6. Hidden Surface Algorithms

Introduction

In the previous lecture, we figured out how to transform the geometry so that the relative sizes will be correct if we drop the $z$ component.

But, how do we decide which geometry actually gets drawn to a pixel?

Known as the **hidden surface elimination problem** or the **visible surface determination problem**.

There are dozens of hidden surface algorithms.

They can be characterized in at least three ways:

- Object-precision vs. image-precision (a.k.a., object-space vs. image-space)
- Object order vs. image order
- Sort first vs. sort last

Object-precision algorithms

Basic idea:

- Operate on the geometric primitives themselves. (We'll use "object" and "primitive" interchangeably.)
- Objects typically intersected against each other
- Tests performed to high precision
- Finished list of visible objects can be drawn at any resolution

Complexity:

- For $n$ objects, can take $O(n^2)$ time to compute visibility.
- For an $mxm$ display, have to fill in colors for $m^2$ pixels.
- Overall complexity can be $O(k_{obj}n^2 + k_{disp}m^2)$.

Implementation:

- Difficult to implement
- Can get numerical problems

Reading

Reading:

- Watt, 6.6 (esp. intro and subsections 1, 4, and 8–10), 12.1.4.

Optional reading:

- Foley, van Dam, Feiner, Hughes, Chapter 15
**Image-precision algorithm**

**Basic idea:**
- Find the closest point as seen through each pixel
- Calculations performed at display resolution
- Does not require high precision

**Complexity:**
- Naïve approach checks all \( n \) objects at every pixel. Then, \( O(n \, m^2) \).
- Better approaches check only the objects that could be visible at each pixel. Let’s say, on average, \( d \) objects are visible at each pixel (a.k.a., depth complexity). Then, \( O(d \, m^2) \).

**Implementation:**
- Very simple to implement.
  - Used a lot in practice.

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**Object order vs. image order**

**Object order:**
- Consider each object only once, draw its pixels, and move on to the next object.
- Might draw the same pixel multiple times.

**Image order:**
- Consider each pixel only once, find nearest object, and move on to the next pixel.
- Might compute relationships between objects multiple times.

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**Sort first vs. sort last**

**Sort first:**
- Find some depth-based ordering of the objects relative to the camera, then draw back to front.
- Build an ordered data structure to avoid duplicating work.

**Sort last:**
- Sort implicitly as more information becomes available.

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**Outline of Lecture**

- Z-buffer
- Ray casting
- Binary space partitioning (BSP) trees
Z-buffer

The Z-buffer or depth buffer algorithm [Catmull, 1974] is probably the simplest and most widely used.

Here is pseudocode for the Z-buffer hidden surface algorithm:

for each pixel $(i,j)$ do
  Z-buffer,$[i,j] ←$ FAR
 Framebuffer$[i,j] ← <$background color$>
end for

for each polygon $A$ do
  for each pixel in $A$ do
    Compute depth $z$ and shade $s$ of $A$ at $(i,j)$
    if $z >$ Z-buffer $[i,j]$ then
      Z-buffer$[i,j] ← z$
     Framebuffer$[i,j] ← s$
    end if
  end for
end for

Q: What should FAR be set to?

Rasterization

The process of filling in the pixels inside of a polygon is called rasterization.

During rasterization, the z value and shade $s$ can be computed incrementally (fast!).

Curious fact:
- Described as the “brute-force image space algorithm” by [SSS]
- Mentioned only in Appendix B of [SSS] as a point of comparison for huge memories, but written off as totally impractical.

Today, Z-buffers are commonly implemented in hardware.

Clipping and the viewing frustum

The center of projection and the portion of the projection plane that map to the final image form an infinite pyramid. The sides of the pyramid are clipping planes.

Frequently, additional clipping planes are inserted to restrict the range of depths. These clipping planes are called the near and far or the hither and yon clipping planes.

All of the clipping planes bound the viewing frustum.

Computing z

In the lecture on projections, we said that we would apply the following 3x4 projective transformation:

\[
\begin{bmatrix}
  x' \\
y' \\
w'
\end{bmatrix} = \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & -1/d & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
w
\end{bmatrix}
\]

and keep around the z-component to do Z-buffering, essentially setting $z' = z$.

Strictly speaking, in order for interpolated $z$ to work correctly, we actually need to map it according to:

\[
z' = A + B/z
\]

For $B < 0$, is depth ordering preserved?

In addition, we have finite precision and would like all of our z bits to be uniformly distributed between the clipping planes.
Computing $z$, cont’d

These requirements lead to the following $4 \times 4$ projective transformation:

$$
\begin{bmatrix}
    x' \\
    y' \\
    z' \\
    w'
\end{bmatrix} = 
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & \frac{f + n}{d(f - n)} & \frac{2fn}{d(f - n)} \\
    0 & 0 & -1/d & 0
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    z \\
    1
\end{bmatrix}
$$

What is $z'$ after the perspective divide?

What do $z = -n$ and $z = -f$ get mapped to?

Z-buffer: Analysis

- Classification?
- Easy to implement?
- Easy to implement in hardware?
- Incremental drawing calculations (uses coherence)?
- Pre-processing required?
- On-line (doesn’t need all objects before drawing begins)?
- If objects move, does it take more work than normal to draw the frame?
- If the viewer moves, does it take more work than normal to draw the frame?
- Typically polygon-based?
- Efficient shading (doesn’t compute colors of hidden surfaces)?
- Handles transparency?
- Handles refraction?

Ray casting

Idea: For each pixel center $P_{ij}$

- Send ray from eye point (COP), $C$, through $P_{ij}$ into scene.
- Intersect ray with each object.
- Select nearest intersection.

Ray casting, cont.

Implementation:

- Might parameterize each ray:
  $$\mathbf{r}(t) = \mathbf{C} + t (\mathbf{P}_{ij} - \mathbf{C})$$

- Each object $O_k$ returns $t_k > 0$ such that first intersection with $O_k$ occurs at $\mathbf{r}(t_k)$.

Q: Given the set $\{t_k\}$, what is the first intersection point?

Note: these calculations generally happen in world coordinates. No projective matrices are applied.
Ray casting: Analysis

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Binary-space partitioning (BSP) trees

![Diagram of BSP tree]

Idea:
- Do extra preprocessing to allow quick display from any viewpoint.

Key observation: A polygon $A$ is painted in correct order if

- Polygons on far side of $A$ are painted first
- $A$ is painted next
- Polygons in front of $A$ are painted last.

BSP tree creation

![BSP tree creation diagram]

procedure $\text{MakeBSPTree}$:
  takes $\text{PolygonList} \ L$
  returns $\text{BSPTree}$
  Choose polygon $A$ from $L$ to serve as root
  Split all polygons in $L$ according to $A$
  node $\leftarrow A$
  $node.\text{neg} \leftarrow \text{MakeBSPTree}(\text{Polygons on neg. side of } A)$
  $node.\text{pos} \leftarrow \text{MakeBSPTree}(\text{Polygons on pos. side of } A)$
  return node
end procedure

Note: Performance is improved when fewer polygons are split --- in practice, best of ~ 5 random splitting polygons are chosen.

Note: BSP is created in world coordinates. No projective matrices are applied.
**BSP tree display**

**procedure** \textit{DisplayBSPTree}:

**Takes** \textit{BSPTree} T

\textbf{if} T is empty \textbf{then return}

\textbf{if} viewer is in front (on pos. side) of \textit{T.node}

\textit{DisplayBSPTree}(T._____)

\textbf{Draw} \textit{T.node}

\textit{DisplayBSPTree}(T._____)

\textbf{else}

\textit{DisplayBSPTree}(T._____)

\textbf{Draw} \textit{T.node}

\textit{DisplayBSPTree}(T._____)

\textbf{end if}

\textbf{end procedure}

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**Cost of Z-buffering**

Z-buffering is the algorithm of choice for hardware rendering, so let’s think about how to make it run as fast as possible…

The steps involved in the Z-buffer algorithm are:

1. Send a triangle to the graphics hardware.
2. Transform the vertices of the triangle using the modeling matrix.
3. Shade the vertices.
4. Transform the vertices using the projection matrix.
5. Set up for incremental rasterization calculations
6. Rasterize and update the framebuffer according to \( z \).

What is the overall cost of Z-buffering?

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**BSP trees: Analysis**

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**Cost of Z-buffering, cont’d**

We can approximate the cost of this method as:

\[
\begin{align*}
\text{cost} &= k_{bus} v_{bus} + k_{shade,xform} v_{shade,xform} + k_{setup} \Delta_{rast} + d m^2
\end{align*}
\]

Where:

- \( k_{bus} \) = bus cost to send a vertex
- \( v_{bus} \) = number of vertices sent over the bus
- \( k_{shade,xform} \) = cost of transforming and shading a vertex
- \( v_{shade,xform} \) = number of vertices transformed and shaded
- \( k_{setup} \) = cost of setting up for rasterization
- \( \Delta_{rast} \) = number of triangles being rasterized
- \( d \) = depth complexity (average times a pixel is covered)
- \( m^2 \) = number of pixels in frame buffer
Visibility tricks for Z-buffers

Given this cost function:

\[ k_{\text{bus}} y_{\text{bus}} + k_{\text{shade,xfm}} y_{\text{shade,xfm}} + k_{\text{setup}} A_{\text{rast}} + d m^2 \]

what can we do to accelerate Z-buffering?

Summary

What to take home from this lecture:

- Classification of hidden surface algorithms
- Understanding of Z-buffer, ray casting, and BSP tree hidden surface algorithms
- Familiarity with some Z-buffer acceleration strategies