CSE 590XYZ - Performance Analysis

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Course Contents

- This course is a descendant of CSE 543 (Computer System Performance Modelling). However, the range of material is significantly expanded.
- There are bound to be some rough points...
- Emphasis is on system performance modelling using stochastic techniques
“System Performance Modeling”

- Vague term intended to mean
  - system modeling → not
    - performance of specific individual applications
    - low-level component (e.g., CPU) performance issues
    - communication network hardware/protocols
  - performance modeling
    - quantitative analysis, excluding reliability/availability/performability issues

“Stochastic Techniques”

- /stochastic/ adj.
  1. determined by a random distribution of probabilities.
  2. (of a process) characterized by a sequence of random variables.
  3. governed by the laws of probability.
- Presented in a restricted scope (primarily queueing networks and more general Markov models)
- Should provide enough background to understand work outside that particular scope
Management Details

• Performance analysis includes some science and some art.
• The science means a significant portion of the class material will involve equations.
  * On the plus side: there is a real right and wrong here
  * On the negative side: not everyone loves equations; they can be hard to assimilate in real-time
• The art means that whether or not a model is “right” is a subjective matter.

More Management

• Lectures will (try to) concentrate on underlying principles.
• Assignments will attempt to provide useful experiences with a number of modelling approaches (simulation and queueing networks, at least).
• I’m planning on using a performance question about web servers as “the problem” for much of this work.
• How much time will this take?
Just Plain Details

- Homeworks: some reading, some paper and pencil, some hands on project
- Not everything will be handed in
- Grading: 50% assignments, 25% midterm, 25% final, 10% intangibles
- Late policy:
  - Please hand paper and pencil exercises in on time
  - The point of the project experience is to have the experience. More flexibility will be easier on all of us.

Course Text

- Nope, but
  - Introduction to Computer System Performance Evaluation, K. Kant, McGraw-Hill.
  - Proceedings ACM Sigmetrics Conference
  - Performance Evaluation, North-Holland
  - ...
Course Web

- There is a course web (under 590yz)
- It’s still under construction (but it’s more complete than you might think…)
- The first assignment is there

Lecture Overview

- Uses of Modeling
- What is a Model?
- The Modeling Process
  - Pose the Question
  - Create the Model
- Model Description Languages
  - Queueing Networks
  - Petri Nets
- The Use of Probability/Statistics
Uses of Modeling

- System Design
- Application (Software) Design
- Capacity Planning
- Tuning
- Understanding

System Design

- Resource allocation issues
  * processor scheduling policies for parallel systems, multimedia systems, etc.
  * memory/cache management policies
- Distributed system issues
  * file system design
  * server configuration
  * distributed databases
Application (Software) Design

- Pre-implementation resource consumption estimation
- Impact of data structure selection
- Choice of locking granularity
- Parallel application performance

Capacity Planning

- Bottleneck assessment
- Adding CPU/Disk/Memory to accommodate workload growth
- Impact of new workloads
- Server configuration/sizing
Tuning

- Performance measurement
- Memory reference behavior/impact
- System call overhead

Understanding

- Often building a model of a system forces you to organize your understanding of it in a way that elucidates the key interactions
- Additionally, in some cases the model solution gives a useful, generally applicable insight
  * Example: Amdahl's Law
Amdahl’s Law

• Gives an upper bound on the “speedup” of a parallel application
  * Speedup on P processors, S(P), is defined as the ratio of the elapsed time to completion on 1 processor to the time on P processor
    \[ S(P) = \frac{T(1)}{T(P)} \]
  
• Amdahl’s Law: Given a parallel application where fraction f is “inherently sequential”, the best speedup possible on P processors is
    \[ S(P) \leq \frac{1}{f + (1-f)/P} \]
    \[ S(\infty) \leq \frac{1}{f} \]
What Is a Model?

- A representation of a system that is
  - simpler than the actual system
  - captures the essential characteristics
  - can be evaluated

“Simpler”

- Easier to construct than the actual system
  - server utilization = .005 * #users
  - Performance in 1 year can be estimated by running a workload generator on the current system

- “Simpler” often means “has fewer parameters”
  - Example: Computational Complexity (O(•) notation): captures almost no details of any real hardware or implementation, yet provides useful information
“Captures the essential”

- Include only those characteristics that affect performance to the level of accuracy desired for the question that has been posed
  - My web server more disk capacity. Can I just add disks or must I add servers?
    - Do you model file buffer space effects or not?
- Deciding whether or not a model captures the essential is subjective
- I personally probably tend to err on the side of simplicity. Your initial inclination will probably be the opposite.

“Can be evaluated”

- Example: (Newtonian) N-Body Simulation
  (From http://www.npac.syr.edu/EDUCATION/PUB/npfe/module5/index.html)

\[
\begin{align*}
\text{Squaring the equation:} \\
\frac{dx}{dt} &= v \\
\frac{dv}{dt} &= a, \quad \text{where } \forall 1 \leq i \leq 5, \quad a_i &= \sum_{j=1 \atop j \neq i}^{5} \frac{m_j (x_j - x_i)}{r_j^3}
\end{align*}
\]
“Can be evaluated”

- **Problem 1: Don’t know how to solve the ODEs exactly**
  - Use an approximate solution technique by discretizing time
- **Problem 2: (Approximate) Computation is O(N²)**
  - Develop faster approximations that have acceptable accuracy

What Is a Model? Revisited

- A representation of a system that is
  - simpler than the actual system
  - captures the essential characteristics
  - can be evaluated
- There is always an interaction among these three
- Particularly in the case of analytical (mathematical) models, what can be evaluated has a strong influence
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The Modeling Process (Given an existing system)

1. Pose the Question
2. Create the Model
3. Parameterize the Model
4. Evaluate the Model
5. Validate / Debug

Existing System

Projected System
Projecting Performance

Modify and Reparameterize the Model
Evaluate the Model
Interpret the Results

Validated Model

Modeling Process Example: Amdahl’s Law

- Imagine we have a parallel application running on a dual processor and want to project performance on larger machines
- Pose the Question
  What will the speedup be on 8 processors?
- Create the Model
  \[ S(2) = \frac{1}{f + \frac{1-f}{2}} \]
- Parameterize the Model
  Suppose measured \( S(2) = 1.79 \). Then we deduce that \( f = 0.12 \).
Modeling Process Example: Amdahl’s Law (cont.)

- **Evaluate / Debug**
  Well, the way we created the model guarantees it evaluates to the correct result.
- **Modify/Reparameterize**
  \[ P = 8 \]
- **Evaluate**
  \[ S(8) = \frac{1}{0.12 + \frac{0.88}{8}} = 4.35 \]
- **Sensitivity Analysis**

Accuracy?

- **Validated Model**
- **Modify and Reparameterize the Model**
- **Evaluate the Model**
- **Interpret the Results**

Accuracy?
The Modeling Process
(Given an existing system)

Pose the Question

- This is a surprisingly difficult step
- One criterion: Is the answer the model is going to give obvious from the problem statement and the model structure?
  * E.g., “Is response time better on a system with a faster CPU?” when the model contains representations of only the workload and the CPU itself
    - Clearly, the answer is “yes”
  * In this case, either problem statement or the model should be changed
Pose the Question

- The question posed strongly affects the kind of accuracy required
  - What will the response time be if the server is upgraded to a 266MHz PII?
    - Must give a single number to within X% of true value
  - How much faster will the 266MHz system be than the current system?
    - If model underestimates response time by 20% on both the original and the projected systems, answer will still be exact

\[
\frac{\text{Modelled Original Performance}}{\text{Modelled Projected Performance}} = \frac{.8 \times \text{Actual Original}}{.8 \times \text{Actual Projected}}
\]

Pose the Question

- Is the 266 MHz system faster than the original?
  - Suppose the model says “yes” and real system in fact has response times that are 80% of the original system. Let \( \alpha \) be the modeling error in the modified system and \( \beta \) be the modeling error in the existing system.

\[
\frac{\alpha}{\beta} < 1.25
\]

\[
\frac{\alpha}{\beta} < 1
\]

Model is “Correct”
Pose the Question

- As a rule, comparing several alternatives to each other is much easier than other sorts of questions.
- On the other hand, you don’t know what the true answers are, and your confidence that the model is behaving accurately in a comparative sense typically decreases as your suspicion about the absolute error grows.

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Create the Model

• “How is the model expressed?” versus “How is the model evaluated?”
• These two are often intimately intertwined (or confused)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>System similar to the projected system</td>
<td>Measurement or Benchmarking</td>
</tr>
<tr>
<td>Arbitrary set of equations</td>
<td>Analytic model</td>
</tr>
<tr>
<td>Arbitrary software description of system behavior (state changes)</td>
<td>Simulation model</td>
</tr>
</tbody>
</table>

Create the Model

• Higher-level description languages can be useful
  * More convenient expression
  * Provide some structure for thinking about how to model the system
  * Determine what needs to be measured
  * “canned evaluations”
• As always, there is a trade-off between convenience and generality: the more tailored a model description language is to the environment, the easier it is to apply there, but the more difficult it is to use for anything else
Model Description Languages

- We're going to look at two description languages, Petri nets and queueing models.
- Only the briefest introductions here - just enough to have some common vocabulary until we look at these topics in more detail.

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Queueing Model Overview

- The basic component is the service center.

[Diagram of a queueing model with labeled components: Customer Arrivals, Queue, Server, Customer Departures, and Service Center.]

- Parameters:
  - Customer arrival rate
  - Average service time
  - Scheduling discipline
  - ...

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Queueing Model Overview

- We can construct networks of service centers

Petri Net Overview

- Basic components are places, transitions, and tokens
Basic PN Operation: Transition Firing

Modeling a Service Center in a Petri Net
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The Use of Probability/Statistics

- Probability distributions are the basic building blocks that allow us to generate complicated sequences of behavior from simple descriptions.
- Statistics take us from observed behavior back to probability distributions (or measures of distributions).
Suppose we want to represent a system as a single service center. How should we describe customer arrivals (say, in a simulation of the system)?

Most faithful representation: a trace of actual arrival times

\[ t_0, t_1, t_2, \ldots, t_N \]

Problems
- This can be a large amount of data
- Was the interval traced representative of “typical behavior”?
- How do you modify the trace to represent the projected system?

What to Do?

If you’re not going to replay a trace, then you must generate the input data somehow. How?

One simple possibility:
- Define \( x_i = t_{i+1} - t_i \) (the \( i \)th inter-arrival time)
- Compute the average inter-arrival time \( \bar{x} = \frac{1}{N} \sum_{i=0}^{N-1} x_i \)
- Create the \( i \)th arrival at time \( \hat{t}_i = i \times \bar{x} \)

Problem: Arrivals are as evenly spread out as possible (so contention for the server is most likely underestimated)
Using Multiple Inter-arrival Times

- Suppose we arbitrarily pick three distinct inter-arrival times: \( \frac{1}{3} \bar{x}, \bar{x}, \) and \( \frac{3}{3} \bar{x} \)
- Now generate the \( i \)th arrival time by picking one of the three possible inter-arrival times “at random” and adding it to the \( i-1 \)st arrival time

\[
\hat{t}_i = X_i + \hat{t}_{i-1}
\]

Moral

- Probability distributions are a convenience
- They’re also almost unavoidable
- Stochastic assumptions provide formalisms that sometimes allow “exact analysis” of models
- Such analyses typically require either restrictions on the distributions used, or quite complicated analyses
- It’s a mistake to dismiss these models for those reasons.
- It’s also a mistake to have blind faith in them.
(Discrete) Probability

- An experiment is the process of making an observation.
  * Running Example: Examining the number of processes in the CPU run queue
- A sample space, $S$, is the set of all possible outcomes of an experiment.
  * $S = \{0, 1, 2, \ldots\}$
- A sample space is discrete if it is countable (which for our purposes means contains no real-valued intervals).

(Discrete) Probability

- A random variable is a variable whose value is the outcome of (some not yet performed) experiment.
  * $X = 4$ is the outcome that 4 runable processes are found in the run queue
- A probability distribution, $f_X(x)$, for a random variable $X$ is a mapping from the possible values of $X$ to probabilities. (We require that $f(x) \geq 0$ and $\sum_x f(x) = 1$)
  * $f(0) = .2$, $f(1) = .2$, $f(2) = .1$, $f(3) = .3$, $f(4) = .2$
(Discrete) Probability Distribution

- Terminology:
  - probability density function (pdf)
  - “X is distributed according to f(x)”
  - “X has distribution f(x)”

\[
\begin{align*}
 f(0) &= 0.2 \\
 f(1) &= 0.2 \\
 f(2) &= 0.1 \\
 f(3) &= 0.3 \\
 f(4) &= 0.2 
\end{align*}
\]

Expected Value

- “Mean of a distribution” or “the expected value, \( E(X) \), of a random variable X”

\[
\sum_x xf(x)
\]

- “Average queue length” = \( 0 \cdot 0.2 + 1 \cdot 0.2 + 2 \cdot 0.1 + 3 \cdot 0.3 + 4 \cdot 0.2 = 2.1 \)
- The mean is often denoted as \( \mu \) or \( \overline{X} \)
- You can also take expected values of functions of random variables

\[
E(X-2) = -2 \cdot 0.2 + -1 \cdot 0.2 + 0 \cdot 0.1 + 1 \cdot 0.3 + 2 \cdot 0.2 = 0.1
\]
Properties of Expected Value

- E(c) = c for any constant c
- Let g(X) be a function of the random variable X, and let c be a constant. Then E(cg(X)) = cE(g(X))
  * E(4(X+2)^2) = 4E((X+2)^2)
- Let g_1(X), g_2(X), ..., g_K(X) be K functions of random variable X. Then
  E(g_1(X)+...+g_K(X)) = E(g_1(X))+...+E(g_K(X))
  * E((X+2)^2) = E(X^2 + 2X + 4) = E(X^2) + 2E(X) + 4

Moments of Distributions

- \text{nth moment of } X \text{ is } E(X^n)
  * The mean is also called "the first moment"
- \text{nth central moment of } X \text{ is }
  E((X-\mu)^n) = \sum (x-\mu)^n f(x)
- The 2\textsuperscript{nd} central moment is called variance, and is often denoted as \sigma^2
  * \sigma^2 = E((X-\mu)^2) = E(X^2) - 2\mu E(X) + \mu^2 = E(X^2) - \mu^2
- The square root of variance, \sigma, is called standard deviation
What Does Variance Tell You?

- Variance is the “mean squared error” of the mean as an estimate of the outcome of a single observation.
- It is a (not very intuitive) measure of how much the outcomes are spread from the mean.
- Is a variance of 10 big? It depends on the mean.
- It’s often useful to use a normalized form of variance:
  * Coefficient of variation: \( CV = \frac{\sigma}{\mu} \)
  * Squared coefficient of variation: \( CV^2 = \frac{\sigma^2}{\mu^2} \)

Our Example

\[
\begin{align*}
\sigma^2 &= 0.2 \times (0-2.1)^2 + 0.2 \times (1-2.1)^2 + 0.1 \times (2-2.1)^2 + 0.3 \times (3-2.1)^2 + 0.2 \times (4-2.1)^2 \\
&= 2.09 \\
CV^2 &= \frac{2.09}{4.41} = 0.473923 \\
CV &= 0.688421
\end{align*}
\]
How Small Can Variance Be?

- Suppose we consider all (discrete) distributions that have the same mean, \( \mu \).
- If \( f(A)=1 \) and \( f(x)=0 \) for all other \( x \), the variance is 0.

\[
\begin{align*}
A &= \mu \\
\sigma^2 &= 0
\end{align*}
\]
(This is the deterministic distribution)

How Large Can Variance Be?

- Consider a family of distributions with constant mean \( \mu \), and for which only two outcomes are possible: \( A \), which occurs with probability \( p \), and \( B \) (which has probability \( 1-p \)).
- Let \( A = \mu - \varepsilon \), for an arbitrary \( \varepsilon > 0 \). Then \( B = \mu + \varepsilon/(1-p) \) (since \( \mu = p(\mu - \varepsilon) + (1-p)B \)).
- The variance of this distribution is
  \[
  \sigma^2 = \varepsilon^2(2-p)/(1-p)
  \]
- Letting \( p \to 1 \) (from below), the variance is unbounded. (Remember that the mean is constant at \( \mu \). However, the value \( B \) is unbounded as \( p \) grows.)
Large Variance

- The variance of this distribution is $\sigma^2 = \varepsilon^2(2-p)/(1-p)$

Letting $p \to 1$ (from below), the variance is unbounded. (Remember that the mean is constant at $\mu$. However, the value $B$ is unbounded as $p$ grows.)

Moral

- Means are in the range $(-\infty, \infty)$.
- Variances are in the range $[0, \infty)$.
- “Deterministic” behavior leads to low variance.
- Many frequently occurring values that are not close to each other lead to high variance.
- However, high variance can also result from rare, very abnormal behavior.
- We’ll often use CV instead of variance.
Conditionals

- It is sometimes useful to consider restricted sets of outcomes for a random variable
  - What is the mean number of processes in the run queue given that the run queue is not empty?

- Imagine running an infinite number of trials, throwing out the results that don’t meet the conditional criterion, and computing the distribution of the trials that do
  - We’ll write $f_{X | \text{criterion}}(x)$ to denote the conditional distribution

Conditional pdf Example

In general, $\text{Prob}[X=x | Y] = \text{Prob}[X=x & Y]/P[Y]$

- $\text{Prob}[X=1 | X>0] = .2/ .8$
- $\text{Prob}[X=0 | X>0] = 0/ .8$
**Independence**

- Suppose $X$ and $Y$ are random variables (i.e., correspond to the outcomes of random experiments). Are they independent?
- **Example**: Let $X$ be run queue length, and $Y$ be the number of processes blocked on disk 0.
- Are $X$ and $Y$ independent?

Are $X$ and $Y$ independent? *It depends...*
Independence

- Two random variables are independent iff
  \[ \text{Prob}[X=x \& Y=y] = \text{Prob}[X=x] \times \text{Prob}[Y=y] \]
- This does not correspond to intuition about “independence”!
- If \( X \) is independent of \( Y \), then
  \[
  \text{Prob}[X=x \mid Y=y] = \frac{\text{Prob}[X=x \& Y=y]}{\text{Prob}[Y=y]}
  = \frac{\text{Prob}[X=x] \text{Prob}[Y=y]}{\text{Prob}[Y=y]}
  = \text{Prob}[X=x]
  \]

This is unlikely for our example, but possible.
Cumulative Distribution Function

- The cumulative distribution function, cdf, is defined by
  \[ F(x) = \sum f(i) \]
- \( F(x) - F(x-1) = f(x) \)
- Clearly, \( 0 \leq F(x) \leq 1 \), and \( F(\infty) = 1 \)

Some Common Distributions

Uniform: \( U(A, B) \)

\[ f(x) = \frac{1}{B-A+1} \]

- \( \mu = \frac{A+B}{2} \)
- \( \sigma^2 = \frac{(B-A)^2}{12} + \frac{B-A}{6} \)
Geometric

- **Example**: Number of tosses until a heads is tossed
- If \( p \) is the probability “of success”, then \( f(x) = p(1-p)^{x-1} \)
  - \( \mu = 1/p \)
  - \( \sigma^2 = (1-p)/p^2 \)
  - The geometric is “memoryless” -- distn of number of future throws is unaffected by how many have taken place so far

```
[Graph showing the geometric distribution]
```

Poisson

- **Example**: Number of radioactive decays during an interval of length \( t \)
- Intuitively, imagine that
  * time is partitioned into small intervals of length \( \varepsilon \)
  * either 0 or 1 event takes place in each interval (but not more than one)
  * each interval (independently) has an event with probability \( p \)
- If \( \lambda \) is the mean rate at which events take place (so \( 1/\lambda \) is the mean time between events), then
  \[
  f(x, t) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}
  \]
Poisson

$\mu = \lambda t$

$\sigma^2 = \lambda t$

$\lambda = 0.5, \ t = 10$