordinate positions of the parts directly; for example, if we wanted a character to pick up a coffee cup, it would be useful to specify the hand position to be located near the cup, instead of figuring out manually what joint angles would achieve the same goal. Determining joint angles automatically is known as inverse kinematics. In this problem, we will explore such a problem for the model described in Problem 6. Use the model from Problem 6 to answer the questions in this problem. **Assume $?_J = 90°$.**

1. (3 points) Suppose there is a black hole at $\left(5\sqrt{3},95\right)$ and the sun is at (0,0). What values for d and $?_G$ will cause Galileo's position to be the same as the black hole?

2. (3 points) Is this the only solution? Why?

Extra Credit: Rotations as Shear Transformations (10 points)

You are ***not*** required to do this problem! But extra points are good things, so we encourage you to give it a try.

In 2D, a rotation transformation by angle θ can be specified as a series of shear transformation matrices. Give these matrices, or if it can't be done, prove it.