Rules:

- The exam is closed-book, closed-note, except for one two-sided 8.5x11in piece of paper.
- Please stop promptly at 8:20.
- You can rip apart the pages.
- There are 100 points total, distributed unevenly among 7 questions.
- The questions have multiple parts.

Advice:

- Read questions carefully. Understand a question before you start writing.
- Write down thoughts and intermediate steps so you can get partial credit.
- The questions are not necessarily in order of difficulty. **Skip around.** In particular, make sure you get to all the problems.
- If you have questions, ask.
- Relax. You are here to learn.
Name: ____________________________

For your reference (page 1 of 2):

\( s := \text{skip} \mid x := e \mid s; s \mid \text{if } e \; s \; s \mid \text{while } e \; s \)

\( e := i \mid x \mid e + e \mid e \ast e \)

\( (i \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}) \)

\( (x \in \{x_1, x_2, \ldots, y_1, y_2, \ldots, z_1, z_2, \ldots, \ldots\}) \)

\[ \begin{array}{l}
\quad H : e \downarrow i \\
\hline
\quad \text{CONST} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ Quad
\[ \Gamma \vdash e : \tau \]

\[
\begin{array}{ccc}
\Gamma \vdash c : \text{int} & \Gamma \vdash x : \Gamma(x) & \Gamma \vdash \lambda x. e : \tau_1 \rightarrow \tau_2 \\
\Gamma \vdash e_1 : \tau_1 & \Gamma \vdash e_2 : \tau_2 & \Gamma \vdash e_1 \cdot e_2 : \tau_1 \\
\Gamma \vdash (e_1, e_2) : \tau_1 \times \tau_2 & \Gamma \vdash e.1 : \tau_1 & \Gamma \vdash e.2 : \tau_2 \\
\Gamma \vdash \text{receive} : T\text{channel} \rightarrow \tau_1 \text{event} & \Gamma \vdash \text{unlock} : T\text{mutex} & \Gamma \vdash \text{lock} : T\text{mutex} \\
\Gamma \vdash \text{join} : T\text{mutex} & \Gamma \vdash \text{create} : (\tau \rightarrow \tau) \rightarrow \tau \\
\end{array}
\]

\[
\begin{array}{ccc}
\Gamma \vdash e_1 : \tau_1 & \Gamma \vdash e_2 : \tau_2 & \Gamma \vdash e_1 \cdot e_2 : \tau_1 \\
\Gamma \vdash e : \tau_1 \rightarrow \tau_2 & \Gamma \vdash e : \tau_1 \times \tau_2 & \Gamma \vdash e_1 : \tau_1 \\
\Gamma \vdash e, e : \tau_1 \rightarrow \tau_2 & \Gamma \vdash e, e : \tau_1 \times \tau_2 & \Gamma \vdash e : \tau_1 \rightarrow \tau_2, x : \tau_1 \vdash e : \tau_2 \\
\Gamma \vdash \text{letrec} f x. e : \tau_1 \rightarrow \tau_2 & \Gamma \vdash e : \{l_1 : \tau_1, \ldots, l_n : \tau_n\} & 1 \leq i \leq n \\
\Gamma \vdash e : \{l_1 : \tau_1, \ldots, l_n : \tau_n\} & \Gamma \vdash e.l_i : \tau_i & \Gamma \vdash e.1 : \tau_1 \\
\Gamma \vdash e : \tau_1 + \tau_2 & \Gamma \vdash e : \tau_1 + \tau_2 & \Gamma \vdash e : \tau_2 \\
\end{array}
\]

\[ \frac{\Gamma \vdash e : \tau \quad \tau \leq \tau'}{\Gamma \vdash e : \tau'} \]

\[
\begin{array}{c}
\{l_1 : \tau_1, \ldots, l_n : \tau_n, l : \tau\} \leq \{l_1 : \tau_1, \ldots, l_n : \tau_n\} \\
\{l_1 : \tau_1, \ldots, l_{i-1} : \tau_{i-1}, l_i : \tau_i, \ldots, l_n : \tau_n\} \leq \{l_1 : \tau_1, \ldots, l_{i-1} : \tau_{i-1}, \ldots, l_n : \tau_n\} \\
\{l_1 : \tau_1, \ldots, l_i : \tau_i, \ldots, l_n : \tau_n\} \leq \{l_1 : \tau_1, \ldots, l_i : \tau_i', \ldots, l_n : \tau_n\} \\
\tau_1 \leq \tau_1' \\
\end{array}
\]

\[
\frac{\tau_3 \leq \tau_1 \quad \tau_2 \leq \tau_1 \quad \tau \leq \tau}{\tau_3 \leq \tau_2 \leq \tau_1 \leq \tau_3} \quad \frac{\tau_1 \leq \tau_2 \quad \tau_2 \leq \tau_3}{\tau_1 \leq \tau_3}
\]

Module Thread:

type t
val create : (\'a \rightarrow \'b) \rightarrow \'a \rightarrow t
val join : t \rightarrow unit

Module Mutex:

type t
val create : unit \rightarrow t
val lock : t \rightarrow unit
val unlock : t \rightarrow unit

Module Event:

type \'a channel
val new : \'a channel
val new_channel : unit \rightarrow \'a channel
val send : \'a channel \rightarrow \'a \rightarrow unit event
val receive : \'a channel \rightarrow \'a event
val choose : \'a event list \rightarrow \'a event
val wrap : \'a event \rightarrow (\'a \rightarrow \'b) \rightarrow \'b event
val sync : \'a event \rightarrow \'a
1. (Caml Programming and Type Inference) (17 points)

(a) Write a function \texttt{wgo\_help1} such that this function:

\begin{verbatim}
let wgo1 max lst = wgo\_help1 0 max lst
\end{verbatim}

behaves as follows: If \texttt{lst} is a list of integers [i1;i2;...;in], then \texttt{wgo1} returns the sum of a prefix of the list \texttt{i1 ... ij} such that:

- The sum is less than \texttt{max}.
- Either the sum of the next-larger prefix \texttt{i1 ... ij i} is not less than \texttt{max} or there is no next-larger prefix (i.e., the entire list has a sum less than \texttt{max}).

Do not define any other helper functions. Note \texttt{wgo} stands for, “without going over.”

(b) What is the type of \texttt{wgo\_help1}?

(c) Write a function \texttt{wgo\_help2} such that this function:

\begin{verbatim}
let wgo2 max lst = wgo\_help2 (fun x -> x < max) (fun x y -> x+y) 0 lst
\end{verbatim}

has the same observable behavior as \texttt{wgo1}. Do not define any other functions. \texttt{wgo\_help2} should not contain an explicit addition.

(d) What is the type of \texttt{wgo\_help2}?

(e) Rewrite \texttt{wgo1} and \texttt{wgo2} to be shorter.
2. (IMP and translation) (15 points)

Here is our Caml abstract syntax for IMP with one new kind of statement described below:

```caml
type exp = Int of int | Var of string | Plus of exp * exp | Times of exp * exp
type stmt = Skip | Assign of string * exp | Seq of stmt * stmt
       | If of exp * stmt * stmt | While of exp * stmt
       | CompareAndSwap of string * exp * exp
```

Recall that in the semantics the expression in an if-statement or while-statement is true if it is not zero.

In this problem, we consider a new kind of statement in IMP. The semantics of \texttt{CompareAndSwap(s, e1, e2)} is as follows:

- If evaluating \texttt{e1} under the current heap produces the same value that variable \texttt{s} holds under the current heap, then update the heap so \texttt{s} holds the value that \texttt{e2} evaluates to under the current heap.
- Otherwise make no change to the heap.

In Caml, write a \textit{translation} from IMP-including-compare-and-swap statements to IMP-not-including-compare-and-swap statements. In other words, write a function \texttt{translate} of type \texttt{stmt -> stmt} such that (1) the result contains no compare-and-swap statements and (2) the result is equivalent to the argument.

Note: Some of you might recognize compare-and-swap as related to concurrency, but this problem has nothing to do with concurrency.
3. (Formal operational semantics) (18 points)

In this problem we define a small language that manipulates a stack of strings. You are given the syntax and the informal semantics.

The language syntax is a command list. A program state contains a command list and a stack of strings (the stack “grows to the right”):

\[
\begin{align*}
\text{string} & : \text{str} ::= \text{(any string)} \\
\text{command} & : \text{c} ::= \text{push str} \mid \text{pop} \mid \text{dup} \mid \text{swap} \\
\text{command-list} & : \text{lst} ::= [] \mid \text{c}\cdot\text{lst} \\
\text{stack} & : \text{stk} ::= \cdot \mid \text{stk}, \text{str}
\end{align*}
\]

Informally, the commands behave as follows:

- \textbf{push str} makes a bigger stack with \textit{str} on top.
- \textbf{pop} makes a smaller stack by removing the top element.
- \textbf{dup} (short for duplicate) makes a bigger stack by placing a copy of the top stack-element on top.
- \textbf{swap} swaps the order of the top two elements on the stack.

A command list executes the commands in order.

(a) Give large-step inference rules for the judgment \( \text{stk}_1; \text{lst} \downarrow \text{stk}_2 \), meaning, “running \text{lst} starting from \text{stk}_1 produces \text{stk}_2.” One rule is given to you as an example. You need to write down 4 other rules.

\[
\frac{}{\text{stk}_1, \text{str}; \text{lst} \downarrow \text{stk}_2} \\
(\text{push str})::\text{lst} \downarrow \text{stk}_2
\]

(b) The semantics can get stuck, i.e., there exists stacks \( \text{stk}_1 \) and command-lists \( \text{lst} \) such that we cannot derive \( \text{stk}_1; \text{lst} \downarrow \text{stk}_2 \) for any \( \text{stk}_2 \). In English, describe why there may be not be a derivation.

(c) Give a complete derivation that concludes \( \cdot; (\text{push “pl”})::\text{dup}::\text{swap}::[] \downarrow \cdot \), “pl”, “pl”
This page is intentionally blank since you might (or might not) want more room to answer problem 3.
4. (Formal typing rules) (15 points)

(a) Recall this typing rule, one of the three rules we added for sums:

$\Gamma \vdash e : \tau_1$

$\Gamma \vdash A(e) : \tau_1 + \tau_2$

Explain this rule in English. In particular, what expressions can this typing rule be used for and what types can it give to such expressions?

(b) Recall this typing rule for functions:

$\Gamma, x : \tau_1 \vdash e : \tau_2$

$\Gamma \vdash \lambda x. e : \tau_1 \rightarrow \tau_2$

Explain this rule in English. In particular, what expressions can this typing rule be used for and what types can it give to such expressions?

(c) Suppose we changed the typing rule for functions to the following:

$\Gamma, x : \tau_2 \vdash e : \tau_2$

$\Gamma \vdash \lambda x. e : \tau_2 \rightarrow \tau_2$

Explain why this change would not violate type safety. Explain why it is a bad idea anyway.
5. (Soundness/Completeness) (10 points)

Suppose you design a new type system for Java to prevent null-pointer dereferences. However, due to poor design, your type system has the strange property that there are exactly 47 programs that your type system accepts; it rejects all others.

**Explain your answers briefly.**

(a) Is it possible that your type system is sound with respect to null-pointer dereferences?
(b) Is it possible that your type system is complete with respect to null-pointer dereferences?
(c) Is it definitely the case given just the information above that your type system is sound with respect to null-pointer dereferences?
(d) Is it definitely the case given just the information above that your type system is complete with respect to null-pointer dereferences?
6. (Concurrent ML) (13 points)

In this problem you will use Concurrent ML to write a “server version” of \texttt{wgo} from problem 1. You will implement this interface:

\begin{verbatim}
  type wgo_adder
  val new_wgo_adder : int -> wgo_adder
  val add : wgo_adder -> int -> int
\end{verbatim}

The argument to \texttt{new\_wgo\_adder} is the new adder’s “max.” Initially an adder has “not reached its max.” While an adder has “not reached its max” each call to \texttt{add} returns the sum of all integers ever passed to \texttt{add} with this particular adder. However, if the argument to \texttt{add} plus the sum of all previous integers exceeds “max” then the “max is reached” and \texttt{add} should return the sum of all \textit{previous} calls. After the “max is reached” all future calls to \texttt{add} with this adder return this same sum no matter what.

You must use Concurrent ML to support multiple threads calling \texttt{add}; do not use other synchronization mechanisms such as locks.

Hints: You do not need \texttt{choose} and \texttt{wrap}. The code you wrote in problem 1 is \emph{not} particularly helpful.
Consider a typical class-based OOP language like we did in class. Suppose somewhere in a program that type-checks we have \texttt{e.m((D)(new C()))} where \texttt{C} is a subclass/subtype of \texttt{D}. Notice the argument in the method call is an explicit upcast. Consider modifying the program by removing this explicit upcast, i.e., replacing the call with \texttt{e.m(new C())}.

**Explain your answers briefly.**

(a) If every class in the program has at most one method named \texttt{m}, can this change cause the program not to type-check?

(b) If every class in the program has at most one method named \texttt{m}, can this change cause the program to produce a different result?

(c) If classes can have multiple methods named \texttt{m} and method calls are resolved with static overloading, can this change cause the program to produce a different result?

(d) If classes can have multiple methods named \texttt{m} and method calls are resolved with multimethods, can this change cause the program to produce a different result?