Question 1. (10 points, 5 each) Regular expressions. Give regular expressions that generate the following sets of strings. You may only use the basic operations of concatenation, choice (|), and repetition (*) plus the derived operators ? and +, and simple character classes like [abc0-9] and [^a-z]. You may use abbreviations like vowels = [aeiou]. You may not use more complex operators found in various software tools that handle extended regular expressions.

(a) All sequences of 0’s and 1’s that contain an odd number of 1’s and contain at least one or more 1’s.

There are many possible solutions. A fairly simple one is 0*1(0|(10*1))*

(b) All strings containing a’s, b’s, and c’s with at least one a and at least one b.

Let abcs = (a|b|c)* (or [abc]*)

abcs a abcs b abcs | abcs b abcs a abcs
Question 2. (10 points) Context-free grammars. Consider the following syntax for expressions involving addition and field selection:

\[
\begin{align*}
\text{expr} & ::= \text{expr} + \text{field} \\
\text{expr} & ::= \text{field} \\
\text{field} & ::= \text{expr} \cdot \text{id} \\
\text{field} & ::= \text{id}
\end{align*}
\]

(a) (6 points) Show that this grammar is ambiguous.

Here are two derivations of id+id.id:

(b) (4 points) Give an unambiguous context-free grammar that fixes the problem(s) with the grammar in part (a) and generates expressions with \text{id}, field selection and addition. As in Java, field selection should have higher precedence than addition and both field selection and addition should be left-associative (i.e., a+b+c means (a+b)+c).

The problem is in the first rule for \text{field}, which creates an ambiguous precedence. Here is a reasonably simple fix.

\[
\begin{align*}
\text{expr} & ::= \text{expr} + \text{field} \\
\text{expr} & ::= \text{field} \\
\text{field} & ::= \text{field} \cdot \text{id} \\
\text{field} & ::= \text{id}
\end{align*}
\]
Question 3. (18 points) The (almost) obligatory LR-parsing question. Consider the following grammar that is meant to capture expressions involving addition (+) and exponentiation (**), where addition is left associative and exponentiation is right associative.

0. \( e' ::= e \) $ \\
1. \( e ::= e + t \) \\
2. \( e ::= t \) \\
3. \( t ::= x \ ** t \) \\
4. \( t ::= x \)

(a) (10 points) Draw the LR(0) state machine for this grammar. (You do not need to include the table with shift/reduce and goto actions, although you can write that out later if you find it useful to answer other parts of the question.)

The operator ** was intended to be a single token. A number of solutions treated it as two adjacent * tokens. If that was done correctly no points were deducted.

(continued next page)
Question 3 (cont.) Grammar repeated for reference

0. $e' ::= e \, \$\$
1. $e ::= e + t$
2. $e ::= t$
3. $t ::= x \mathit{**} t$
4. $t ::= x$

(b) (2 points) Is this grammar LR(0)? Why or why not?

No. The state labeled * in the diagram has a shift-reduce conflict if the input is **.

(c) (4 points) Compute First, Follow, and Nullable for each of the non-terminals in this grammar.

<table>
<thead>
<tr>
<th></th>
<th>first</th>
<th>follow</th>
<th>nullable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e'$</td>
<td>x</td>
<td></td>
<td>no</td>
</tr>
<tr>
<td>$e$</td>
<td>x</td>
<td>+, $$</td>
<td>no</td>
</tr>
<tr>
<td>$t$</td>
<td>x</td>
<td>+, $$</td>
<td>no</td>
</tr>
</tbody>
</table>

(d) (2 points) Is this grammar SLR? Why or why not?

Yes. The operator ** is not contained in Follow(t) so we can resolve the shift-reduce conflict in favor of a shift on input **.
Question 4. (22 points) We would like to add a new loop to MiniJava that is similar to a while loop except that it can have multiple conditions and statement sequences in a loop.

The simple case has one condition and a sequence of statements. For example, the loop

```java
    do i<n => sum=sum+a[i]; i++; od
```

is equivalent to the traditional loop

```java
    while (i<n) { sum=sum+a[i]; i++; }
```

The difference is that parentheses are not required around the condition, the terminal symbol => separates the condition from the loop body, the end of the loop is indicated by the keyword od with no semicolon or other punctuation after it, and the loop body can have several statements without requiring curly braces because the => and od symbols indicate the extent of the loop body.

These loops can be nested in the obvious way since the new loop is just another kind of statement. For example, if we had 2-D arrays, a n x n array could be initialized with

```java
    i=0;
    do i<n => j=0;
        do j<n => a[i][j]=0; j++; od
        i++;
    od
```

More interesting, a loop may contain more than one condition plus statement sequence group. The loop below changes positive integers x and y so that on termination both of them contain the greatest common divisor of their original values.

```java
    do x>y => x=x-y; [] y>x => y=y-x; od
```

If a loop contains more than one condition/statement sequence group, they are separated by a box made up of left and right brackets with no space between ([]). To execute the loop the conditions are evaluated in order from the beginning. If a condition evaluates to true, the corresponding statement(s) following the => symbol up to the next [] or od are executed, and then that iteration of the loop is finished. Execution of the next iteration of the loop begins back at the top after do. If no condition is true, then execution of the loop terminates. In other words, on each iteration of the loop, at most one sequence of statements is executed – the statements following the first true condition in the list – and later conditions and statements in the loop are not executed on that iteration.

(You may detach this page from the exam while working, but must turn it in with the rest of the exam.)
Question 4 (cont.) Answer the following questions about the new loop statement.

(a) (6 points) Give an unambiguous context free grammar rule or rules to add this new kind of loop to the MiniJava grammar for Statement. Your answer should include additional new terminals and non-terminals as needed and can include additional grammar rules for non-terminals if appropriate. You only need to give the additions and changes to the MiniJava grammar; you do not need to write CUP or other parser-generator source code.

[Historical note: the do-od loop and a similar if-fi conditional statement were introduced by Dijkstra in 1976. The condition=>statements pair was called a guarded command – the condition being the guard that determined whether the statements were executed. Thus the (possibly unexpected) names of the nonterminals in the sample solution below.]

```
Statement ::= “do” Guards “od”
Guards ::= Guard | Guards “[ ]” Guard
Guard ::= Expression “=>” (Statement)*
```

(b) (3 points) What changes need to be made to the MiniJava scanner to add this new loop statement to the language? Again, just describe the changes. You do not need to write JFlex/CUP or other source code.

We need to add new tokens for do, od, =>, and [. (Note that we do need [ ] to be a new, separate token even though [ and ] are already MiniJava tokens. No space is allowed between the brackets when [ ] is used to separate sequences of guards.)
Question 4 (cont.) (c) (4 points) What changes or additions need to be made to the MiniJava Abstract Syntax Tree classes or node definitions to include this new loop statement in the MiniJava abstract grammar? Your answer should give a description of the kinds of nodes that need to be added or changed and their contents. It does not need to be detailed Java, C#, or other code.

There are many possible ways to do this and answers that were plausible received credit. The simplest solution would be to add a new DoStatement class extending Statement containing a list of Expression/Statement-list pairs as its children.

(d) (3 points) What checks need to be added to the static semantics/type checking phase of the compiler for this new loop statement?

Verify that the expressions to the left of each => have type Boolean.
Question 4. (cont.) (e) (6 points) Outline the code shape (essentially the pseudo-assembly language) needed to implement this new loop statement. Your answer should show the code shape needed for a loop like the following one with two condition/statement sequence pairs:

```
do cond1 => stmt1
[] cond2 => stmt2
od
```

The answer should be similar to the examples given in class for other control constructs showing where labels and jumps would appear and where the code for the conditions and statements inside the loop would be placed.

```
loop:
  <code for cond1>
  jmp_false test2
  <code for stmt1>
  jmp loop
test2:
  <code for cond2>
  jmp_false done
  <code for stmt2>
  jmp loop
done:
```
Question 5. (17 points) Consider the following C data structure definition and function. (This code uses the gcc convention that long is a 64-bit integer type):

```c
struct node { // node with 64-bit int and pointer
    long val;
    struct node * next;
};
long sum(struct node * p) {
    long first, rest;
    if (p == NULL) {
        return 0;
    } else {
        first = p->val;
        rest = sum(p->next);
        return first+rest;
    }
}
```

This question involves translating this function to x86-64 assembler code. Ground rules:

- You may use either Linux/gcc or Microsoft/masm assembly language, and must follow the corresponding register linkage and stack frame conventions.
  - Linux argument registers: rdi, rsi, rdx, rcx, r8, r9
  - Linux: called function must save/restore rbx, rbp, r12-r15 if used.
  - Microsoft argument registers: rcx, rdx, r8, r9; caller must provide a save area for these four parameter registers at the bottom of the caller’s stack frame that the called function can use if desired.
  - Microsoft: called function must save/restore rbx, rsi, rdi, rbp, r12-r15 if used.
  - Both: function result returned in rax.
  - Both: rsp must be aligned on a 16-byte boundary when a call instruction is executed to call the function.

- Pointers and ints are 64 bits (8 bytes) each.
- Your code should implement all of the statements in the original function. In particular, it should include the recursive function call and include store instructions for assignments to local variables, and may not rewrite the function into something different like a loop that produces the same result. Other than that, you can use any reasonable x86-64 code that follows the standard function call and register conventions. (In particular, if a previously computed value is still in a register when needed later in the code, you don’t need to include an additional instruction to reload it from memory.)

- Assume the representation of a NULL pointer is an 8-byte binary zero (0) value.

- A C struct is a simple record type. It is not an object with a method table pointer or other hidden data fields.

(You may detach this page from the exam while working, but must turn it in with the rest of the exam.)
Question 5 (cont.) Code repeated for reference.

```c
long sum(struct node * p) {
    struct node {
        long first, rest;
    } val;
    if (p == NULL) {
        return 0;
    } else {
        first = p->val;
        rest = sum(p->next);
        return first + rest;
    }
}
```

(a) (0 points) Which conventions are you using (circle)

Linux/gcc

(b) (5 points) Draw the stack frame for function `sum` as it would appear in an x86-64 program using the function call conventions you indicated above. Your picture should show the locations of function parameters (if they occupy storage), local variables, temporaries, and the stack pointer and frame pointer registers as they exist after the function prologue has executed and has allocated the stack frame, but before any of the statements in the body of the function have been executed. Be sure to show the numeric offsets from the frame pointer register to each local variable and other assigned storage locations.

```
<table>
<thead>
<tr>
<th>caller's stack frame</th>
<th>return address</th>
</tr>
</thead>
<tbody>
<tr>
<td>rbp -&gt;</td>
<td>old rbp or alignment pad</td>
</tr>
<tr>
<td></td>
<td>first</td>
</tr>
<tr>
<td>rsp -&gt;</td>
<td>rest</td>
</tr>
</tbody>
</table>
```

Notes:
- Solutions that omitted `rbp` from the calling sequence and used `rsp` to reference locals were fine. However, the total size of the stack frame still needs to be a multiple of 16 to preserve alignment.
- The variables can be allocated in any order, and if `rbp` is not saved as part of the calling convention the padding can be anywhere relative to the local variables.
- Solutions that used the Microsoft/masm conventions needed to provide an additional 32-byte argument register save area at the bottom of the stack frame (`rsp+0 to rsp+31`) as part of the Microsoft calling convention.

(continued next page)
Question 5 (cont.) (c) (12 points) Translate function \texttt{sum} to x86-64 assembly language. Be sure to follow the rules given on the first page of the question (hint: read the rules again after you’ve finished your answer.) Code repeated for reference.

```c
long sum(struct node * p) {
    struct node {
        long val;
        struct node * next;
    } first, rest;
    if (p == NULL) {
        return 0;
    } else {
        first = p->val;
        rest = sum(p->next);
        return first+rest;
    }
}
```

This solution uses the Linux/gcc conventions and uses \texttt{rbp} as a frame pointer. If the frame pointer is not used then the amount subtracted from \texttt{rsp} to allocate the stack frame needs to be increased by 8. Solutions using the Microsoft/masm conventions would be similar with the expected differences in syntax, argument registers (\texttt{rcx} instead of \texttt{rdi}), and a larger stack frame including the argument register save area.

Obviously there are many possible solutions. This is a fairly straightforward translation of the original code.

```
sum: pushq %rbp
    # function prologue
    movq %rsp,%rbp
    subq $16,%rsp
    movq $0,%rax
    tstq %rdi,%rdi
    beq exit
    movq 0(%rdi),%rax
    # first = p->val
    movq %rax,-8(%rbp)
    movq 8(%rdi),%rdi
    # set argument to p->next
    call sum
    movq %rax,-16(%rbp)
    # rest = function result
    addq -8(%rbp),%rax
    # result = first+rest
exit:
    movq %rbp,%rsp
    # return - function epilog
    popq %rbp
    ret
```

The \texttt{movq/popq} instruction pair at the end can be replaced with \texttt{leave} assuming that \texttt{rbp} is used as a base register.
Question 6. (13 points) Dominators and loops. Consider the following flow graph.

(a) (9 points) For each node, list the nodes that are its dominators and its immediate dominator. A is the initial node in the flow graph.

<table>
<thead>
<tr>
<th>Node</th>
<th>Dominators</th>
<th>IDOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>B</td>
<td>A, B</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>A, C</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>A, B, D</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>A, B, E</td>
<td>B</td>
</tr>
<tr>
<td>F</td>
<td>A, B, F</td>
<td>B</td>
</tr>
</tbody>
</table>

(b) (4 points) List the back edges in the flow graph (i.e., x->y where the edge x->y is a back edge). For each back edge, list the set of nodes that form the natural loop associated with that back edge. (There are likely more rows in the table below than you need, but add more if you need them.)

<table>
<thead>
<tr>
<th>Back edge</th>
<th>Nodes in the associated natural loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>C -&gt; A</td>
<td>A, C</td>
</tr>
<tr>
<td>E -&gt; B</td>
<td>B, D, E, F</td>
</tr>
</tbody>
</table>
**Question 7.** (10 points) Register coloring. Suppose we have the following interference graph involving five live ranges A-E.

We would like to use the graph coloring register allocation algorithm to discover if there is a way to successfully allocate these five live ranges to three registers R1, R2, and R3.

(a) (5 points) List the nodes in the order they would be removed from the graph and placed on the stack during the simplify phase of the graph coloring algorithm. If there is more than one possible ordering you should list any one of them. If the simplify algorithm stops because it is not possible to remove any of the remaining nodes from the graph, indicate which nodes remain in the graph when the algorithm terminates.

There are many possible orderings, however no node can be removed from the graph and added to the list if it has 3 or more neighbors remaining in the graph. One ordering is C, A, B, D, E

(b) (5 points) Is it possible to assign the live ranges to three registers R1, R2, and R3 without interference? If so give an assignment based on your answer to part (a). If it is not possible, explain why not.

Yes. R1: E, C; R2: D; R3: B, A