CSE P505, Spring 2006, IMP Formal Semantics

$$
\begin{array}{rcl}
s & ::= & \mathsf{skip} \mid x := e \mid s;s \mid \mathsf{if}\ e\ s\ s \mid \mathsf{while}\ e\ s \\
e & ::= & i \mid x \mid e+e \mid e*e \\
(i & \in & \{\ldots,-2,-1,0,1,2,\ldots\}) \\
(x & \in & \{\mathtt{x}_1,\mathtt{x}_2,\ldots,\mathtt{y}_1,\mathtt{y}_2,\ldots,\mathtt{z}_1,\mathtt{z}_2,\ldots,\ldots\})
\end{array}
$$

We write $H(x)$ to indicate the value of x in heap H.
We write $H, x \mapsto i$ to represent the heap that is just like H except $H(x) = i$.
A program evaluates to $H'(ans)$ if $H_0\ ;\ e\ \Downarrow\ H'$ wher H_0 is the heap where $H_0(x) = 0$ for all x.

$\boxed{H\ ;\ e\ \Downarrow\ i}$

$$
\textsc{const}\ \frac{}{H\ ;\ c\ \Downarrow\ c}
\qquad
\textsc{var}\ \frac{}{H\ ;\ x\ \Downarrow\ H(x)}
\qquad
\textsc{add}\ \frac{H\ ;\ e_1\ \Downarrow\ c_1 \qquad H\ ;\ e_2\ \Downarrow\ c_2}{H\ ;\ e_1+e_2\ \Downarrow\ c_1+c_2}
\qquad
\textsc{mult}\ \frac{H\ ;\ e_1\ \Downarrow\ c_1 \qquad H\ ;\ e_2\ \Downarrow\ c_2}{H\ ;\ e_1*e_2\ \Downarrow\ c_1*c_2}
$$

$\boxed{H\ ;\ s\ \Downarrow\ H'}$

$$
\textsc{skip}\ \frac{}{H\ ;\ \mathsf{skip}\ \Downarrow\ H}
\qquad
\textsc{assign}\ \frac{H\ ;\ e\ \Downarrow\ i}{H\ ;\ x := e\ \Downarrow\ H, x \mapsto i}
\qquad
\textsc{seq}\ \frac{H\ ;\ s_1\ \Downarrow\ H'' \qquad H''\ ;\ s_2\ \Downarrow\ H'}{H\ ;\ s_1; s_2\ \Downarrow\ H'}
$$

$$
\textsc{if1}\ \frac{H\ ;\ e\ \Downarrow\ i \qquad i \neq 0 \qquad H\ ;\ s_1\ \Downarrow\ H'}{H\ ;\ \mathsf{if}\ e\ s_1\ s_2\ \Downarrow\ H'}
\qquad
\textsc{if2}\ \frac{H\ ;\ e\ \Downarrow\ i \qquad i = 0 \qquad H\ ;\ s_2\ \Downarrow\ H'}{H\ ;\ \mathsf{if}\ e\ s_1\ s_2\ \Downarrow\ H'}
$$

$$
\textsc{while}\ \frac{H\ ;\ \mathsf{if}\ e\ (s; \mathsf{while}\ e\ s)\ \mathsf{skip}\ \Downarrow\ H'}{H\ ;\ \mathsf{while}\ e\ s\ \Downarrow\ H'}
$$

Note: Instead of doing one rule for loops that "unrolls to an if" we could have had two rules like we have two rules for if. One rule (when the result of evaluating e is 0) would produce no heap change; the other would evaluate s and then the whole loop again.