CSE P 501 – Compilers

LR Parsing
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Agenda

• LR Parsing
• Table-driven Parsers
• Parser States
• Shift-Reduce and Reduce-Reduce conflicts
Bottom-Up Parsing

• Idea: Read the input left to right
• Whenever we've matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
• The upper edge of this partial parse tree is known as the *frontier*
Example

• Grammar

\[ S ::= aAB\ e \]
\[ A ::= Abc \mid b \]
\[ B ::= d \]

• Bottom-up Parse

\[ a \ b \ b \ c \ d \ e \]
LR(1) Parsing

• We’ll look at LR(1) parsers
  – Left to right scan, Rightmost derivation, 1 symbol lookahead
  – Almost all practical programming languages have a LR(1) grammar
  – LALR(1), SLR(1), etc. – subsets of LR(1)
    • LALR(1) can parse most real languages, tables are more compact, and is used by YACC/Bison/CUP/etc.
LR Parsing in Greek

- The bottom-up parser reconstructs a reverse rightmost derivation
- Given the rightmost derivation
  \[ S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \ldots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = w \]
  the parser will first discover \( \beta_{n-1} \Rightarrow \beta_n \), then \( \beta_{n-2} \Rightarrow \beta_{n-1} \), etc.
- Parsing terminates when
  - \( \beta_1 \) reduced to \( S \) (start symbol, success), or
  - No match can be found (syntax error)
How Do We Parse with This?

• Key: given what we’ve already seen and the next input symbol (the lookahead), decide what to do.
• Choices:
  – Perform a reduction
  – Look ahead further
• Can reduce $A \Rightarrow \beta$ if both of these hold:
  – $A \Rightarrow \beta$ is a valid production, and
  – $A \Rightarrow \beta$ is a step in this rightmost derivation
• This is known as a shift-reduce parser
Sentential Forms

• If $S \Rightarrow^* \alpha$, the string $\alpha$ is called a *sentential form* of the grammar

• In the derivation $S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \ldots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = \mathbf{w}$ each of the $\beta_i$ are sentential forms

• A sentential form in a rightmost derivation is called a right-sentential form (similarly for leftmost and left-sentential)
Handles

• Informally, a production whose right hand side matches a substring of the tree frontier *that is part of the rightmost derivation of the current input string* (i.e., the “correct” production)
  – Even if $A ::= \beta$ is a production, it is a handle only if $\beta$ matches the frontier at a point where $A ::= \beta$ was used in *this specific* derivation
  – $\beta$ may appear in many other places in the frontier without designating a handle

• Bottom-up parsing is all about finding handles
Handle Examples

• In the derivation
  \[ S \Rightarrow aABe \Rightarrow aAde \Rightarrow aAbcde \Rightarrow abbcde \]
  – \( abbcde \) is a right sentential form whose handle is \( A::=b \) at position 2
  – \( aAbcde \) is a right sentential form whose handle is \( A::=Abc \) at position 4
    • Note: some books take the left end of the match as the position
Handles – The Dragon Book Defn.

• Formally, a *handle* of a right-sentential form $\gamma$ is a production $A ::= \beta$ and a position in $\gamma$ where $\beta$ may be replaced by $A$ to produce the previous right-sentential form in the rightmost derivation of $\gamma$
Implementing Shift-Reduce Parsers

• Key Data structures
  – A stack holding the frontier of the tree
  – A string with the remaining input (tokens)

• We also need something to encode the rules that tell us what action to take next, given the state of the stack and the lookahead symbol
  – Typically a table that encodes a finite automata
Shift-Reduce Parser Operations

• **Reduce** – if the top of the stack is the right side of a handle $A::=\beta$, pop the right side $\beta$ and push the left side $A$

• **Shift** – push the next input symbol onto the stack

• **Accept** – announce success

• **Error** – syntax error discovered
Shift-Reduce Example

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>abbcde$</td>
<td>shift</td>
</tr>
</tbody>
</table>

$S ::= aABe$
$A ::= Abc | b$
$B ::= d$
How Do We Automate This?

• Cannot use clairvoyance in a real parser (alas...)
• Defn. **Viable prefix** – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
  
  – Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
  
  – In Greek: \( \gamma \) is a **viable prefix** of \( G \) if there is some derivation \( S \Rightarrow^{*}_{rm} \alpha Aw \Rightarrow^{*}_{rm} \alpha \beta w \) and \( \gamma \) is a prefix of \( \alpha \beta \).
  
  – The occurrence of \( \beta \) in \( \alpha \beta w \) is a **handle** of \( \alpha \beta w \)
How Do We Automate This?

• Fact: the set of viable prefixes of a CFG is a regular language(!)

• Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
  – Perform reductions when we recognize them
DFA for prefixes of

\[
S ::= aABe \\
A ::= Abc \mid b \\
B ::= d
\]

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Trace

Stack   Input
$    abbcde$
Observations

• Way too much backtracking
  – We want the parser to run in time proportional to the length of the input

• Where the heck did this DFA come from anyway?
  – From the underlying grammar
  – Defer construction details for now
Avoiding DFA Rescanning

- Observation: no need to restart DFA after a shift. Stay in the same state and process next token.
- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
  - ∴ Scanning the stack will take us through the same transitions as before until the last one
  - ∴ If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack
Stack

• Change the stack to contain pairs of states and symbols from the grammar
  \[ s_0 \ X_1 \ s_1 \ X_2 \ s_2 \ \ldots \ X_n \ s_n \]
  - State \( s_0 \) represents the accept (start) state
    (Not always explicitly on stack – depends on particular presentation)
  - When we push a symbol on the stack, push the symbol plus the FA state
  - When we reduce, popping the handle will reveal the state of the FA just prior to reading the handle

• Observation: in an actual parser, only the state numbers are needed since they implicitly contain the symbol information. But for explanations / examples it can help to show both.
A shift-reduce parser’s DFA can be encoded in two tables

- One row for each state
- \textit{action} table encodes what to do given the current state and the next input symbol
- \textit{goto} table encodes the transitions to take after a reduction
Actions (1)

• Given the current state and input symbol, the main possible actions are
  – $si$ – shift the input symbol and state $i$ onto the stack (i.e., shift and move to state $i$)
  – $rj$ – reduce using grammar production $j$
    • The production number tells us how many $<\text{symbol, state}>$ pairs to pop off the stack
      ($=$ number of symbols on rhs of production)
Actions (2)

• Other possible *action* table entries
  – *accept*
  – *blank* – no transition – syntax error
    • A LR parser will detect an error as soon as possible on a left-to-right scan
    • A real compiler needs to produce an error message, recover, and continue parsing when this happens
Goto

• When a reduction is performed using $A ::= \beta$, we pop $|\beta| <\text{symbol, state}>$ pairs from the stack revealing a state $uncovered_s$ on the top of the stack

• $\text{goto}[uncovered_s, A]$ is the new state to push on the stack when reducing production $A ::= \beta$ (after popping handle $\beta$ and pushing $A$)
Reminder: DFA for

\[ S ::= aABe \]
\[ A ::= Abc | b \]
\[ B ::= d \]

\[ A ::= b \]
\[ B ::= d \]
LR Parse Table for

1. \( S ::= aABe \)
2. \( A ::= Abc \)
3. \( A ::= b \)
4. \( B ::= d \)

<table>
<thead>
<tr>
<th>State</th>
<th>action</th>
<th>goto</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>s2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>s4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>s6</td>
<td>s5</td>
</tr>
<tr>
<td>4</td>
<td>r3</td>
<td>r3</td>
</tr>
<tr>
<td>5</td>
<td>r4</td>
<td>r4</td>
</tr>
<tr>
<td>6</td>
<td>s7</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>r2</td>
<td>r2</td>
</tr>
<tr>
<td>8</td>
<td>r1</td>
<td>r1</td>
</tr>
<tr>
<td>9</td>
<td>r1</td>
<td>r1</td>
</tr>
</tbody>
</table>
LR Parsing Algorithm

tok = scanner.getToken();
while (true) {
    s = top of stack;
    if (action[s, tok] = si ) {
        push tok; push i (state);
        tok = scanner.getToken();
    } else if (action[s, tok] = rj ) {
        pop 2 * length of right side of production j (2*|β|);
        uncovered_s = top of stack;
        push left side A of production j ;
        push state goto[uncovered_s, A];
    } else if (action[s, tok] = accept ) {
        return;
    } else {
        // no entry in action table
        report syntax error;
        halt or attempt recovery;
    }
Example

1. $S ::= aABe$
2. $A ::= Abc$
3. $A ::= b$
4. $B ::= d$

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
<th>action</th>
<th>goto</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\</td>
<td>abbcde$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S</th>
<th>action</th>
<th>goto</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>s2</td>
<td>ac</td>
</tr>
<tr>
<td>1</td>
<td>s2</td>
<td>g0</td>
</tr>
<tr>
<td>2</td>
<td>s4</td>
<td>g3</td>
</tr>
<tr>
<td>3</td>
<td>s6 s5</td>
<td>g8</td>
</tr>
<tr>
<td>4</td>
<td>r3 r3 r3 r3 r3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>r4 r4 r4 r4 r4 r4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>s7</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>r2 r2 r2 r2 r2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>s9</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>r1 r1 r1 r1 r1 r1</td>
<td></td>
</tr>
</tbody>
</table>
LR States

• Idea is that each state encodes
  – The set of all possible productions that we could be looking at, given the current state of the parse, and
  – Where we are in the right hand side of each of those productions
Items

- An *item* is a production with a dot in the right hand side.
- Example: Items for production $A ::= X \ Y$
  
  $A ::= \ . \ X \ Y$
  
  $A ::= X \ . \ Y$
  
  $A ::= X \ Y \ .$

- Idea: The dot represents a position in the production.
DFA for

\[
S ::= aABe \\
A ::= Abc \mid b \\
B ::= d
\]
Problems with Grammars

• Grammars can cause problems when constructing a LR parser
  – Shift-reduce conflicts
  – Reduce-reduce conflicts
Shift-Reduce Conflicts

• Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)

• Classic example: if-else statement

  \[ S ::= \text{ifthen } S \mid \text{ifthen } S \text{ else } S \]
Parser States for

1. \( S ::= \text{ifthen } S \)
2. \( S ::= \text{ifthen } S \text{ else } S \)

- State 3 has a shift-reduce conflict
  - Can shift past else into state 4 (s4)
  - Can reduce (r1)
    \( S ::= \text{ifthen } S \)

(Note: other \( S ::= \text{. ifthen} \) items not included in states 2-4 to save space)
Solving Shift-Reduce Conflicts

• Fix the grammar
  — Done in Java reference grammar, others
• Use a parse tool with a “longest match” rule – i.e., if there is a conflict, choose to shift instead of reduce
  — Does exactly what we want for if-else case
  — Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want (and that this behavior is guaranteed by the tool specification)
Reduce-Reduce Conflicts

• Situation: two different reductions are possible in a given state
• Contrived example
  \[ S ::= A \]
  \[ S ::= B \]
  \[ A ::= x \]
  \[ B ::= x \]
Parser States for

1. $S ::= A$
2. $S ::= B$
3. $A ::= x$
4. $B ::= x$

- State 2 has a reduce-reduce conflict ($r3$, $r4$)
Handling Reduce-Reduce Conflicts

• These normally indicate a serious problem with the grammar.

• Fixes
  – Use a different kind of parser generator that takes lookahead information into account when constructing the states
    • Most practical tools use this information
  – Fix the grammar
Another Reduce-Reduce Conflict

• Suppose the grammar tries to separate arithmetic and boolean expressions

\[
\begin{align*}
expr & ::= aexp \mid bexp \\
aexp & ::= aexp \ast aident \mid aident \\
bexp & ::= bexp \&\& bident \mid bident \\
aident & ::= id \\
bident & ::= id
\end{align*}
\]

• This will create a reduce-reduce conflict after recognizing \textit{id}
Covering Grammars

• A solution is to merge \textit{aident} and \textit{bident} into a single non-terminal like \textit{ident} (or just use \textit{id} in place of \textit{aident} and \textit{bident} everywhere they appear)

• This is a \textit{covering grammar}
  – Will generate some programs (sentences) that are not generated by the original grammar
  – Use the type checker or other static semantic analysis to weed out illegal programs later
Coming Attractions

• Constructing LR tables
  – We’ll present a simple version (SLR(0)) in lecture, then talk about adding lookahead and then a little bit about how this relates to LALR(1) used in most parser generators
• LL parsers and recursive descent
• Continue reading ch. 3