Structure from Motion with Unknown Correspondence
(aka: How to combine EM, MCMC, Nonlinear LS!)

Frank Dellaert, Steve Seitz, Chuck Thorpe, and Sebastian Thrun

Traditionally: 2 Problems

1. Correspondence
2. Compute 3D positions, camera motion

The Correspondence Problem

The Structure from Motion Problem

• Find the most likely structure and motion \( \Theta \)

Question

How to recover structure and motion with unknown correspondence?

Aside: related subproblems

- known correspondence, unknown structure, motion
- known motion, unknown correspondence, structure
- stereo
- known structure, unknown correspondence, motion
- 3D registration, ICP
Without Correspondence:
Try Tracking Features

Lucas-Kanade Tracker (figures from Tommasini et. al. 98)

New Idea: try all correspondences

Likelihood: \( P(\Theta; U, J) \)

Marginalize over \( J \): \( P(\Theta; U) = \sum_j P(\Theta; U, J) \)

Combinatorial Explosion
• 3 images, 4 features: \( 4!^3 = 13,824 \)
• 5 images, 30 features: \( 30!^5 = 1.3131 \times 10^{162} \)
• (number of stars: \( 10^{20} \), atoms: \( 10^{79} \))

• Computing \( P(\Theta; U, J) \) is intractable!

EM for Correspondence

Expectation Maximization

E-Step: Soft Correspondences

\( P(J | U, \Theta^t) \)

View 1
View 2
View 3
M-Step: SFM

• Compute expected SFM solution

\[ \sum_j P(J | U, \Theta^t) \sum_{i, k} \sum \| u_{jk} - h(m_i, x_{jk}) \|^2 \]

• Problem: this requires solving an exponential number of SFM problems

• If we assume correspondences \( j_{ik} \) are independent, we can simplify...

Virtual Measurements

Virtual Measurements

Structure from Motion without Correspondence via EM:

1. Generate an initial structure and motion estimate \( \Theta^0 \).
2. In each image, calculate the \( n^2 \) "soft correspondences"
3. Calculate the virtual measurements \( v_{ij} \)
4. Find the new estimate \( \Theta^{t+1} \) for structure and motion using the virtual measurements \( v_{ij} \)
5. If not converged, return to step 2.

Key Issue: Calculating Weights

Calculating Weights

Weights = Marginal Probabilities
Mutual Exclusion

Monte Carlo EM

Monte Carlo Approximation

Sampling Assignments using the Metropolis Algorithm (1953)

To sample from \( P(J_i | U, \Theta^j) \) we use the Metropolis algorithm:

1. Start with a valid initial assignment \( J_0^i \).
2. Propose a new valid assignment \( J_i^j \).
3. Accept with ratio
   \[
   \alpha = \min \left( 1, \frac{P(J_i | U, \Theta^j)}{P(J_i | U, \Theta^0)} \right) = \frac{P(\Theta^j | U, J_i)}{P(\Theta^0 | U, J_i)}
   \]

Swap Proposals (inefficient)

‘Chain Flipping’ Proposals
Efficient Sampling (Example)

MCMC to deal with mutex

Input Images
- Manual measurements
- Random order

Results: Cube

Soft Correspondences