Projection

Readings
- Szeliski 2.1

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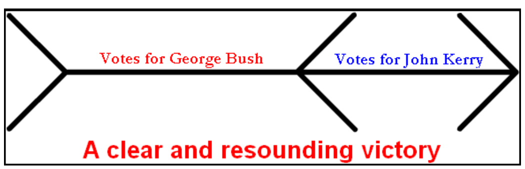
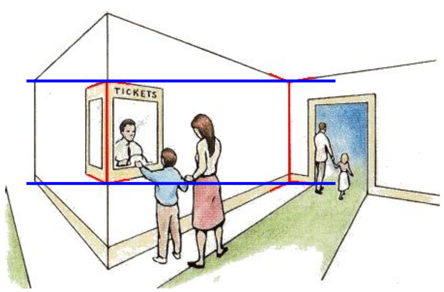
Müller-Lyer Illusion

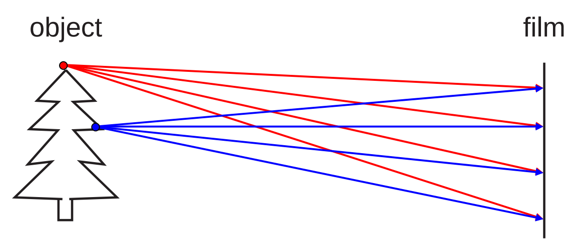


by Pravin Bhat



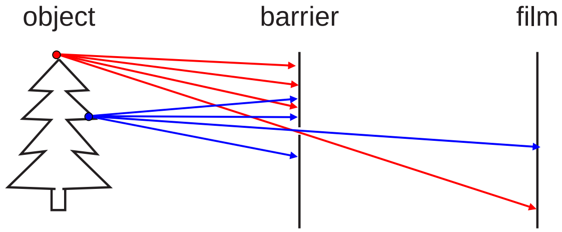
http://www.michaelbach.de/ot/sze_muelue/index.html

Image formation



Let's design a camera
- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

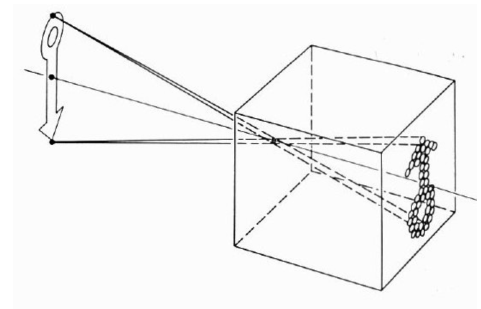
Pinhole camera



Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**
- How does this transform the image?

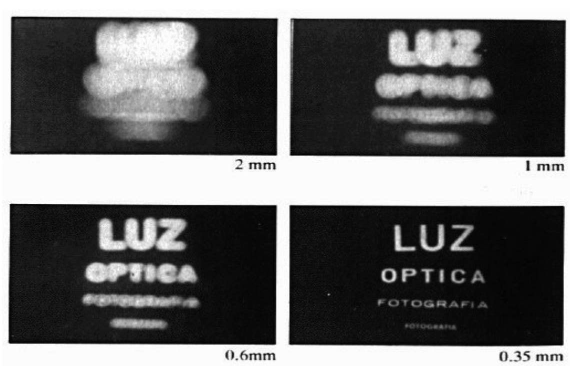
Camera Obscura



The first camera

- Known to Aristotle
- How does the aperture size affect the image?

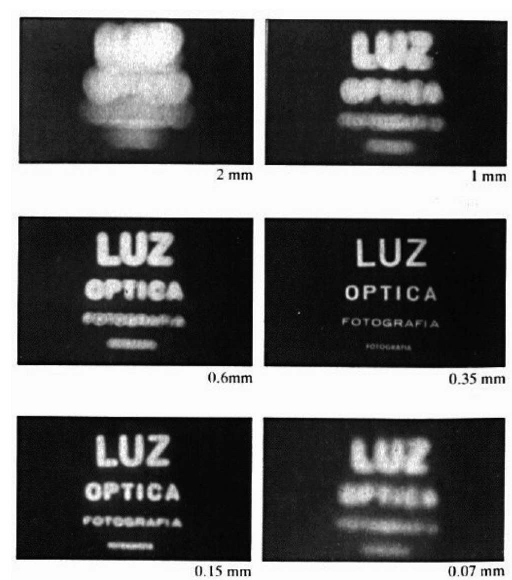
Shrinking the aperture



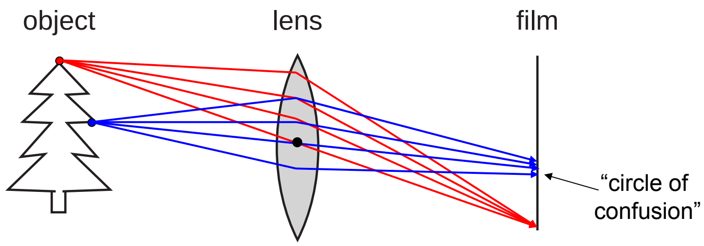
Why not make the aperture as small as possible?

- Less light gets through
- *Diffraction* effects...

Shrinking the aperture



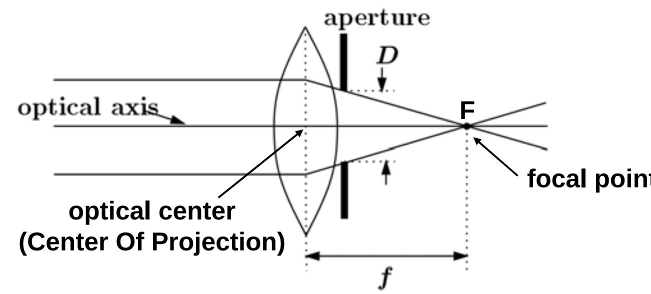
Adding a lens



A lens focuses light onto the film

- There is a specific distance at which objects are "in focus"
  - other points project to a "circle of confusion" in the image
- Changing the shape of the lens changes this distance

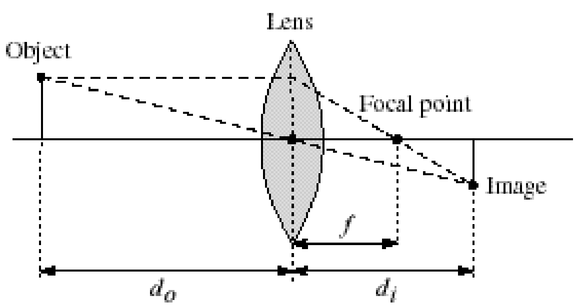
Lenses



A lens focuses parallel rays onto a single focal point

- focal point at a distance *f* beyond the plane of the lens
  - *f* is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
  - aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)

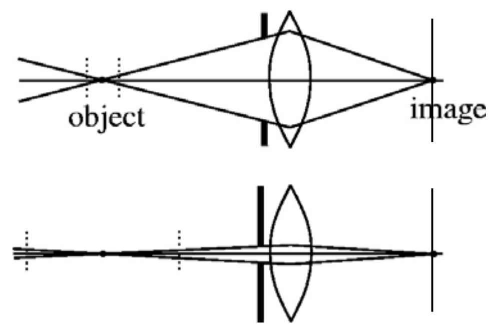
Thin lenses

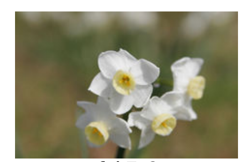


Thin lens equation: $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

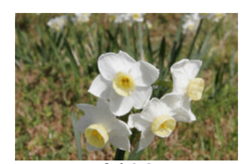
- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
- Thin lens applet: http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html (by Fu-Kwun Hwang)

Depth of field





f / 5.6



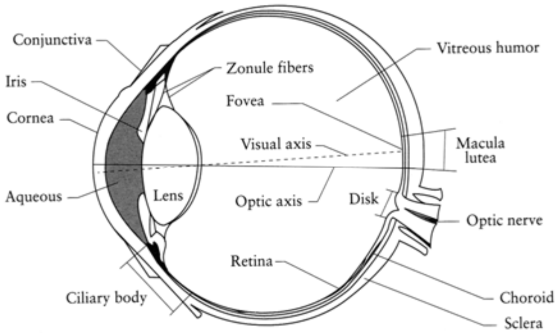
f / 32

Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

The eye



The human eye is a camera

- **Iris -** colored annulus with radial muscles
- **Pupil -** the hole (aperture) whose size is controlled by the iris
- What's the "film"?
  – photoreceptor cells (rods and cones) in the **retina**

Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is a **C**harge **C**oupled **D**evice
  – light-sensitive diode that converts photons to electrons
  – other variants exist: CMOS is becoming more popular
  – http://electronics.howstuffworks.com/digital-camera.htm

Issues with digital cameras

Noise
– big difference between consumer vs. SLR-style cameras
– low light is where you most notice noise

Compression
– creates artifacts except in uncompressed formats (tiff, raw)

Color
– color fringing artifacts from Bayer patterns

Blooming
– charge overflowing into neighboring pixels

In-camera processing
– oversharpening can produce halos

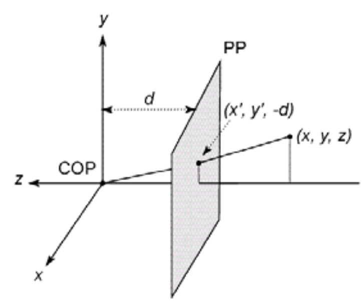
Interlaced vs. progressive scan video
– even/odd rows from different exposures

Are more megapixels better?
– requires higher quality lens
– noise issues

Stabilization
– compensate for camera shake (mechanical vs. electronic)

More info online, e.g.,
• http://electronics.howstuffworks.com/digital-camera.htm
• http://www.dpreview.com/

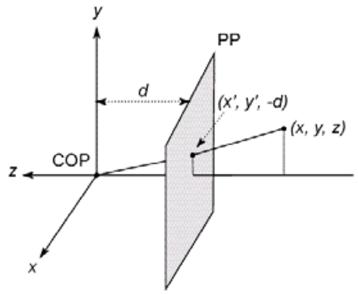
Modeling projection



The coordinate system

- We will use the pin-hole model as an approximation
- Put the optical center (**C**enter **O**f **P**rojection) at the origin
- Put the image plane (**P**rojection **P**lane) *in front* of the COP
  – Why?
- The camera looks down the *negative* z axis
  – we need this if we want right-handed-coordinates

Modeling projection



Projection equations

- Compute intersection with PP of ray from (x,y,z) to COP
- Derived using similar triangles (on board)

$$(x, y, z) \to (-d\frac{x}{z},\ -d\frac{y}{z},\ -d)$$

- We get the projection by throwing out the last coordinate:

$$(x, y, z) \to (-d\frac{x}{z},\ -d\frac{y}{z})$$

Homogeneous coordinates

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous image coordinates

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- Can also formulate as a 4x4 (today's reading does this)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

divide by fourth coordinate

Perspective Projection

How does scaling the projection matrix change the transformation?

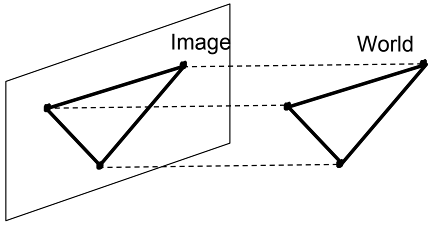
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

$$\begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \end{bmatrix} \Rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

Orthographic projection

Special case of perspective projection

- Distance from the COP to the PP is infinite

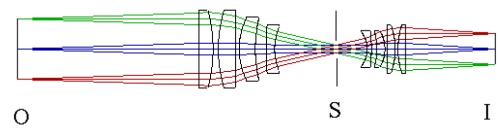


- Good approximation for telephoto optics
- Also called "parallel projection": (x, y, z) → (x, y)
- What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic ("telecentric") lenses

Navitar telecentric zoom lens



http://www.lhup.edu/~dsimanek/3d/telecent.htm

Variants of orthographic projection

Scaled orthographic

- Also called "weak perspective"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

Affine projection

- Also called "paraperspective"

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

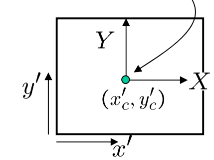
Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

$$\mathbf{X} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$



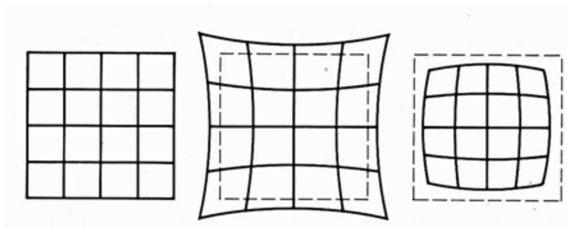
- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{translation}}$$

- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

Distortion



No distortion   Pin cushion   Barrel

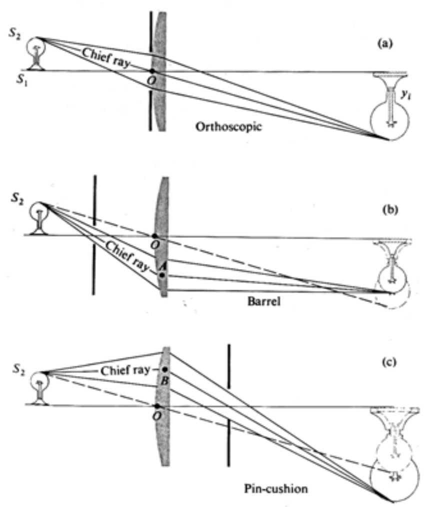
Radial distortion of the image

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

Correcting radial distortion

from [Helmut Dersch]

Distortion



Modeling distortion

Project $(\hat{x}, \hat{y}, \hat{z})$ to "normalized" image coordinates

$$x'_n = \hat{x}/\hat{z}$$
$$y'_n = \hat{y}/\hat{z}$$

Apply radial distortion

$$r^2 = {x'_n}^2 + {y'_n}^2$$
$$x'_d = x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$
$$y'_d = y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$

Apply focal length translate image center

$$x' = f x'_d + x_c$$
$$y' = f y'_d + y_c$$

To model lens distortion

- Use above projection operation instead of standard projection matrix multiplication

Other types of projection

Lots of intriguing variants…

(I'll just mention a few fun ones)

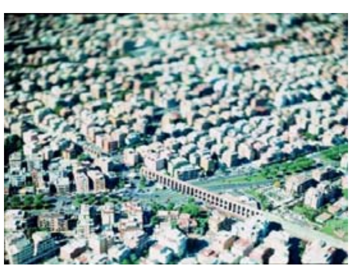
360 degree field of view…

Basic approach

- Take a photo of a parabolic mirror with an orthographic lens (Nayar)
- Or buy one a lens from a variety of omnicam manufacturers…
  - See http://www.cis.upenn.edu/~kostas/omni.html

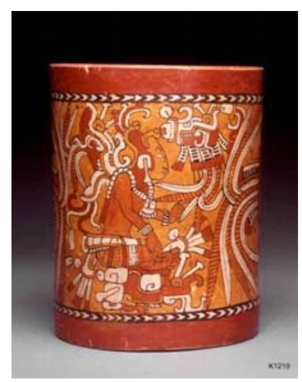
Tilt-shift

http://www.northlight-images.co.uk/article_pages/tilt_and_shift_ts-e.html



Titlt-shift images from Olivo Barbieri
and Photoshop imitations

Rotating sensor (or object)



Rollout Photographs © Justin Kerr
http://research.famsi.org/kerrmaya.html

Also known as "cyclographs", "peripheral images"

Photofinish