Image filtering

Images by Pawan Sinha
What is an image?

We can think of an image as a function, $f$, from $\mathbb{R}^2$ to $\mathbb{R}$:

- $f(x, y)$ gives the intensity at position $(x, y)$
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a “vector-valued” function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$
Images as functions
What is a digital image?

In computer vision we usually operate on digital (discrete) images:

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)

If our samples are $\Delta$ apart, we can write this as:

$$f[i, j] = \text{Quantize}\{ f(i \Delta, j \Delta) \}$$

The image can now be represented as a matrix of integer values

```
   j
  i

  62  79  23  119  120  105  4   0
  10  10   9  62   12  78  34  0
  10  58  197  46  46   0   0  48
 176 135   5 188  191  68  0  49
   2   1   1  29  26  37   0  77
   0  89 144 147 187 102  62 208
255 252   0 166  123  62   0  31
166  63 127  17   1   0  99  30
```
Filtering Operations Use Masks

- Masks operate on a neighborhood of pixels.

- A mask of coefficients is centered on a pixel.

- The mask coefficients are multiplied by the pixel values in its neighborhood and the products are summed.

- The result goes into the corresponding pixel position in the output image.
Noise

Image processing is useful for noise reduction...

Common types of noise:

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise**: contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution
Practical noise reduction

How can we “smooth” away noise in a single image?

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Mean filtering

\[
F[x, y]
\]

\[
G[x, y]
\]
Mean filtering

\[ F[x, y] \]

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 0 \\
0 & 0 & 0 & 90 & 0 & 90 & 90 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 0 \\
0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[ G[x, y] \]

\[
\begin{array}{cccccccc}
0 & 10 & 20 & 30 & 30 & 30 & 20 & 10 \\
0 & 20 & 40 & 60 & 60 & 60 & 40 & 20 \\
0 & 30 & 60 & 90 & 90 & 90 & 60 & 30 \\
0 & 30 & 50 & 80 & 80 & 90 & 60 & 30 \\
0 & 30 & 50 & 80 & 80 & 90 & 60 & 30 \\
0 & 20 & 30 & 50 & 50 & 60 & 40 & 20 \\
10 & 30 & 30 & 30 & 30 & 30 & 20 & 10 \\
10 & 10 & 10 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
Effect of mean filters

Gaussian noise

Salt and pepper noise

3x3

5x5

7x7
Cross-correlation filtering

Let’s write this down as an equation. Assume the averaging window is \((2k+1)\times(2k+1)\):

\[
G[i, j] = \frac{1}{(2k + 1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i + u, j + v]
\]

We can generalize this idea by allowing different weights for different neighboring pixels:

\[
G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v]F[i + u, j + v]
\]

This is called a **cross-correlation** operation and written:

\[
G = H \otimes F
\]

H is called the “filter,” “kernel,” or “mask.”

The above allows negative filter indices. When you implement need to use: \(H[u+k,v+k]\) instead of \(H[u,v]\)
Mean kernel

What’s the kernel for a 3x3 mean filter?

\[
F[x, y] = H[u, v]
\]
Gaussian Filtering

A Gaussian kernel gives less weight to pixels further from the center of the window.

$$F[x, y]$$

This kernel is an approximation of a Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2}e^{-\frac{u^2 + v^2}{\sigma^2}}$$
Mean vs. Gaussian filtering
Convolution

A convolution operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

\[ G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i - u, j - v] \]

It is written: \( G = H \ast F \)

Suppose \( H \) is a Gaussian or mean kernel. How does convolution differ from cross-correlation?
Median filters

A **Median Filter** operates over a window by selecting the median intensity in the window.

What advantage does a median filter have over a mean filter?

Is a median filter a kind of convolution?
Comparison: salt and pepper noise

Mean | Gaussian | Median
-----|----------|---------
3x3  | ![Image]  | ![Image] |
5x5  | ![Image]  | ![Image] |
7x7  | ![Image]  | ![Image] |
Comparison: Gaussian noise

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<th>Gaussian</th>
<th>Median</th>
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