

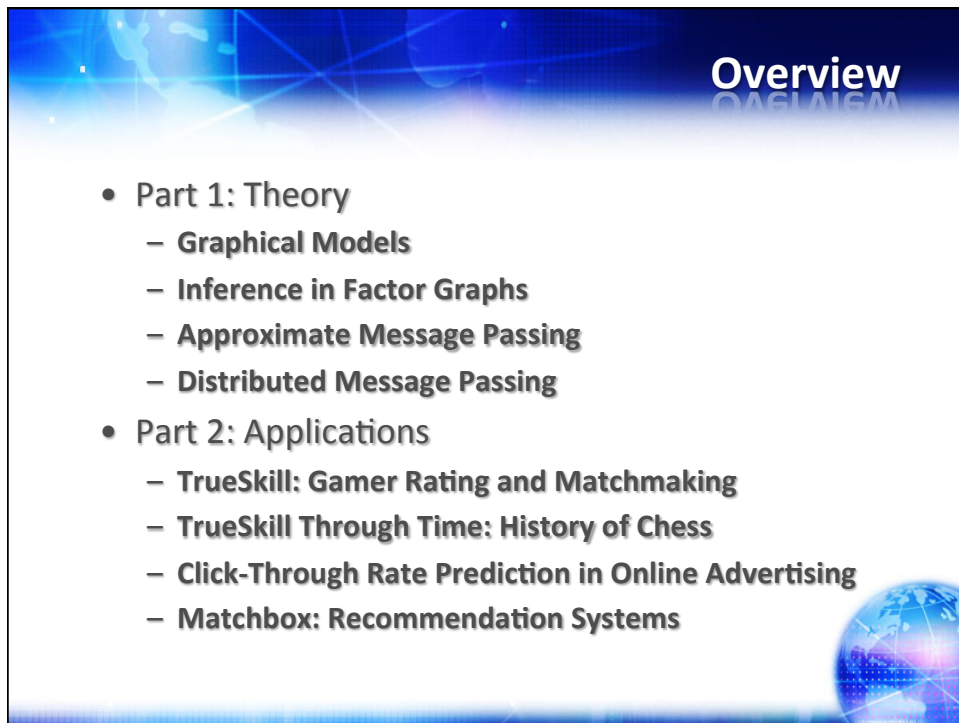
Distributed, Real-Time Bayesian Learning in Online Services

Ralf Herbrich
Amazon



Overview

- Part 1: Theory
 - Graphical Models
 - Inference in Factor Graphs
 - Approximate Message Passing
 - Distributed Message Passing
- Part 2: Applications
 - TrueSkill: Gamer Rating and Matchmaking
 - TrueSkill Through Time: History of Chess
 - Click-Through Rate Prediction in Online Advertising
 - Matchbox: Recommendation Systems



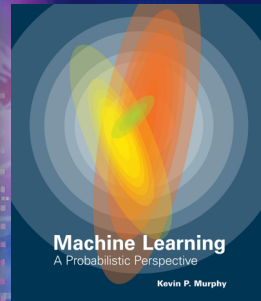
Part 1: Theory

Coursera

10110110101
1101000 10101010101
MACHINE
LEARNING
David Barber

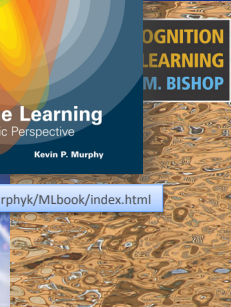
<http://www.coursera.org>

<http://www.cs.ucl.ac.uk/staff/d.barber/brml/>



<http://www.cs.ubc.ca/~murphyk/MLbook/index.html>

COGNITION
LEARNING
M. BISHOP



<http://research.microsoft.com/en-us/um/people/cmbishop/PRML/index.htm>

Overview

- Graphical Models
- Inference in Factor Graphs
- Approximate Message Passing
- Distributed Message Passing

Probabilities and Beliefs

- **Design:** System must assign degree of plausability $P(A)$ to each logical statement A .

- **Axiom:**

P must be a probability measure!

3. $P(A|C') > P(A|C)$ and $P(B|AC') = P(B|AC)$ then $P(AB|C') \geq P(AB|C)$

Infer-Predict-Decide Cycle

Decision Making:

$\text{Loss}(\text{Action}, \text{Data}) + P(\text{Data})$
 $\rightarrow \text{Action}$

- Business-loss not learning-loss!
- Often involves optimization!



Inference:

$P(\text{Parameters}) + \text{Data} \rightarrow$
 $P(\text{Parameters} | \text{Data})$

- Requires a (structural) model $P(\text{Data} | \text{Parameters})$
- Allows to incorporate prior information $P(\text{Parameters} | \text{Data})$

Prediction:

$P(\text{Parameters}) +$
 $\text{Data} \rightarrow P(\text{Data})$

- Requires integration/summation of parameter uncertainty
- Does not change state!



Graphical Models

- **Definition:** Graphical representation of joint probability distribution
 - Nodes: ○ = Variables
 - Edges: Relationship between variables
- **Variables:**
 - Observed Variables: Data
 - Unobserved Variables: 'Causes' + Temporary/Latent
- **Key Questions:**
 - (Conditional) *Dependency*: $p(a, b|c) \stackrel{?}{=} p(a|c) \cdot p(b|c)$
 - *Inference/Marginalisation*: $p(a, b) = \sum_c p(a, b, c)$

Directed Models: Bayesian Networks

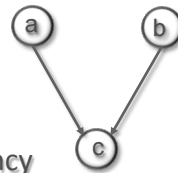
- **Definition:** Graphical representation of joint probability distribution (Pearl, 1988)
 - Nodes: ○ = Variables
 - Directed Edges: Conditional probability distribution

- **Semantic:**

$$p(x) = \prod_i p(x_i | x_{\text{parents}(i)})$$

- Ancestral relationship of dependency

$$p(a, b, c) = p(a) \cdot p(b) \cdot p(c|a, b)$$



Undirected Models: Markov Networks

- **Definition:** Graphical representation of joint probability distribution (Pearl, 1988)

- Nodes: ○ = Variables
- Edges: Dependency between variables

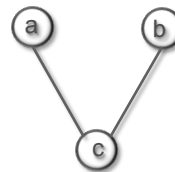
- **Semantic:**

$$p(\mathbf{x}) = \frac{1}{Z} \cdot \prod_c \phi(x_c) \quad \phi \geq 0$$

- Local potentials over cliques

$$p(a, b, c) = \frac{1}{Z} \cdot \phi_{ac}(a, c) \cdot \phi_{bc}(b, c)$$

$$Z = \sum_a \sum_b \sum_c \phi_{ac}(a, c) \cdot \phi_{bc}(b, c)$$



Factor Graphs

- **Definition:** Graphical representation of product structure of a function (Wiberg, 1996)

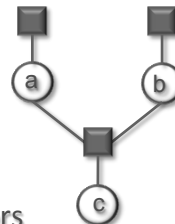
- Nodes: ■ = Factors ○ = Variables
- Edges: Dependencies of factors on variables.

- **Semantic:**

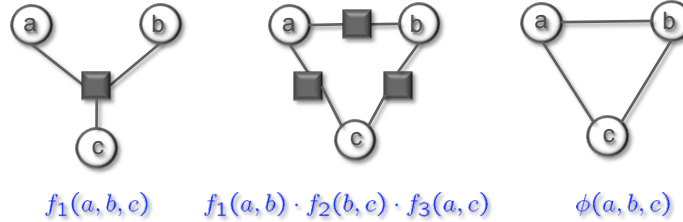
$$p(\mathbf{x}) = \prod_f f(x_{V(f)})$$

- Local variable dependency of factors

$$p(a, b, c) = f_1(a) \cdot f_2(b) \cdot f_3(a, b, c)$$



Factor Graphs are Powerful!



Undirected graphical models can hide the factorisation within a clique!

Factor Graphs and Bayes' Law

- Bayes' law

$$p(s|y) \propto p(y|s) \cdot p(s)$$

- Factorising prior

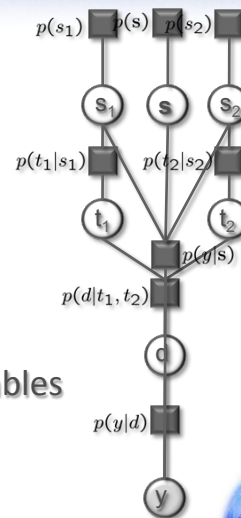
$$p(s) = p(s_1) \cdot p(s_2)$$

- Factorising likelihood

$$p(y, \mathbf{t}, d|\mathbf{s}) = \prod_i p(t_i|s_i) \cdot p(d|t_1, t_2) \cdot p(y|d)$$

- Inference: Sum out latent variables

$$p(y|s) = \sum_{\mathbf{t}} \sum_d p(y, \mathbf{t}, d|\mathbf{s})$$



Summary

	Dependency	Efficient Inference	Usage
Bayesian Networks	Yes	Somewhat	Ancestral Generative Process
Markov Networks	Yes	No	Local Couplings and Potentials
Factor Graphs	No	Yes	Efficient, distributed inference

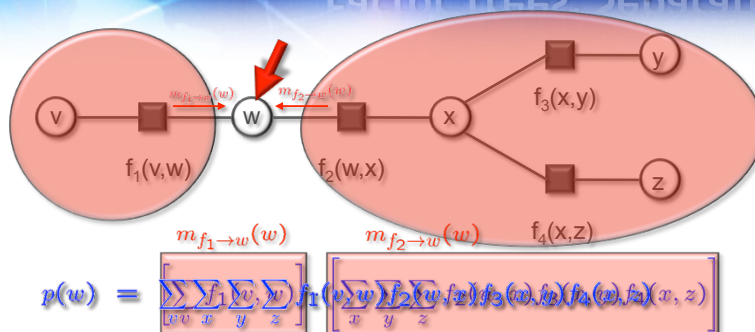
Overview

- Graphical Models
- **Inference in Factor Graphs**
- Approximate Message Passing
- Distributed Message Passing

Factor Graphs and Factor Trees

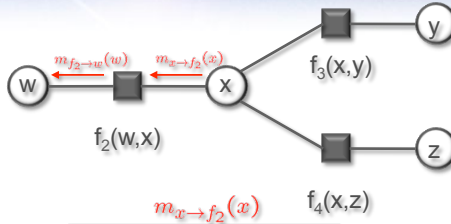
- **Factor Graphs:** Arbitrary functions
 - Bayesian Networks
 - Markov Networks
- **Factor Trees:** Functions where the variable indices never decrease from left to right
- **Factor Graph \rightarrow Factor Tree:**
 1. Pick an arbitrary node
 2. Build the spanning tree

Factor Trees: Separation



Observation: Sum of products becomes product of sums of all messages from neighbouring factors to variable!

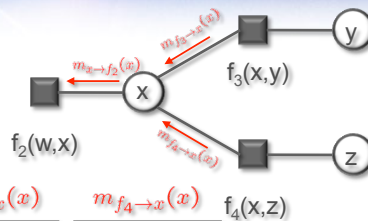
Messages: From Factors To Variables



$$m_{f_2 \rightarrow w}(w) = \sum_x \sum_y \sum_z f_2(w, x) \left[\sum_y \sum_z f_3(x, y) f_4(x, z) \right]$$

Observation: Factors only need to sum out all their local variables!

Messages: From Variables To Factors



$$m_{w \rightarrow f_2}(w) = \sum_y \sum_z (f_3(y, x) \cdot f_4(x, z)) \cdot f_2(w, x)$$

Observation: Variables pass on the product of all incoming messages!

The Sum-Product Algorithm

- Three update equations (Aji & McEliece, 1997)

$$p(t) = \prod_{f \in F_t} m_{f \rightarrow t}(t)$$
$$m_{f \rightarrow t_1}(t_1) = \sum_{t_2} \sum_{t_3} \cdots \sum_{t_n} f(t_1, t_2, t_3, \dots) \prod_{i>1} m_{t_i \rightarrow f}(t_i)$$
$$m_{t \rightarrow f}(t) = \prod_{f_j \in F_t \setminus \{f\}} m_{f_j \rightarrow t}(t)$$

- Update equations can be directly derived from the distributive law.
- Calculate all marginals at the same time!
- Only need to pass messages twice along each edge!

Practical Considerations I

- **Log-Transform:** $\lambda_{f \rightarrow t}(t) := \log [m_{f \rightarrow t}(t)]$

$$\log [p(t)] = \sum_{f \in F_t} \lambda_{f \rightarrow t}(t)$$

$$\lambda_{f \rightarrow t_1}(t_1) = \sum_{t_2} \sum_{t_3} \cdots \sum_{t_n} f(t_1, t_2, t_3, \dots) \exp \left[\sum_{i>1} \lambda_{t_i \rightarrow f}(t_i) \right]$$

$$\lambda_{t \rightarrow f}(t) = \sum_{f_j \in F_t \setminus \{f\}} \lambda_{f_j \rightarrow t}(t)$$

- **Exponential Family Messages:**

$$m(t) \propto \exp(\psi(t) \cdot \theta)$$

- Message updates are just additions of the parameters θ !

Exponential Families

- (Univariate) Gaussian: $\theta := \left(\frac{\mu}{\sigma^2}, \frac{1}{\sigma^2} \right)$
- Bernoulli: $\theta := \log \left(\frac{p}{1-p} \right)$
- Binomial: $\theta := \log \left(\frac{p}{1-p} \right)$
- Beta: $\theta := (\alpha, \beta)$
- Gamma: $\theta := \left(\alpha, \frac{1}{\beta} \right)$

Practical Considerations II

- **Redundant computations:**

$$\begin{aligned} p(t) &= \prod_{f \in F_t} m_{f \rightarrow t}(t) \\ m_{t \rightarrow f}(t) &= \prod_{f_j \in F_t \setminus \{f\}} m_{f_j \rightarrow t}(t) \end{aligned} \quad \Rightarrow \quad p(t) = m_{t \rightarrow f}(t) \cdot m_{f \rightarrow t}(t)$$

- **Caching:** Only store $p(t)$ and $m_{f \rightarrow t}(t)$, then

$$m_{t \rightarrow f}(t) = \frac{p(t)}{m_{f \rightarrow t}(t)}$$

Overview

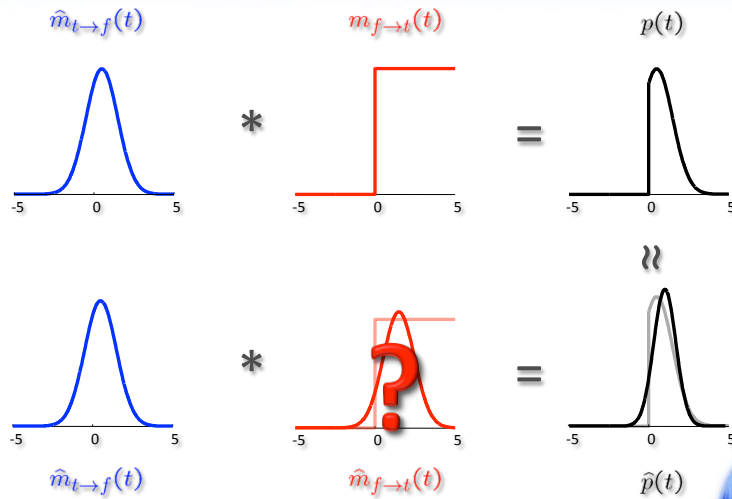
- Graphical Models
- Inference in Factor Graphs
- **Approximate Message Passing**
- Distributed Message Passing

Approximate Message Passing

- **Problem:** The exact messages from factors to variables may not be closed under products.
- **Solution:** Approximate *each* marginal as well as possible in using a divergence measure on beliefs.
- **General Idea:** Leave-one out approximation

$$\hat{p}(t) = \operatorname{argmin}_{\hat{p}} D \left[m_{f \rightarrow t} \cdot \hat{m}_{t \rightarrow f}, \hat{p} \right]$$
$$\hat{m}_{f \rightarrow t}(t) = \frac{\hat{p}(t)}{\hat{m}_{t \rightarrow f}(t)}$$

Approximate Message Passing



Divergence Measures

- **Kullback-Leibler Divergence:** Expected log-odd ratio between two distributions:

$$\text{KL}(p, q) := \sum_t p(t) \log \left(\frac{p(t)}{q(t)} \right)$$

- **Minimizer for Exponential Families:** Matching the moments of the distribution $p(t)$!
- **General α -Divergence:**

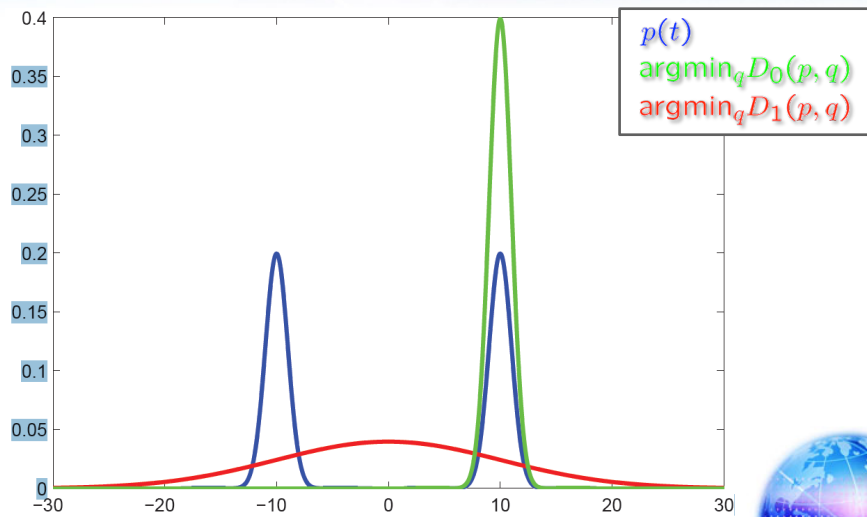
$$D_\alpha(p, q) := \frac{1 - \sum_t \frac{p^{\alpha-1}(t)}{q^{\alpha-1}(t)}}{\alpha(1 - \alpha)}$$

- **Special Cases:**

$$D_0(p, q) = \text{KL}(q, p)$$

$$D_1(p, q) = \text{KL}(p, q)$$

α -Divergence in Pictures



Overview

- Graphical Models
- Inference in Factor Graphs
- Approximate Message Passing
- **Distributed Message Passing**

Large-Data Challenge

- **Large Data (e.g. Facebook user actions)**
 - 500m daily users
 - 3 bln daily likes & comments
- **Two types of variables**
 - Observed → Data Factors
 - Latent → Model parameters
- **Discriminative Models**
 - Given the model parameters, data variables are independent

$$p(\theta|X, Y) \propto \prod_i p(y_i|\theta, \mathbf{x}_i) \cdot \prod_j p(\theta_j)$$

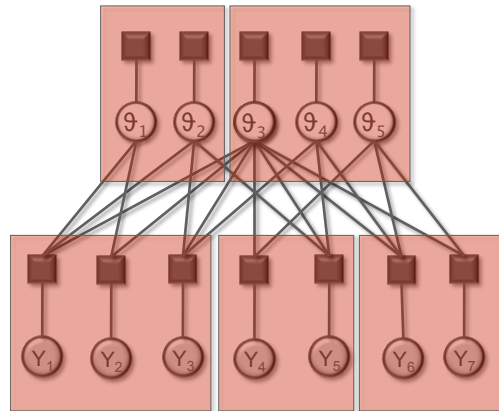
Distributed Message Passing

- **Idea:** Group variables and send messages across system boundaries

$$\prod_i p(y_i|\theta, \mathbf{x}_i) \cdot p(\theta) = \prod_k \underbrace{\prod_{j=1}^{n_k} p(y_{k,j}|\theta, \mathbf{x}_{k,j})}_{f_k(\mathbf{X}_k, \mathbf{Y}_k, \theta)} \cdot \prod_l \underbrace{\prod_{r=1}^{m_l} p(\theta_{l,r})}_{g_l(\theta_l)}$$

- **Data factors:** $f_k(\mathbf{X}_k, \mathbf{Y}_k, \theta)$
 - Know exactly which model parameter messages get updated
- **Parameter factors:** $g_l(\theta_l)$
 - Need to keep track of which data factors need message update

Distributed Conditional Models

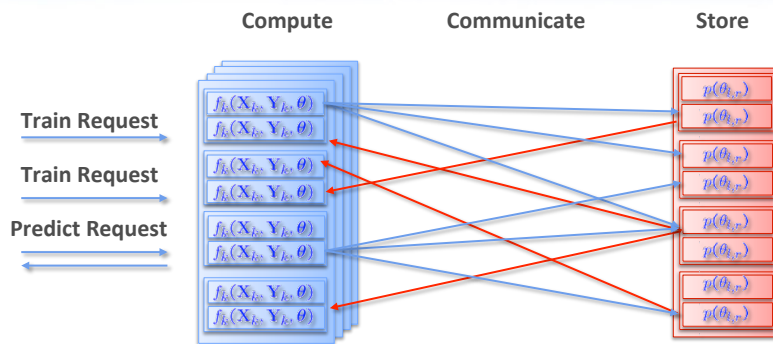


Belief Store
("Memory")

Message Passing
("Communicate")

Data Messages
("Compute")

A Systems Service View



Relation to Map-Reduce

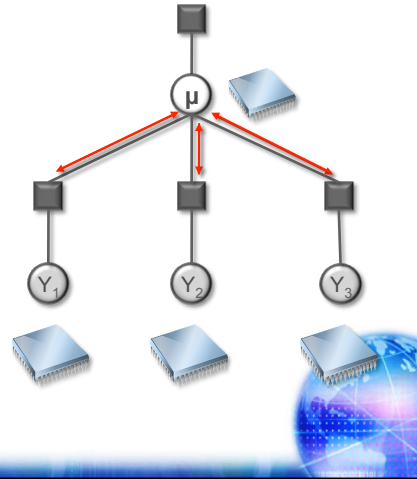
- **Map-Reduce**

- **Map:** Data nodes compute messages $m_{F_k \rightarrow \mu}$ from data y_i and $m_{\mu \rightarrow F_k}$
- **Reduce:** Combine messages $m_{F_k \rightarrow \mu}$ into p_{μ} by multiplication
- Vanilla MR is a single pass only!

- **Caveats:**

- Approximate data factors need all incoming message $m_{F_k \rightarrow \mu}$!
- Each machine needs to be able to store the belief over μ

$$p(\theta | x, y) \propto \prod_k f_k(Y_k | \theta, X_k) \cdot p(\theta)$$

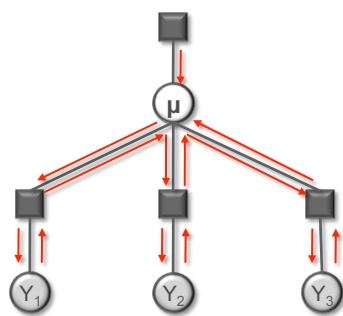


Approximation Quality

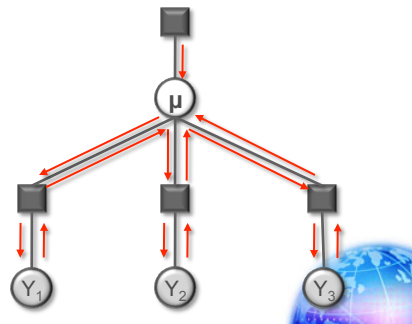
$$p(y_i | \theta, \mathbf{x}_i) = \Phi(y_i \theta^T \mathbf{x}_i)$$

$$p(\theta) = \prod_j \mathcal{N}(\theta_j; \mu_j, \sigma_j^2)$$

Sequential



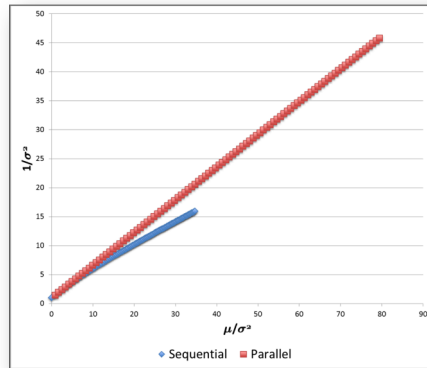
Parallel



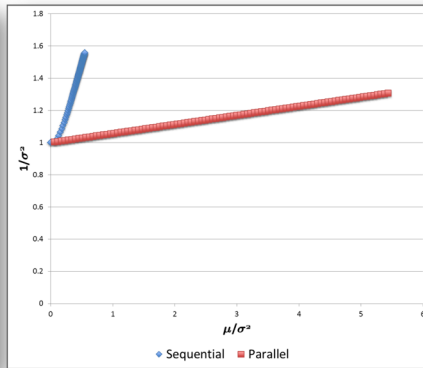
Approximation Quality

$$x = [1; 1; \dots; 1]^T$$

Single Bias Feature



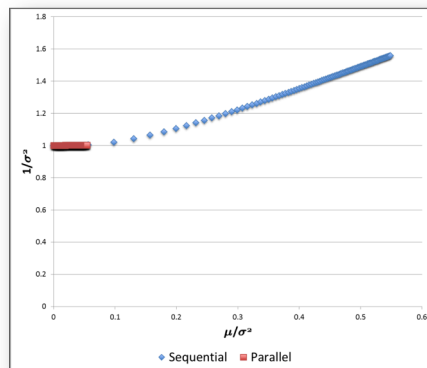
100 Bias Features



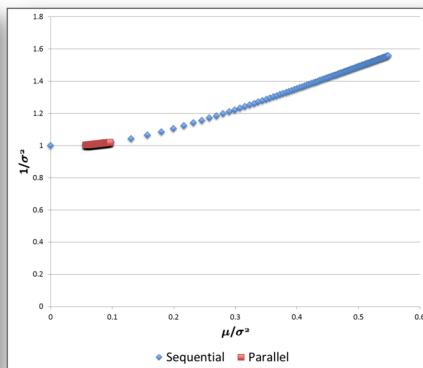
Solution : Dampening!

$$\lambda_{f \rightarrow \theta} \Rightarrow \alpha \cdot \lambda_{f \rightarrow \theta}$$

First Step



Second Step





Overview

- TrueSkill: Gamer Rating and Matchmaking
- Click-Through Rate Prediction in Online Advertising
- Matchbox: Recommendation Systems

TrueSkill™

Joint work with Thore Graepel, Tom Minka & Phillip Trelford

Motivation

- Competition is central to our lives
 - Innate biological trait
 - Driving principle of many sports
- Chess Rating for fair competition
 - ELO: Developed in 1960 by Árpád Imre Élő
 - Matchmaking system for tournaments
- Challenges of online gaming
 - Learn from few match outcomes efficiently
 - Support multiple teams and multiple players per team



The Skill Rating Problem

- Given:
 - Match outcomes: Orderings among k teams

• Q

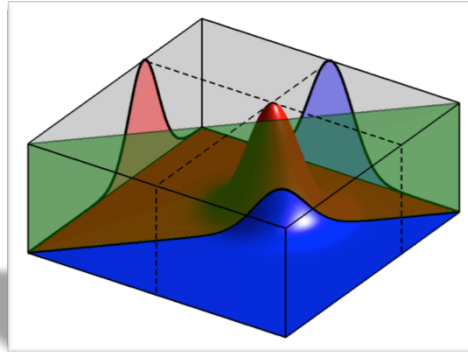
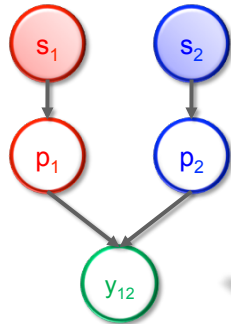
Team	Score
1st Red Team	50

Level	Gamertag	Avg. Life	Best Spree	Score
1st	SniperEye	N/A	N/A	25
2nd	xxHALDOxx	N/A	N/A	24
3rd	AjaySandhu	N/A	N/A	15
3rd	AjaySandhu(G)	N/A	N/A	15
5th	Robot115	N/A	N/A	11
5th	TurboNego84(G)	N/A	N/A	11
7th	TurboNego84	N/A	N/A	5
8th	SniperEye(G)	N/A	N/A	1

1	27	SEWICSYDE OWNS
2	26	FATAL REVENGE
3	25	Paranoia 1
4	25	Paulk
5	25	ixX OMG XxI
6	25	BittyTom
7	24	brian 2007
8	24	SEXY MOZES
9	24	droplates
10	24	jaCKdsSaMuRai
11	24	Il Me Il
12	24	iamNightMare
13	24	a retarded007
14	24	Perfected Brit
15	24	THE MUFFIN MANx
16	23	TheVunit
17	23	Mr Sush87

Two Player Match Outcome Model

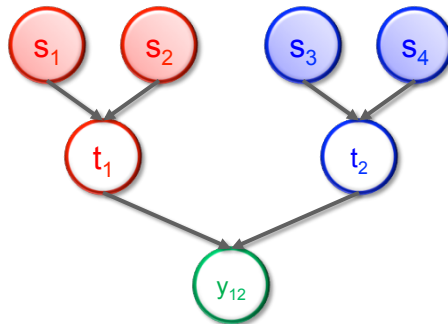
- Latent Gaussian performance model for fixed skills
- Possible outcomes: Player 1 wins over 2 (and vice versa)



$$\mathbf{P}(y_{12} = (1, 2) | p_1, p_2) = \mathbb{I}(p_1 > p_2)$$

Two Team Match Outcome Model

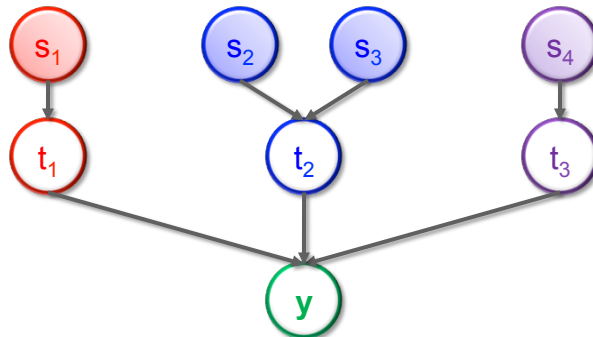
- Skill of a team is the sum of the skills of its members



$$\mathbf{P}(t_1 | s_1, s_2) = \mathcal{N}(t_1; s_1 + s_2, 2 \cdot \beta^2)$$

Multiple Team Match Outcome Model

- Possible outcomes: Permutations of the teams

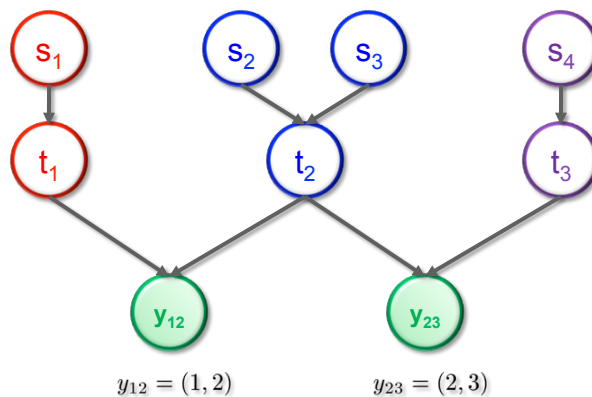


$$\mathbf{P}(\mathbf{y}|t_1, t_2, t_3) = \mathbb{I}(\mathbf{y} = (i, j, k)) \text{ where } t_i > t_j > t_k$$

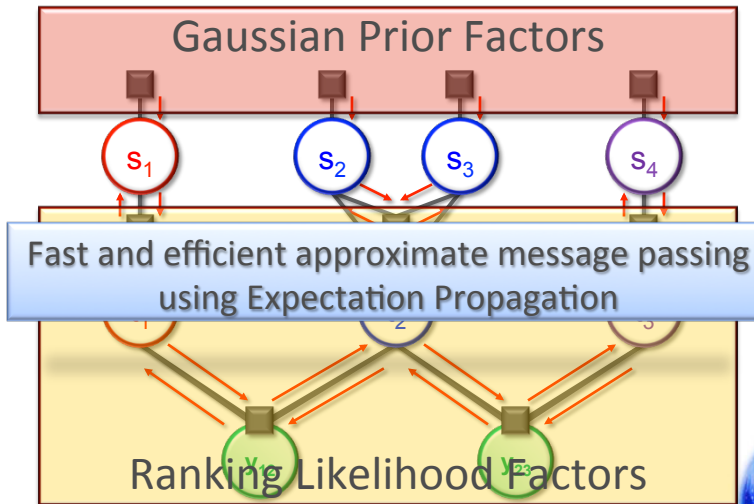
Multiple Team Match Outcome Model

- But we are interested in the (Gaussian) posterior!

$$\mathbf{P}(s_i|\mathbf{y} = (1, 2, 3)) = \mathcal{N}(s_i; \mu_i, \sigma_i^2)$$



Efficient Approximate Inference



Applications to Online Gaming

- **Leaderboard**
 - Global ranking of all players

$$\mu_i - 3 \cdot \sigma_i$$

- **Matchmaking**
 - For gamers: Most uncertain outcome

	Level	Gamertag	Avg. Life	Best Spree	Score
1st	10	BlueBot	00:00:49	6	15
1st	7	SniperEye	00:00:41	4	14
1st	9	ProThePirate	00:01:07	3	13
1st	10	dazdemon	00:00:59	3	8
2nd	10	WastedHarry	00:00:41	4	17
2nd	3	Ascla	00:00:57	2	5
2nd	9	AntidotedLosing	00:00:44	2	5
2nd	12	BlackDops	00:00:46	2	5

1	27	SEWCYSYDE OWNS
2	26	FATAL REVENGE
3	25	Paranoia 1
4	25	Paulk
5	25	lxX OMG Xxl
6	25	BittyTom
7	24	brian 2007
		SEXY MOZES
		droplates
		jaCKdaSaMuRai
		ll Me ll
		iamNightMare
		a retarded007
		Perfected Brit
		THE MUFFIN MANX
		TheVunit
		Mr Sushi87

$$P(p_i \approx p_j | \mu_i, \sigma_i, \mu_j, \sigma_j^2 + \sigma_j^2)$$

$$P(p_i \approx p_j | \mu_i - \mu_j = 0, \sigma_i^2 + \sigma_j^2 = 0)$$

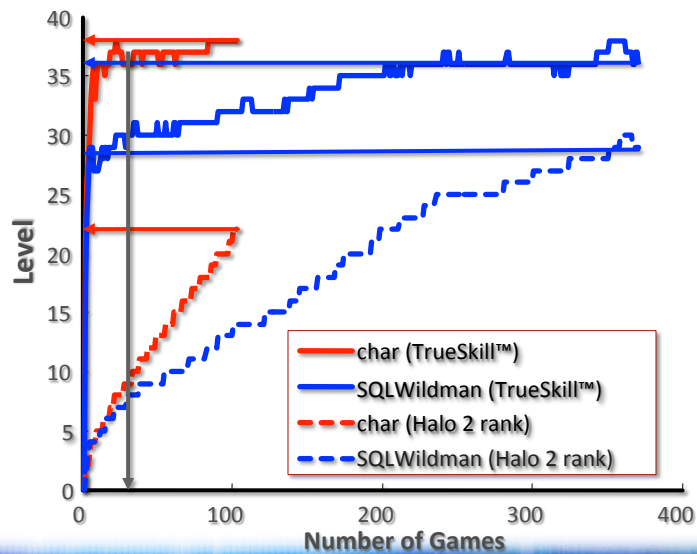
Experimental Setup

- **Data Set: Halo 2 Beta**

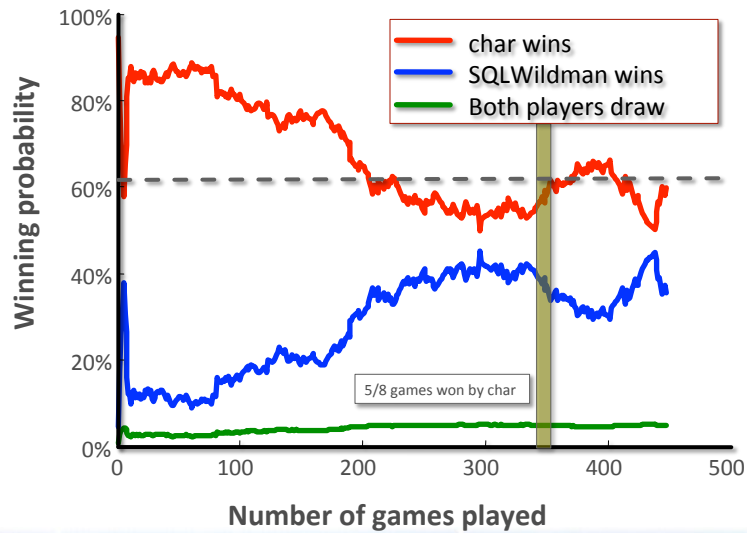
- 3 game modes
 - Free-for-All
 - Two Teams
 - 1 vs. 1
- > 60,000 match outcomes
- ≈ 6,000 players
- 6 weeks of game play
- Publically available



Convergence Speed



Convergence Speed (ctd.)



Xbox 360 & Halo 3

- **Xbox 360 Live**

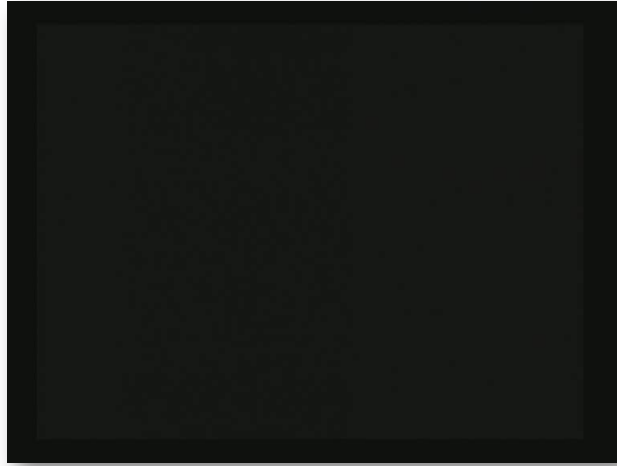
- Launched in September 2005
- Every game uses TrueSkill™ to match players
- > 10 million players
- > 2 million matches per day
- > 2 billion hours of gameplay



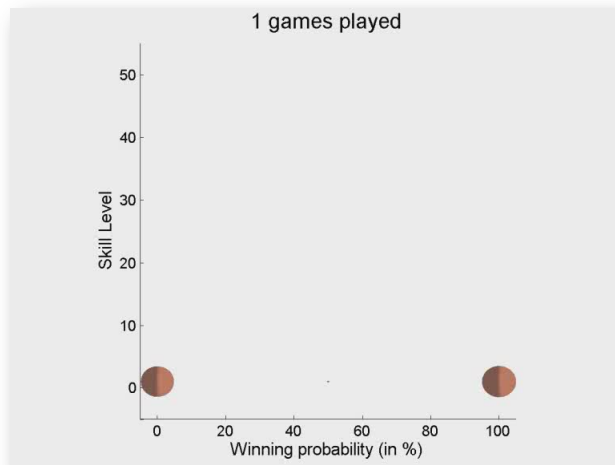
- **Halo 3**

- Launched on 25th September 2007
- Largest entertainment launch in history
- > 200,000 player concurrently (peak: 1,000,000)

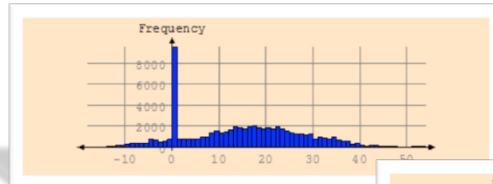
Halo 3 in Action



Halo 3 Public Beta Analysis

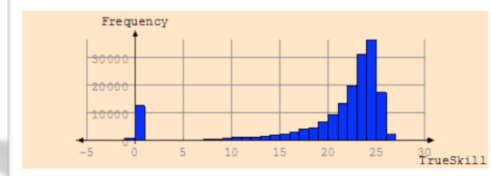
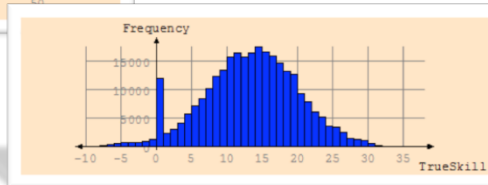


Skill Distributions of Online Games



Golf (18 holes): 60 levels

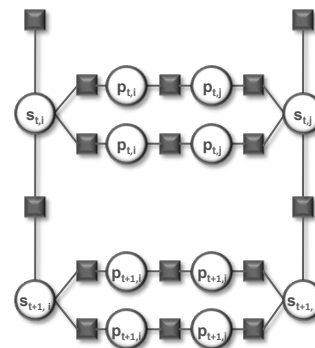
Car racing (3-4 laps): 40 levels



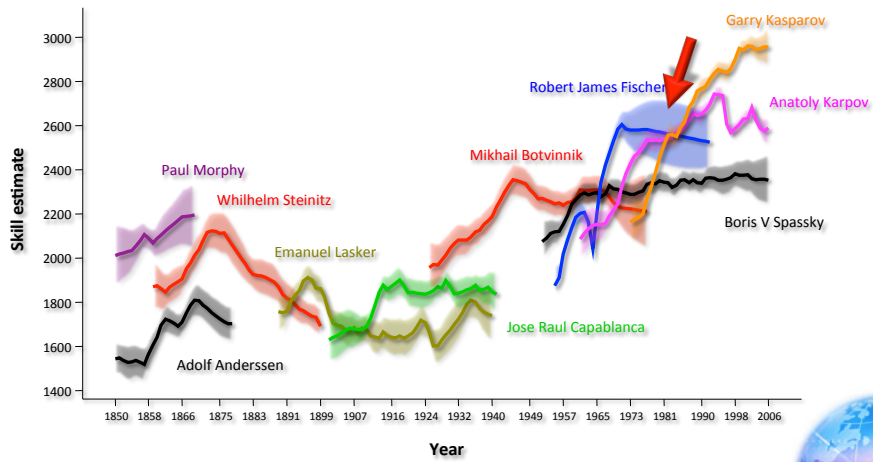
UNO (chance game): 10 levels

TrueSkill™ Through Time: Chess

- Model time-series of skills by smoothing across time
- History of Chess
 - 3.5M game outcomes (ChessBase)
 - 20 million variables (each of 200,000 players in each year of lifetime + latent variables)
 - 40 million factors



ChessBase Analysis: 1850 - 2006



Online Advertising

Joint work with Thore Graepel, Joaquin Quiñonero Candela, Onno Zoeter, Tom Borchert, Philip Treford

Why Predict Probability-of-Click?

The screenshot shows a Live Search results page for the query 'Seattle'. The page displays several search results, including 'Seattle Flights', 'Visiting Seattle?', and 'seattle'. Each result is accompanied by a price and a percentage change. Handwritten mathematical formulas are overlaid on the page:

- $b_1 \cdot p_1 \geq b_2 \cdot p_2$ is written above the search bar.
- $c_i = b_i + 1 \cdot \frac{p_i + 1}{p_i}$ is written in the middle of the page.

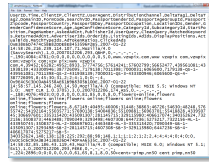
Result	Price	Change	Value
Seattle Flights	\$1.00	* 10%	=\$0.10
Visiting Seattle?	\$2.00	* 4%	=\$0.08
seattle	\$0.10	* 50%	=\$0.05

Other elements on the page include a search bar, navigation links (Home, Hotmail, Spaces), and a 'Sign out' button. The page also features a 'Sponsored sites' section with links to 'Seattle Washington Rates' and 'Seattle Washington Hotel Bargains!'.

The Scale of Things

- **Several weeks of data in training:**
7,000,000,000 impressions
- **2 weeks of CPU time during training:**
2 wks \times 7 days \times 86,400 sec/day =
1,209,600 seconds
- **Learning algorithm speed requirement:**
 - **5,787** impression updates / sec
 - **172.8 μ s** per impression update

The Flow of Information



User interaction



Raw Logs

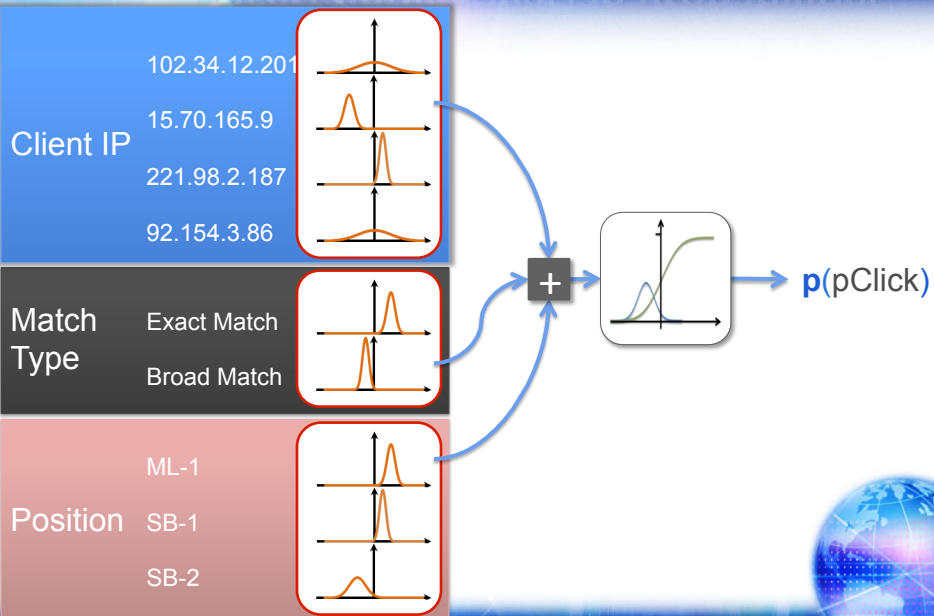


Structured Data

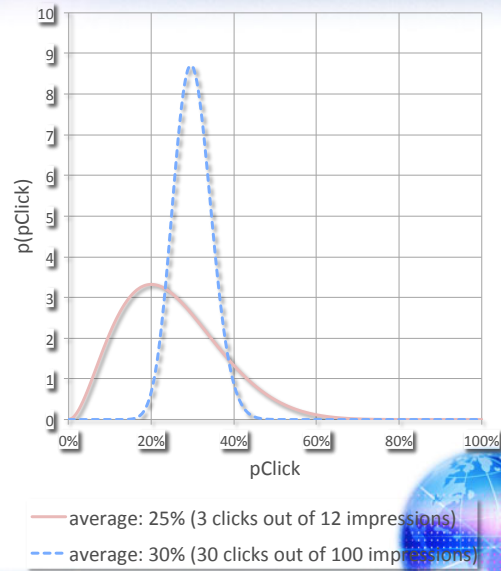
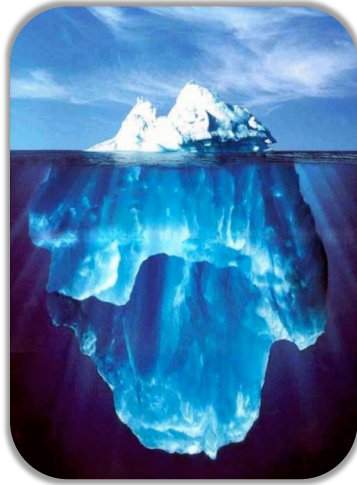
- Why structured data?
 - Data validation and cleaning
 - Principled feature transformations



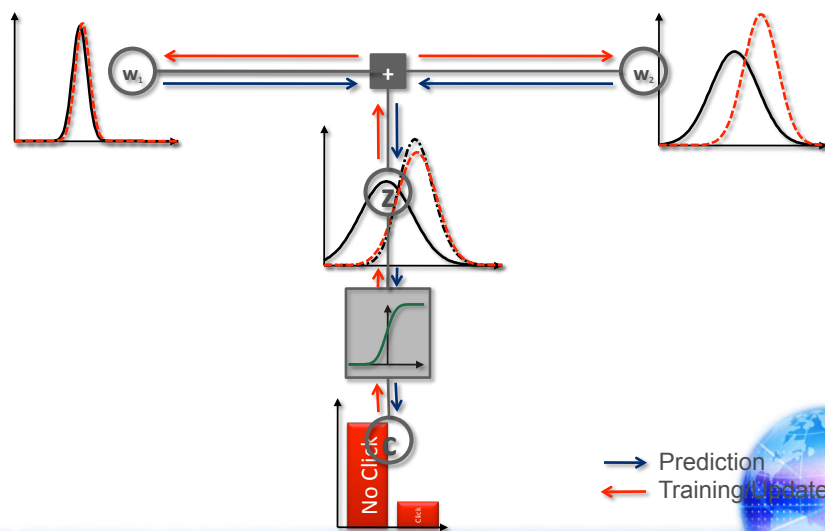
Uncertainty: Bayesian Probabilities



Principled Exploration



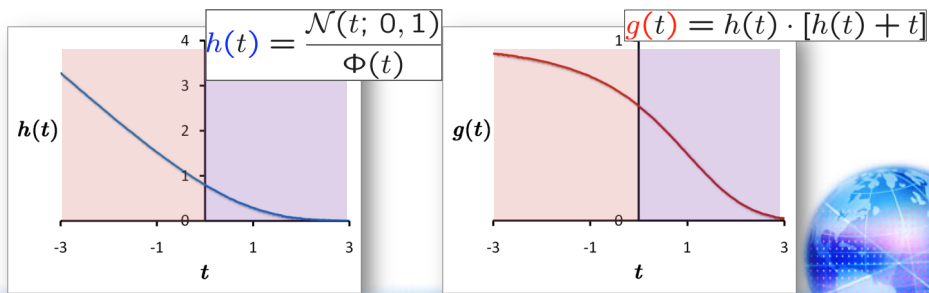
Training Algorithm in Action



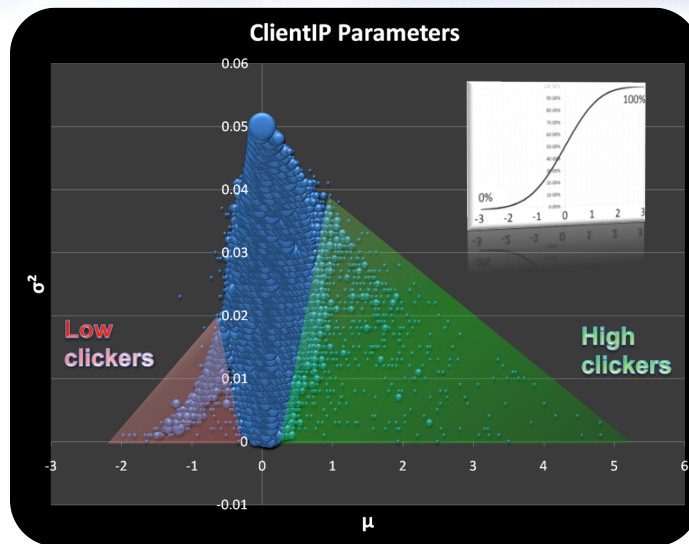
Inference: An Optimization View

$$\mu_i \leftarrow \mu_i + \frac{\sigma_i^2}{s} \cdot h \left[\frac{\sum_{j=1}^d \mu_j}{s} \right] \quad \sigma_i^2 \leftarrow \sigma_i^2 \left(1 - \frac{\sigma_i^2}{s^2} \cdot g \left[\frac{\sum_{j=1}^d \mu_j}{s} \right] \right)$$

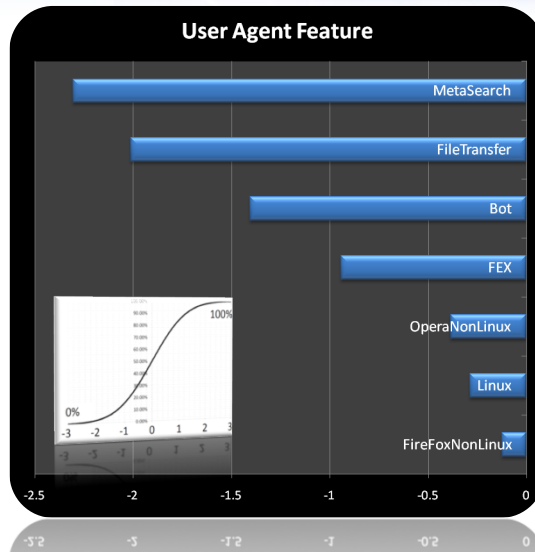
$$s^2 = \beta^2 + \sum_{j=1}^d \sigma_j^2$$



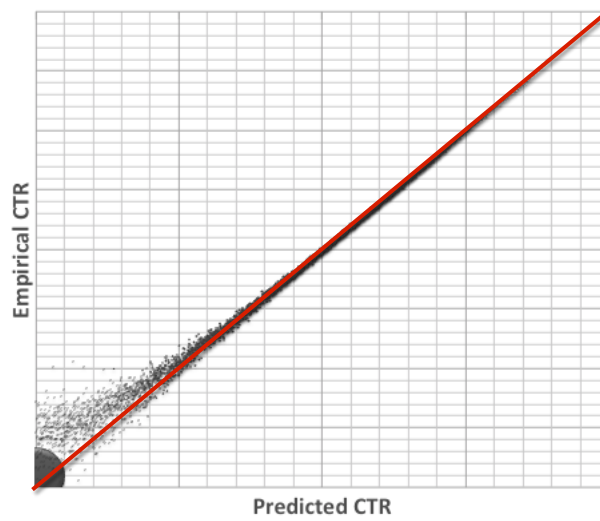
Client IP: Mean & Variance



UserAgent: Mean Posterior Effects

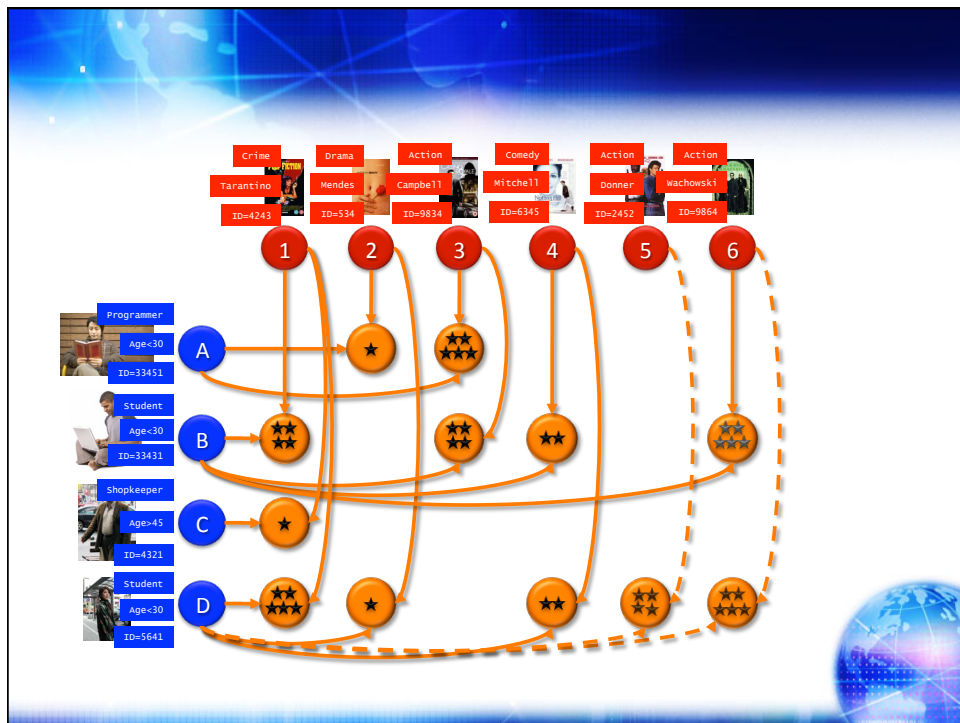


Accuracy

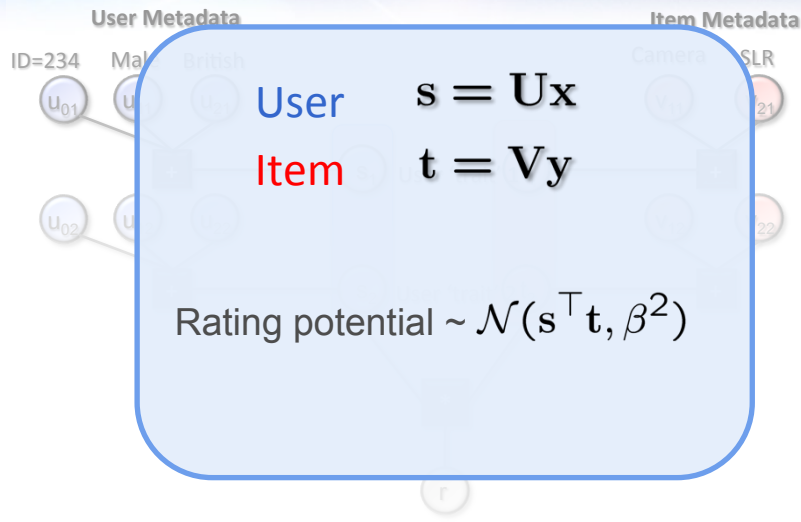


MatchBox

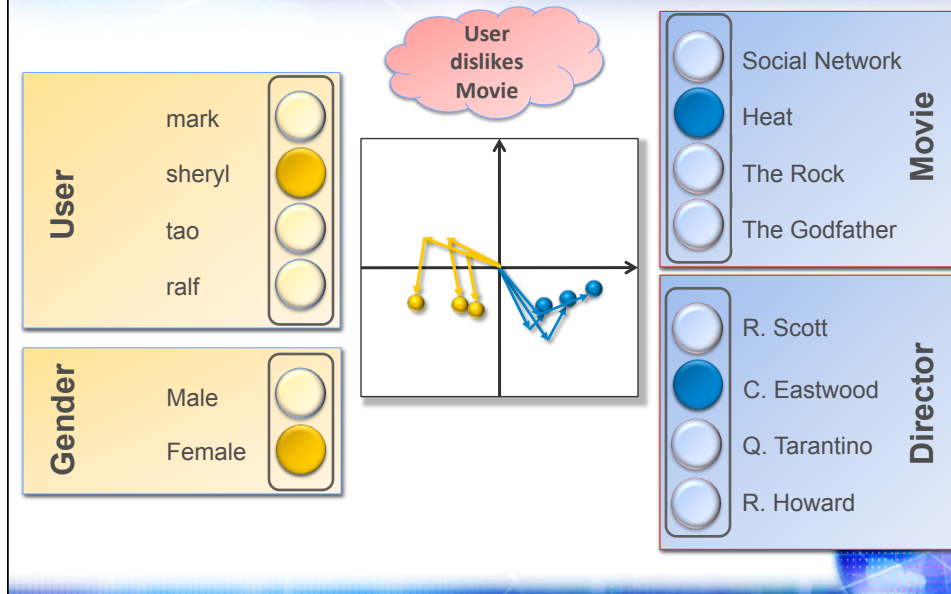
Joint work with Thore Graepel, Joaquin Quiñero Candela, David Stern

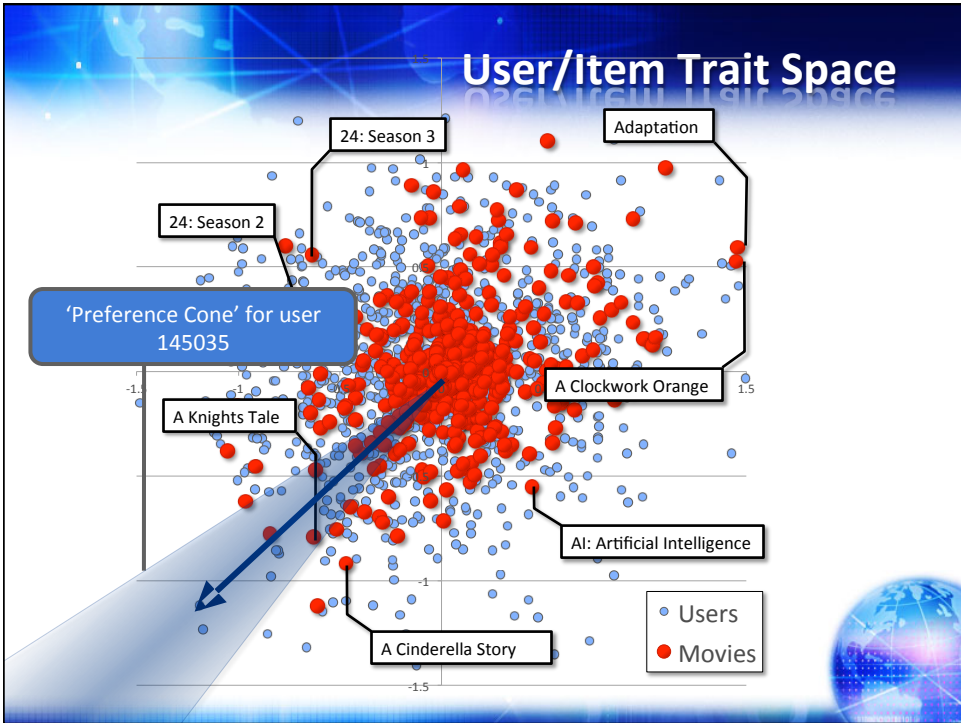
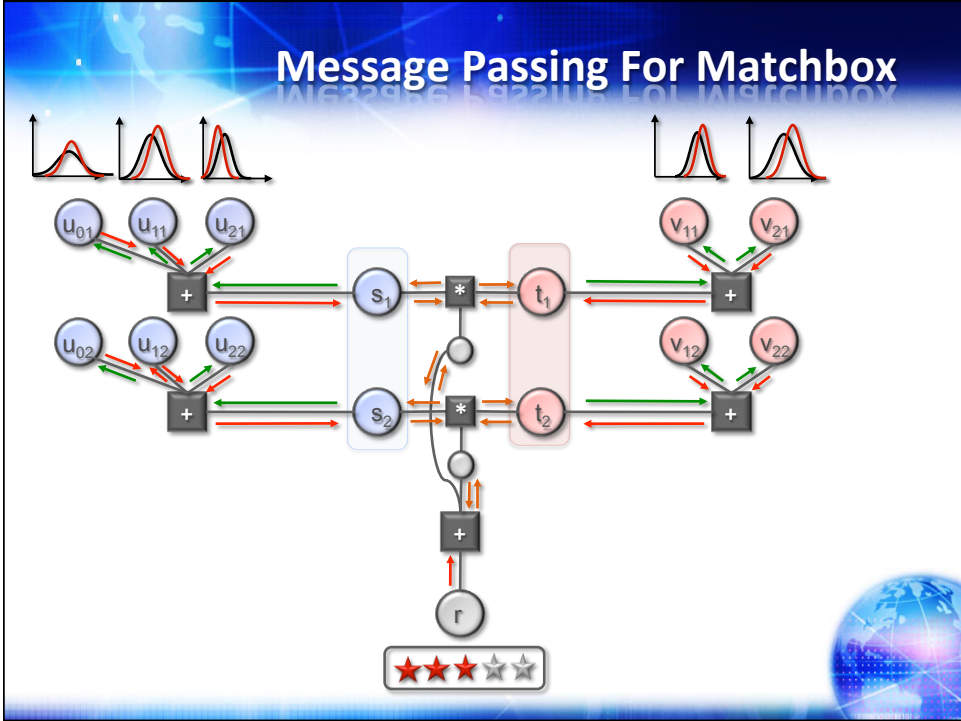


Matchbox With Metadata

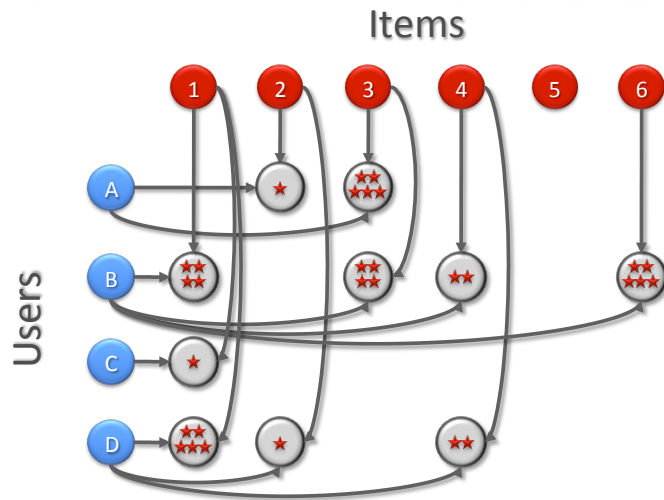


Recommender System: MatchBox

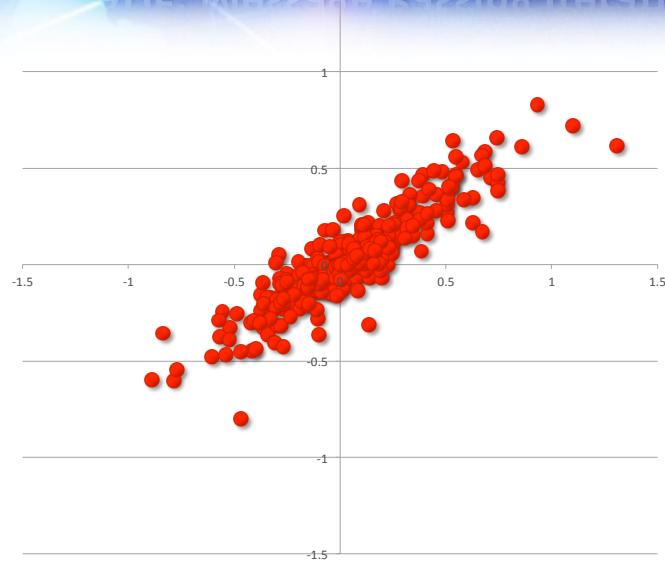




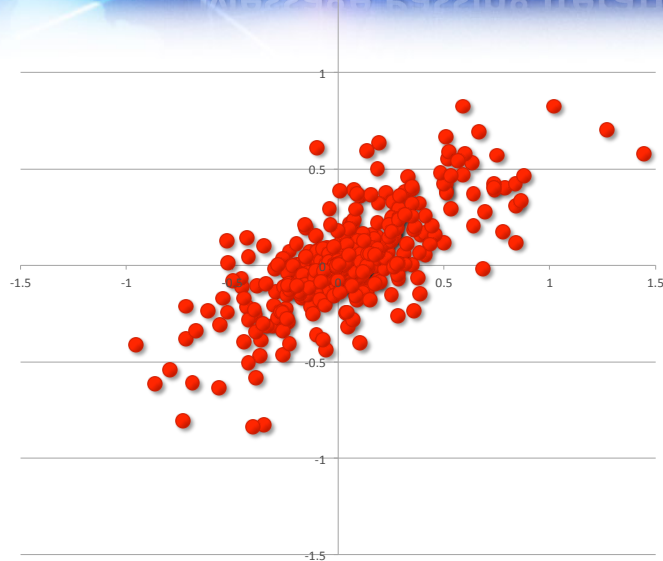
Incremental Training with ADF



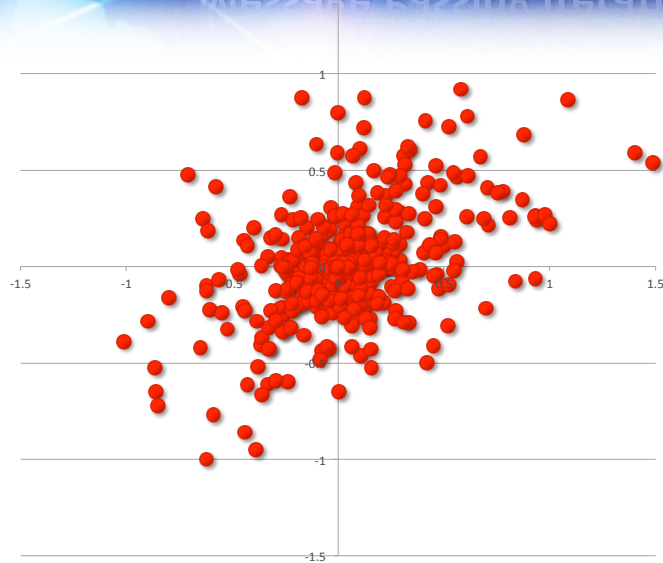
ADF: Message Passing Iteration 1



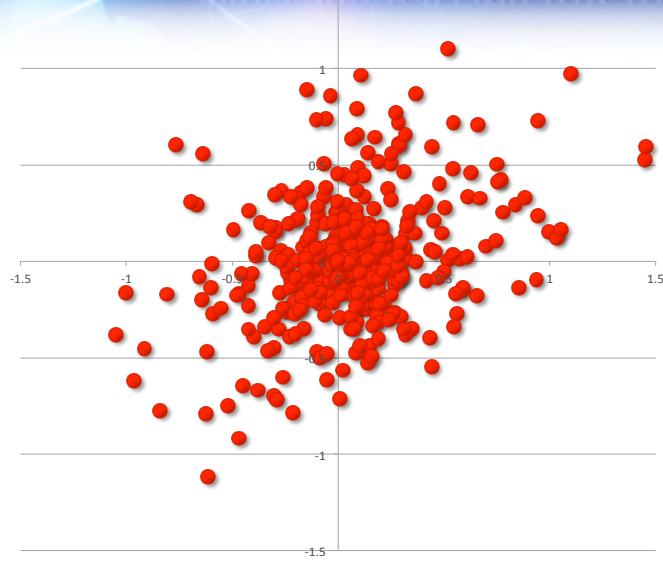
Message Passing Iteration 2



Message Passing Iteration 3

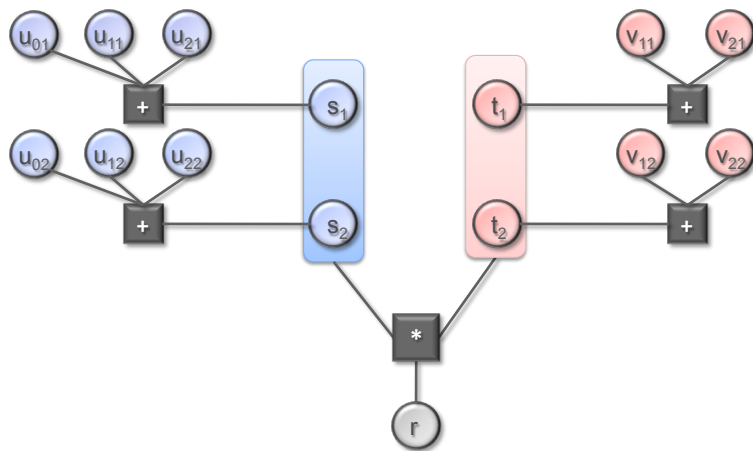


Message Passing Iteration 4

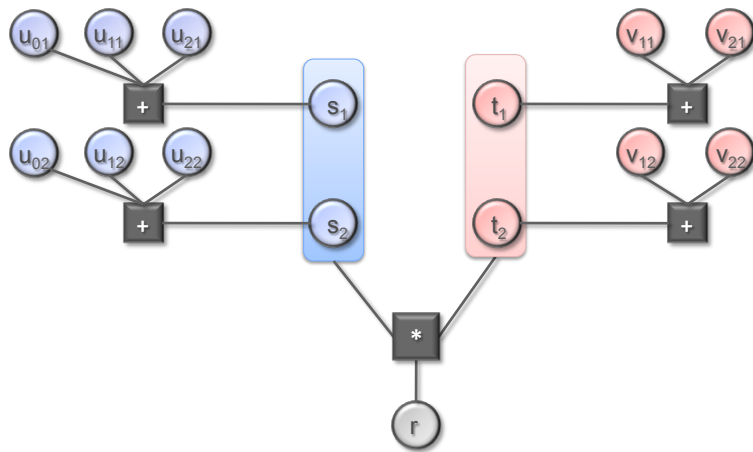


feedback models

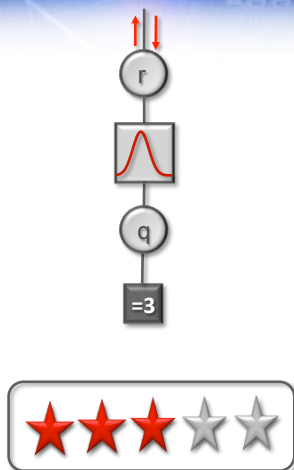
Feedback Models



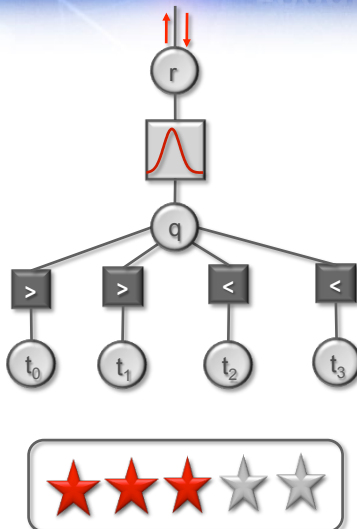
Feedback Models



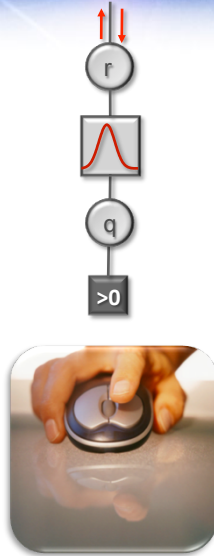
Feedback Models



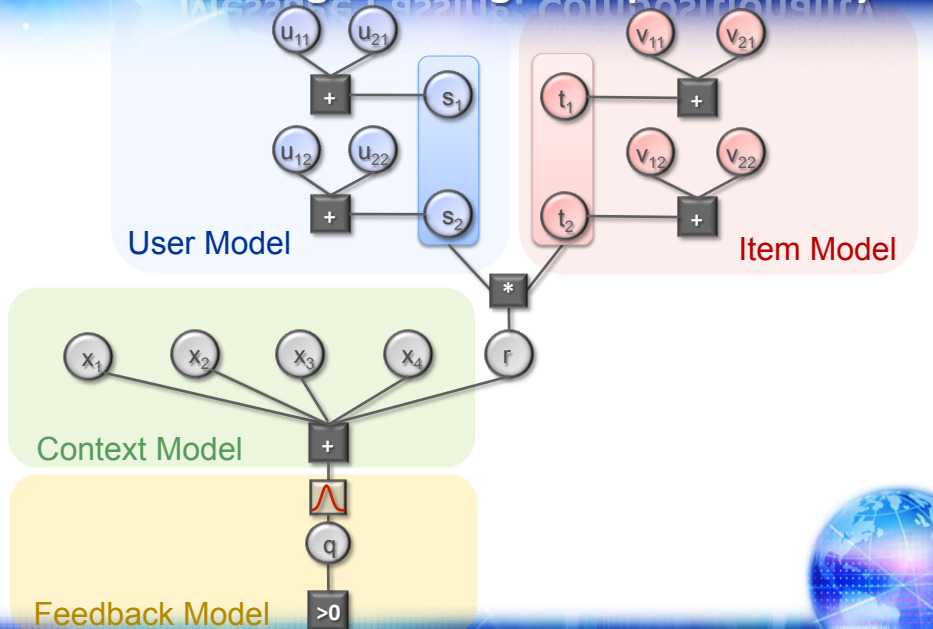
Feedback Models



Feedback Models



Message Passing: Compositionality



accuracy

Performance and Accuracy



MovieLens Data

- 1 million ratings
- 3,900 movies / 6,040 users
- User / movie metadata

MovieLens – 1,000,000 ratings

6,040 users

3,900 movies

User ID	
User Job	
Other	Lawyer
Academic	Programmer
Artist	Retired
Admin	Sales
Student	Scientist
Customer Service	Self-Employed
Health Care	Technician
Managerial	Craftsman
Farmer	Unemployed
Homemaker	Writer

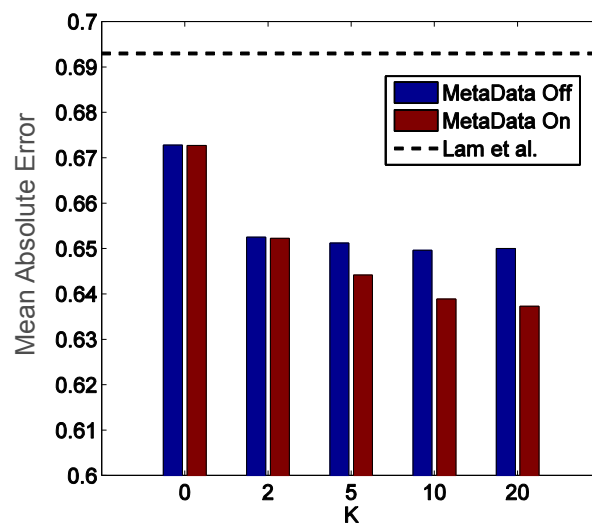
User Age
<18
18-25
25-34
35-44
45-49
50-55
>55

User Gender
Male
Female

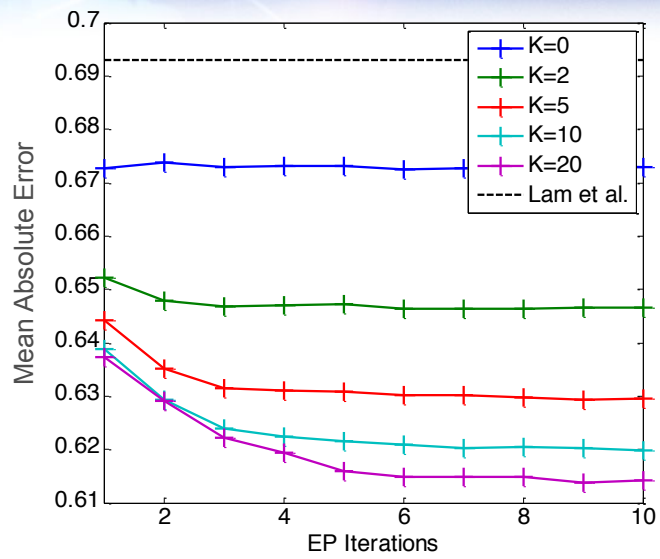
Movie ID	
Movie Genre	
Action	Horror
Adventure	Musical
Animation	Mystery
Children's	Romance
Comedy	Thriller
Crime	Sci-Fi
Documentary	War
Drama	Western
Fantasy	Film Noir

MovieLens with Thresholds Model

(ADF), Training Time= 1 Minute



MovieLens Error with Thresholds



Recommendation Speed

Recommendation Speed

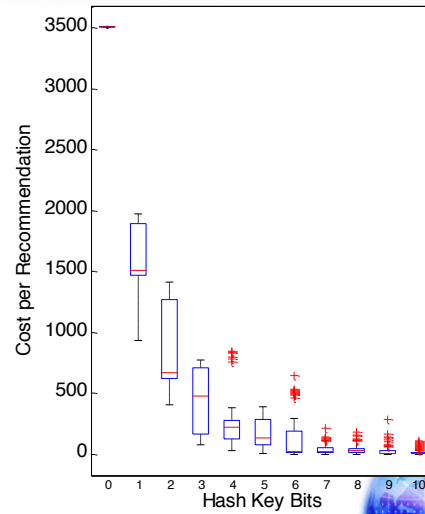
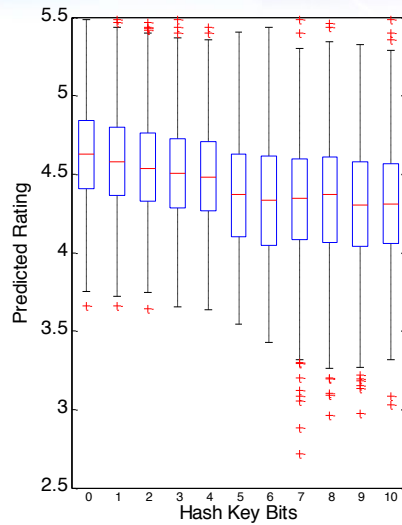
- **Goal:**
find N items with highest predicted rating.
- **Challenge:**
potentially have to consider all items.
- Two approaches to make this faster:
 - Locality Sensitive Hashing
 - KD Trees
- **Locality Sensitive Hash:**

$$P(h(x) = h(y)) = \text{sim}(x, y)$$

Random Projection Hashing

- **Random Projections:**
 - Generate random hyper planes
(m random vectors, a_i).
 - Gives m bit hash, $\{x_0, x_1, \dots, x_m\}$, by:
$$x_i = \mathbf{1}[a_i \cdot t > 0]$$
- $p(\text{all bits match}) \propto \text{cosine similarity}$.
- Store items in buckets indexed by keys.
- Given a user trait vector:
 1. Generate key, q.
 2. Search buckets by hamming distance from q until find N items.

Accuracy and Speedup



Thanks!