# Image Segmentation

From Sandlot Science

Today's Readings

- Forsyth & Ponce, Chapter 14
- (plus lots of optional references in the slides)

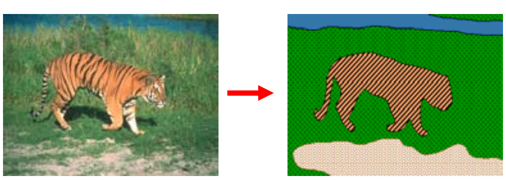
# Announcements

Status reports next Thursday

- ~5min presentations in class

Project3 voting

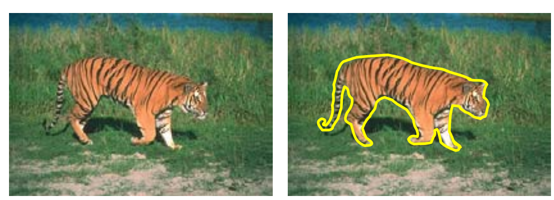
# From images to objects



What Defines an Object?

- Subjective problem, but has been well-studied
- Gestalt Laws seek to formalize this
  - proximity, similarity, continuation, closure, common fate
  - see notes by Steve Joordens, U. Toronto

# Extracting objects



How could this be done?

## Image Segmentation

Many approaches proposed

- cues: color, regions, contours
- automatic vs. user-guided
- no clear winner
- we'll consider several approaches today

## Intelligent Scissors (demo)

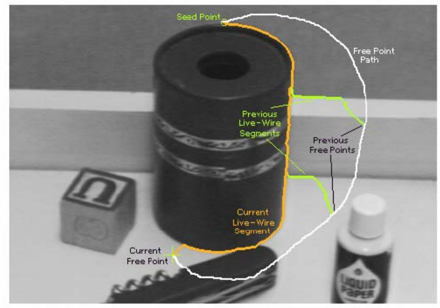



**Figure 2:** *Image demonstrating how the live-wire segment adapts and snaps to an object boundary as the free point moves (via cursor movement). The path of the free point is shown in white. Live-wire segments from previous free point positions ($t_0$, $t_1$, and $t_2$) are shown in green.*

## Intelligent Scissors [Mortensen 95]

Approach answers a basic question

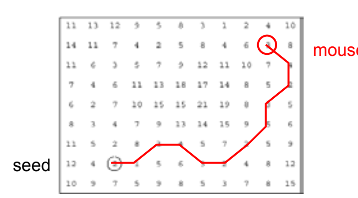
- Q: how to find a path from seed to mouse that follows object boundary as closely as possible?

**Figure 2:** *Image demonstrating how the live-wire segment adapts and snaps to an object boundary as the free point moves (via cursor movement). The path of the free point is shown in white. Live-wire segments from previous free point positions ($t_0$, $t_1$, and $t_2$) are shown in green.*

## Intelligent Scissors

Basic Idea

- Define edge score for each pixel
  - edge pixels have low cost
- Find lowest cost path from seed to mouse




Questions

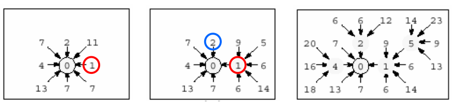
- How to define costs?
- How to find the path?

## Path Search (basic idea)

Graph Search Algorithm

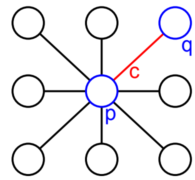
- Computes minimum cost path from seed to *all other pixels*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 13 | 12 | 9 | 5 | 8 | 3 | 1 | 2 | 4 | 10 |
| 14 | 11 | 7 | 4 | 2 | 5 | 8 | 4 | 6 | 3 | 8 |
| 11 | 6 | 3 | 5 | 7 | 9 | 12 | 11 | 10 | 7 | 4 |
| 7 | 4 | 6 | 11 | 13 | 18 | 17 | 14 | 8 | 5 | 2 |
| 6 | 2 | 7 | 10 | 15 | 15 | 21 | 19 | 8 | 3 | 5 |
| 8 | 3 | 4 | 7 | 9 | 13 | 14 | 15 | 9 | 5 | 6 |
| 11 | 5 | 2 | 8 | 3 | 4 | 5 | 7 | 2 | 5 | 9 |
| 12 | 4 | 2 | 1 | 5 | 6 | 3 | 2 | 4 | 8 | 12 |
| 10 | 9 | 7 | 5 | 9 | 8 | 5 | 3 | 7 | 8 | 15 |



## How does this really work?

Treat the image as a graph




Graph

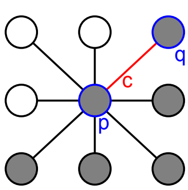
- node for every pixel **p**
- link between every adjacent pair of pixels, **p**,**q**
- cost **c** for each link

Note: each *link* has a cost

- this is a little different than the figure before where each pixel had a cost

## Defining the costs

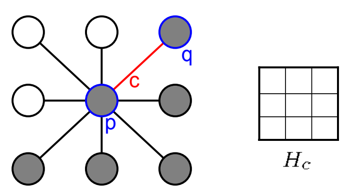
Treat the image as a graph




Want to hug image edges: how to define cost of a link?

- the link should follow the intensity edge
  - want intensity to change rapidly ⊥ to the link
- c ≈ - |difference of intensity ⊥ to link|

## Defining the costs



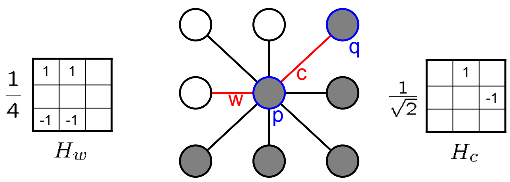

c can be computed using a cross-correlation filter

- assume it is centered at p

Also typically scale c by its length

- set c = (max-|filter response|)
  - where max = maximum |filter response| over all pixels in the image

## Defining the costs

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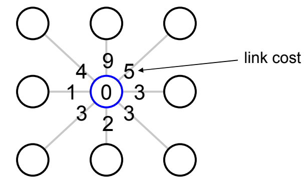
## Dijkstra's shortest path algorithm




Algorithm

1. init node costs to ∞, set p = seed point, cost(p) = 0
2. expand p as follows:

   for each of p's neighbors q that are not expanded

   » set cost(q) = min( cost(p) + $c_{pq}$, cost(q) )

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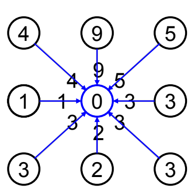

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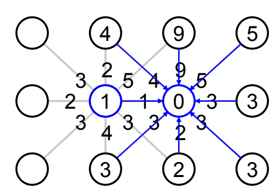

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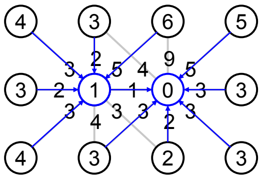


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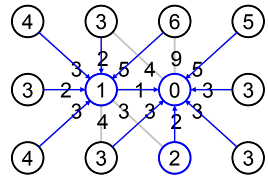


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# Dijkstra's shortest path algorithm

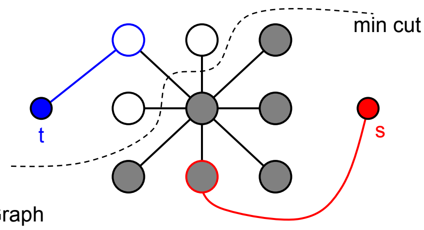
Properties

- It computes the minimum cost path from the seed to every node in the graph. This set of minimum paths is represented as a *tree*
- Running time, with N pixels:
  - O($N^2$) time if you use an active list
  - O(N log N) if you use an active priority queue (heap)
  - takes fraction of a second for a typical (640x480) image
- Once this tree is computed once, we can extract the optimal path from any point to the seed in O(N) time.
  - it runs in real time as the mouse moves
- What happens when the user specifies a new seed?

# Intelligent Scissors Results

http://www.cs.washington.edu/education/courses/455/04wi/projects/project1/artifacts/index.html

## Segmentation by min (s-t) cut [Boykov 2001]



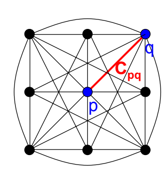

Graph
- node for each pixel, link between pixels
- specify a few pixels as foreground and background
  - create an infinite cost link from each bg pixel to the "t" node
  - create an infinite cost link from each fg pixel to the "s" node
- compute min cut that separates s from t
- how to define link cost between neighboring pixels?

## Grabcut [Rother et al., SIGGRAPH 2004]

## Is user-input required?

Our visual system is proof that automatic methods are possible
- classical image segmentation methods are automatic

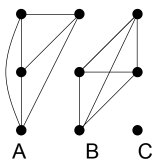
## Automatic graph cut [Shi & Malik]





*Fully-connected* graph
- node for every pixel
- link between *every* pair of pixels, **p**,**q**
- cost $\mathbf{c_{pq}}$ for each link
  - $\mathbf{c_{pq}}$ measures *similarity*
    - » similarity is *inversely proportional* to difference in color and position

## Segmentation by Graph Cuts

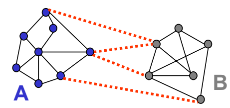




Break Graph into Segments

- Delete links that cross between segments
- Easiest to break links that have low cost (similarity)
  - similar pixels should be in the same segments
  - dissimilar pixels should be in different segments

## Cuts in a graph




Link Cut

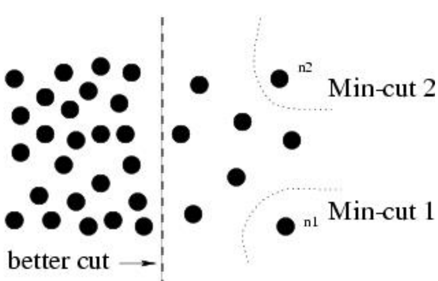
- set of links whose removal makes a graph disconnected
- cost of a cut:

$$cut(A,B) = \sum_{p \in A, q \in B} c_{p,q}$$

Find minimum cut

- gives you a segmentation

## But min cut is not always the best cut...

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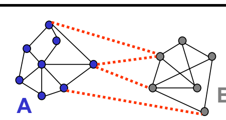


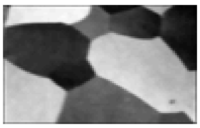

Normalized Cut

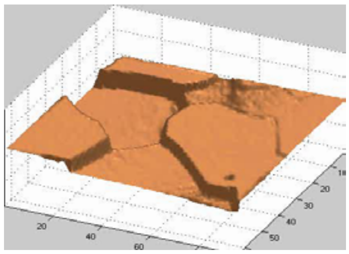
- a cut penalizes large segments
- fix by normalizing for size of segments

$$Ncut(A,B) = \frac{cut(A,B)}{volume(A)} + \frac{cut(A,B)}{volume(B)}$$

- volume(A) = sum of costs of all edges that touch A

## Interpretation as a Dynamical System






Treat the links as springs and shake the system
- elasticity proportional to cost
- vibration “modes” correspond to segments
  - can compute these by solving an eigenvector problem
  - http://www.cis.upenn.edu/~jshi/papers/pami_ncut.pdf

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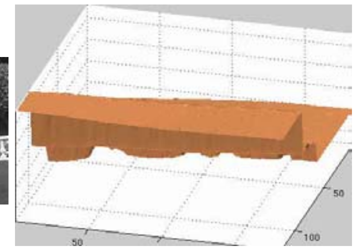


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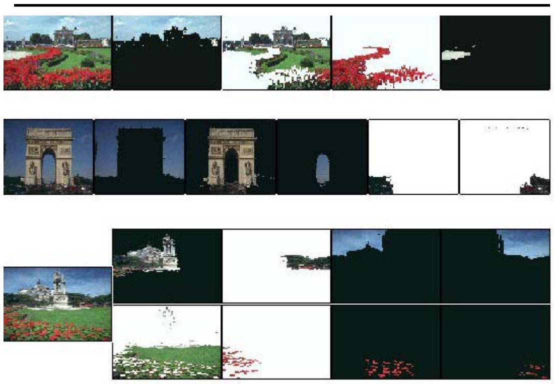
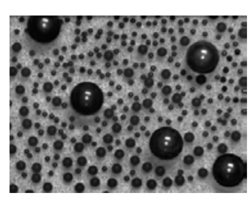
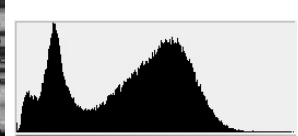
## Color Image Segmentation



## Color histogram-based segmentation

Goal
- Break the image into K regions (segments)
- Solve this by reducing the number of colors to K and mapping each pixel to the closest color
  - photoshop demo





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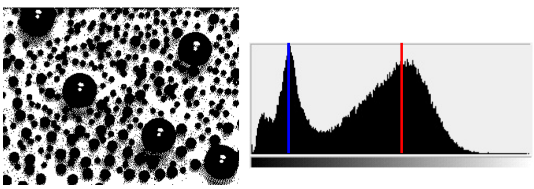
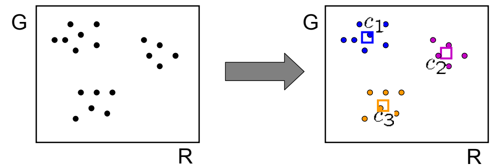


Here’s what it looks like if we use two colors

## Clustering

How to choose the representative colors?

- This is a clustering problem!




Objective

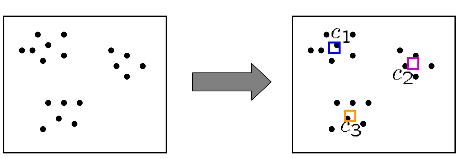
- Each point should be as close as possible to a cluster center
  - Minimize sum squared distance of each point to closest center

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} \|p - c_i\|^2$$

## Break it down into subproblems

Suppose I tell you the cluster centers $c_i$

- Q: how to determine which points to associate with each $c_i$?
- A: for each point p, choose closest $c_i$




Suppose I tell you the points in each cluster

- Q: how to determine the cluster centers?
- A: choose $c_i$ to be the mean of all points in the cluster

## K-means clustering

K-means clustering algorithm

1. Randomly initialize the cluster centers, $c_1$, ..., $c_K$
2. Given cluster centers, determine points in each cluster
   - For each point p, find the closest $c_i$. Put p into cluster i
3. Given points in each cluster, solve for $c_i$
   - Set $c_i$ to be the mean of points in cluster i
4. If $c_i$ have changed, repeat Step 2

Java demo: http://www.elet.polimi.it/upload/matteucc/Clustering/tutorial_html/AppletKM.html

Properties

- Will always converge to *some* solution
- Can be a “local minimum”
  - does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} \|p - c_i\|^2$$

# Probabilistic clustering

Basic questions

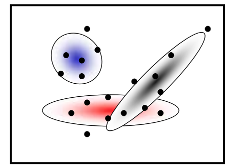
- what's the probability that a point **x** is in cluster m?
- what's the shape of each cluster?

K-means doesn't answer these questions

Basic idea

- instead of treating the data as a bunch of points, assume that they are all generated by sampling a continuous function
- This function is called a **generative model**
  - defined by a vector of parameters **θ**

# Mixture of Gaussians



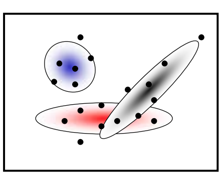
One generative model is a mixture of Gaussians (MOG)

- K Gaussian blobs with means $\boldsymbol{\mu}_\mathbf{b}$ covariance matrices $\mathbf{V}_\mathbf{b}$, dimension d
  - blob *b* defined by: $P(x|\mu_b, V_b) = \frac{1}{\sqrt{(2\pi)^d|V_b|}} e^{-\frac{1}{2}(x-\mu_b)^T V_b^{-1}(x-\mu_b)}$
- blob *b* is selected with probability $\alpha_b$
- the likelihood of observing **x** is a weighted mixture of Gaussians

$$P(x|\theta) = \sum_{b=1}^{K} \alpha_b P(x|\theta_b)$$

- where $\theta = [\mu_1, \ldots, \mu_n, V_1, \ldots, V_n]$

# Expectation maximization (EM)



Goal

- find blob parameters θ that maximize the likelihood function:

$$P(data|\theta) = \prod_x P(x|\theta)$$

Approach:

1. E step: given current guess of blobs, compute ownership of each point
2. M step: given ownership probabilities, update blobs to maximize likelihood function
3. repeat until convergence

# EM details

E-step

- compute probability that point **x** is in blob i, given current guess of **θ**

$$P(b|x, \mu_b, V_b) = \frac{\alpha_b P(x|\mu_b, V_b)}{\sum_{i=1}^{K} \alpha_i P(x|\mu_i, V_i)}$$

M-step

- compute probability that blob b is selected

$$\alpha_b^{new} = \frac{1}{N}\sum_{i=1}^{N} P(b|x_i, \mu_b, V_b) \qquad \text{N data points}$$

- mean of blob b

$$\mu_b^{new} = \frac{\sum_{i=1}^{N} x_i P(b|x_i, \mu_b, V_b)}{\sum_{i=1}^{N} P(b|x_i, \mu_b, V_b)}$$

- covariance of blob b

$$V_b^{new} = \frac{\sum_{i=1}^{N} (x_i - \mu_b^{new})(x_i - \mu_b^{new})^T P(b|x_i, \mu_b, V_b)}{\sum_{i=1}^{N} P(b|x_i, \mu_b, V_b)}$$

## EM demo

http://www.cs.ucsd.edu/users/ibayrakt/java/em/

## Applications of EM

Turns out this is useful for all sorts of problems

- any clustering problem
- any model estimation problem
- missing data problems
- finding outliers
- segmentation problems
  - segmentation based on color
  - segmentation based on motion
  - foreground/background separation
- ...

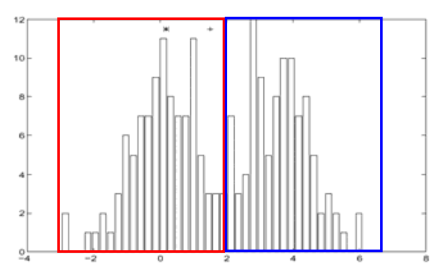
## Problems with EM

Local minima

Need to know number of segments

Need to choose generative model

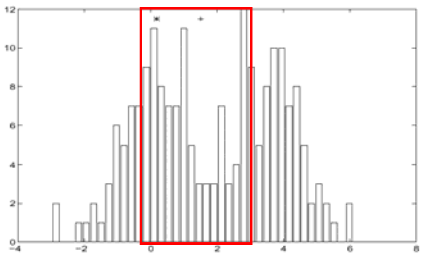
## Finding Modes in a Histogram



How Many Modes Are There?

- Easy to see, hard to compute

## Mean Shift [Comaniciu & Meer]



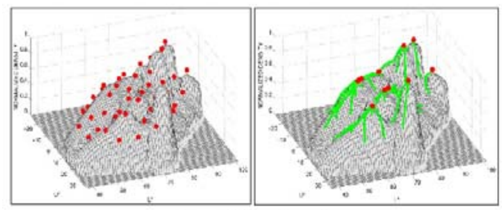

### Iterative Mode Search

1. Initialize random seed, and window W
2. Calculate center of gravity (the "mean") of W: $\sum_{x \in W} xH(x)$
3. Translate the search window to the mean
4. Repeat Step 2 until convergence

## Mean-Shift

### Approach

- Initialize a window around each point
- See where it shifts—this determines which segment it's in
- Multiple points will shift to the same segment



Mean shift trajectories

## Mean-shift for image segmentation

### Useful to take into account spatial information

- instead of (R, G, B), run in (R, G, B, x, y) space
- D. Comaniciu, P. Meer, Mean shift analysis and applications, *7th International Conference on Computer Vision*, Kerkyra, Greece, September 1999, 1197-1203.
  - http://www.caip.rutgers.edu/riul/research/papers/pdf/spatmsft.pdf

More Examples: http://www.caip.rutgers.edu/~comanici/segm_images.html

## References


- Mortensen and Barrett, "Intelligent Scissors for Image Composition," Proc. *SIGGRAPH* 1995.
- Boykov and Jolly, "Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D images," Proc. ICCV, 2001.
- Shi and Malik, "Normalized Cuts and Image Segmentation," Proc. CVPR 1997.
- Comaniciu and Meer, "Mean shift analysis and applications," Proc. *ICCV* 1999.