## Announcements

- Project 2 due today
- Project 3 out today
  - help session today

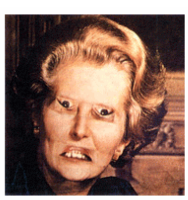
## Recognition


The “Margaret Thatcher Illusion”, by Peter Thompson

### Readings

- C. Bishop, “Neural Networks for Pattern Recognition”, Oxford University Press, 1998, Chapter 1.
- Forsyth and Ponce, 22.3 (eigenfaces)

## Recognition

The “Margaret Thatcher Illusion”, by Peter Thompson

### Readings

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## Recognition problems

What is it?

- Object detection

Who is it?

- Recognizing identity

What are they doing?

- Activities

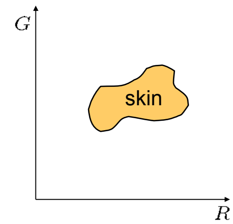
All of these are **classification** problems

- Choose one class from a list of possible candidates

# Face detection

How to tell if a face is present?

# One simple method: skin detection



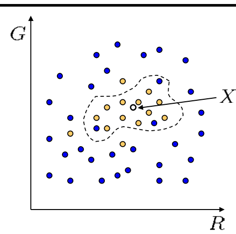

Skin pixels have a distinctive range of colors

- Corresponds to region(s) in RGB color space
  - for visualization, only R and G components are shown above

Skin classifier

- A pixel X = (R,G,B) is skin if it is in the skin region
- But how to find this region?

# Skin detection



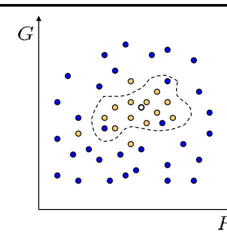

**Learn** the skin region from examples

- Manually label pixels in one or more “training images” as skin or not skin
- Plot the training data in RGB space
  - skin pixels shown in orange, non-skin pixels shown in blue
  - some skin pixels may be outside the region, non-skin pixels inside. Why?

Skin classifier

- Given X = (R,G,B): how to determine if it is skin or not?

# Skin classification techniques



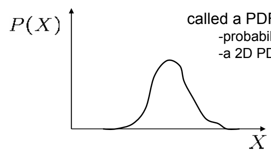

Skin classifier

- Given X = (R,G,B): how to determine if it is skin or not?
- Nearest neighbor
  - find labeled pixel closest to X
  - choose the label for that pixel
- Data modeling
  - fit a model (curve, surface, or volume) to each class
- Probabilistic data modeling
  - fit a probability model to each class

# Probability

## Basic probability

- X is a random variable
- P(X) is the probability that X achieves a certain value



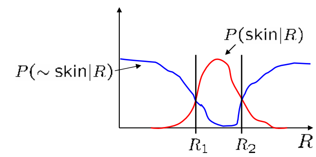

called a PDF
-probability distribution/density function
-a 2D PDF is a surface, 3D PDF is a volume

- $0 \le P(X) \le 1$
- $\int_{-\infty}^{\infty} P(X)dX = 1$ or $\sum P(X) = 1$

  continuous X    discrete X

- Conditional probability: P(X | Y)
  - probability of X given that we already know Y

# Probabilistic skin classification




Now we can model uncertainty

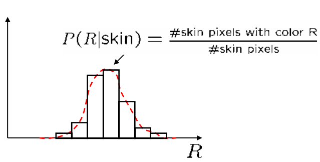
- Each pixel has a probability of being skin or not skin
  - $P(\sim \text{skin}|R) = 1 - P(\text{skin}|R)$

Skin classifier

- Given X = (R,G,B): how to determine if it is skin or not?
- Choose interpretation of highest probability
  - set X to be a skin pixel if and only if $R_1 < X \le R_2$

Where do we get $P(\text{skin}|R)$ and $P(\sim \text{skin}|R)$ ?

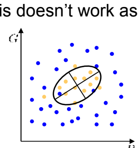
# Learning conditional PDF's




We can calculate P(R | skin) from a set of training images

- It is simply a histogram over the pixels in the training images
  - each bin $R_i$ contains the proportion of skin pixels with color $R_i$

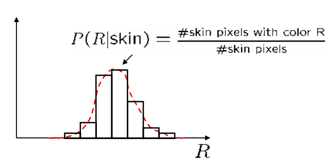
This doesn't work as well in higher-dimensional spaces. Why not?




Approach: fit parametric PDF functions

- common choice is rotated Gaussian
  - center $\mathbf{c} = \overline{X}$
  - covariance $\sum_X (X - \overline{X})(X - \overline{X})^T$
    - » orientation, size defined by eigenvecs, eigenvals

# Learning conditional PDF's




We can calculate P(R | skin) from a set of training images

- It is simply a histogram over the pixels in the training images
  - each bin $R_i$ contains the proportion of skin pixels with color $R_i$

But this isn't quite what we want

- Why not? How to determine if a pixel is skin?
- We want P(skin | R) not P(R | skin)
- How can we get it?

## Bayes rule

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

In terms of our problem:

what we measure (**likelihood**)

domain knowledge (**prior**)

$$P(\text{skin}|R) = \frac{P(R|\text{skin})\ P(\text{skin})}{P(R)}$$

what we want (**posterior**)

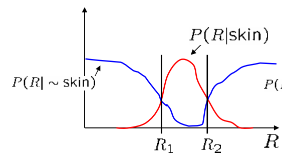
**normalization** term

$P(R) = P(R|\text{skin})P(\text{skin}) + P(R|\sim\text{skin})P(\sim\text{skin})$

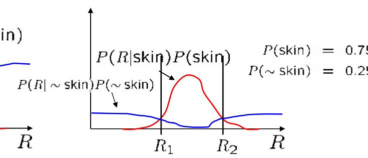
The prior: P(skin)

- Could use domain knowledge
  - P(skin) may be larger if we know the image contains a person
  - for a portrait, P(skin) may be higher for pixels in the center
- Could learn the prior from the training set. How?
  - P(skin) may be proportion of skin pixels in training set

## Bayesian estimation




likelihood




posterior (unnormalized)

Bayesian estimation = minimize probability of misclassification

- Goal is to choose the label (skin or ~skin) that maximizes the posterior
  - this is called **Maximum A Posteriori (MAP) estimation**
- Suppose the prior is uniform: P(skin) = P(~skin) = 0.5
  - in this case $P(\text{skin}|R) = cP(R|\text{skin}), \quad P(\sim\text{skin}|R) = cP(R|\sim\text{skin})$
  - maximizing the posterior is equivalent to maximizing the likelihood
    - » $P(\text{skin}|R) > P(\sim\text{skin}|R)$ if and only if $P(R|\text{skin}) > P(R|\sim\text{skin})$
  - this is called **Maximum Likelihood (ML) estimation**

## Skin detection results

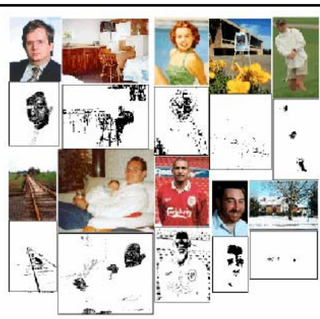
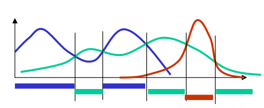


**Figure 25.3.** The figure shows a variety of images together with the output of the skin detector of Jones and Rehg applied to the image. Pixels marked black are skin pixels, and white are background. Notice that this process is relatively effective, and could certainly be used to focus attention on, say, faces and hands. *Figure from "Statistical color models with application to skin detection," M.J. Jones and J. Rehg, Proc. Computer Vision and Pattern Recognition, 1999 © 1999, IEEE*

## General classification

This same procedure applies in more general circumstances

- More than two classes
- More than one dimension



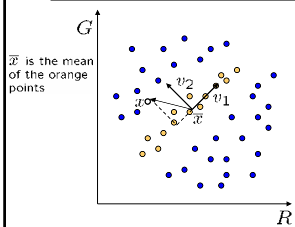
Example: face detection

- Here, X is an image region
  - dimension = # pixels
  - each face can be thought of as a point in a high dimensional space

H. Schneiderman and T.Kanade

H. Schneiderman, T. Kanade. "A Statistical Method for 3D Object Detection Applied to Faces and Cars". IEEE Conference on Computer Vision and Pattern Recognition (CVPR 2000) http://www-2.cs.cmu.edu/afs/cs.cmu.edu/user/hws/www/CVPR00.pdf

## Linear subspaces




convert **x** into $\mathbf{v}_1$, $\mathbf{v}_2$ coordinates

$$\mathbf{x} \rightarrow ((\mathbf{x}-\overline{x})\cdot \mathbf{v}_1, (\mathbf{x}-\overline{x})\cdot \mathbf{v}_2)$$

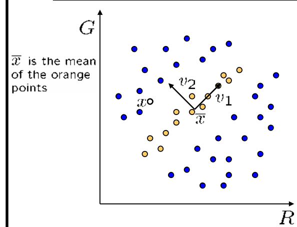
What does the $\mathbf{v}_2$ coordinate measure?
- distance to line
- use it for classification—near 0 for orange pts

What does the $\mathbf{v}_1$ coordinate measure?
- position along line
- use it to specify which orange point it is

Classification can be expensive
- Must either search (e.g., nearest neighbors) or store large PDF's

Suppose the data points are arranged as above
- Idea—fit a line, classifier measures distance to line

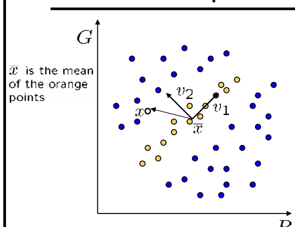
## Dimensionality reduction




How to find $\mathbf{v}_1$ and $\mathbf{v}_2$ ?
- work out on board

Dimensionality reduction
- We can represent the orange points with *only* their $\mathbf{v}_1$ coordinates
  – since $\mathbf{v}_2$ coordinates are all essentially 0
- This makes it much cheaper to store and compare points
- A bigger deal for higher dimensional problems

## Linear subspaces




Consider the variation along direction **v** among all of the orange points:

$$var(\mathbf{v}) = \sum_{\text{orange point } \mathbf{x}} \|(\mathbf{x}-\overline{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\|^2$$

What unit vector **v** minimizes *var*?

$$\mathbf{v}_2 = min_{\mathbf{v}}\ \{var(\mathbf{v})\}$$

What unit vector **v** maximizes *var*?

$$\mathbf{v}_1 = max_{\mathbf{v}}\ \{var(\mathbf{v})\}$$

$$\begin{aligned} var(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x}-\mathbf{x})^{\mathrm{T}} \cdot \mathbf{v}\| \\ &= \sum_{\mathbf{x}} \mathbf{v}^{\mathrm{T}}(\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^{\mathbf{T}}\mathbf{v} \\ &= \mathbf{v}^{\mathbf{T}}\left[\sum_{\mathbf{x}}(\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^{\mathrm{T}}\right]\mathbf{v} \\ &= \mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}}(\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^{\mathbf{T}} \end{aligned}$$

Solution: $\mathbf{v}_1$ is eigenvector of **A** with *largest* eigenvalue
$\mathbf{v}_2$ is eigenvector of **A** with *smallest* eigenvalue

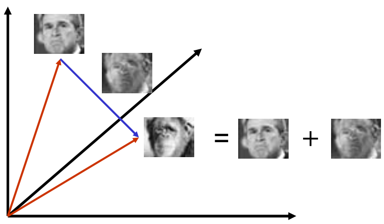
## Principal component analysis

Suppose each data point is N-dimensional
- Same procedure applies:

$$\begin{aligned} var(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x}-\overline{\mathbf{x}})^{\mathrm{T}} \cdot \mathbf{v}\| \\ &= \mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}}(\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^{\mathbf{T}} \end{aligned}$$

- The eigenvectors of **A** define a new coordinate system
  – eigenvector with largest eigenvalue captures the most variation among training vectors **x**
  – eigenvector with smallest eigenvalue has least variation
- We can compress the data by only using the top few eigenvectors
  – corresponds to choosing a "linear subspace"
    » represent points on a line, plane, or "hyper-plane"
  – these eigenvectors are known as the ***principal components***

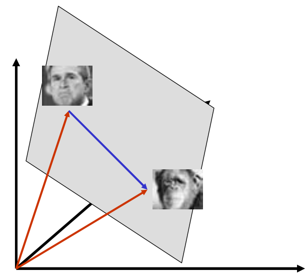
## The space of faces



An image is a point in a high dimensional space

- An N x M image is a point in $R^{NM}$
- We can define vectors in this space as we did in the 2D case

## Dimensionality reduction



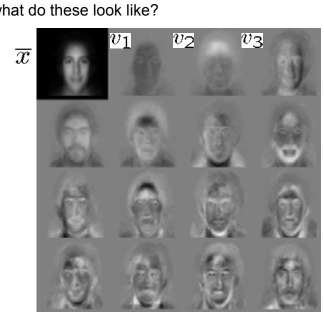
The set of faces is a "subspace" of the set of images

- Suppose it is K dimensional
- We can find the best subspace using PCA
- This is like fitting a "hyper-plane" to the set of faces
  - spanned by vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_K}$
  - any face $\mathbf{x} \approx \overline{\mathbf{x}} + a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \ldots + a_k\mathbf{v}_k$

## Eigenfaces

PCA extracts the eigenvectors of **A**

- Gives a set of vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, ...$
- Each one of these vectors is a direction in face space
  - what do these look like?



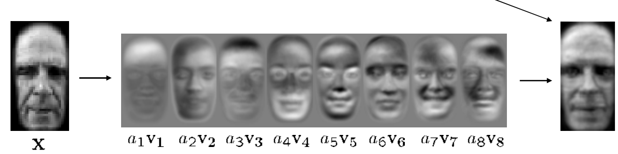

## Projecting onto the eigenfaces

The eigenfaces $\mathbf{v_1}, ..., \mathbf{v_K}$ span the space of faces

- A face is converted to eigenface coordinates by

$$\mathbf{x} \rightarrow (\underbrace{(\mathbf{x}-\overline{\mathbf{x}})\cdot\mathbf{v}_1}_{a_1}, \underbrace{(\mathbf{x}-\overline{\mathbf{x}})\cdot\mathbf{v}_2}_{a_2}, \ldots, \underbrace{(\mathbf{x}-\overline{\mathbf{x}})\cdot\mathbf{v}_K}_{a_K})$$

$$\mathbf{x} \approx \overline{\mathbf{x}} + a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \ldots + a_K\mathbf{v}_K$$



## Recognition with eigenfaces

Algorithm

1. Process the image database (set of images with labels)
   - Run PCA—compute eigenfaces
   - Calculate the K coefficients for each image
2. Given a new image (to be recognized) **x**, calculate K coefficients

$$\mathbf{x} \rightarrow (a_1, a_2, \ldots, a_K)$$

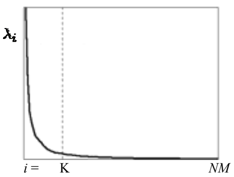
3. Detect if x is a face

$$\|\mathbf{x} - (\overline{\mathbf{x}} + a_1\mathbf{v_1} + a_2\mathbf{v_2} + \ldots + a_K\mathbf{v_K})\| < \text{threshold}$$

4. If it is a face, who is it?
   - Find closest labeled face in database
     - nearest-neighbor in K-dimensional space

## Choosing the dimension K

eigenvalues $\lambda_i$




How many eigenfaces to use?

Look at the decay of the eigenvalues

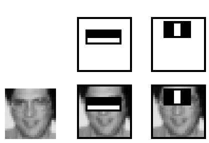
- the eigenvalue tells you the amount of variance “in the direction” of that eigenface
- ignore eigenfaces with low variance

## Issues: metrics

What’s the best way to compare images?

- need to define appropriate features
- depends on goal of recognition task

**exact matching**
complex features work well
(SIFT, MOPS, etc.)



**classification/detection**
simple features work well
(Viola/Jones, etc.)

## Metrics

Lots more feature types that we haven’t mentioned

- moments, statistics
  – metrics: Earth mover’s distance, ...
- edges, curves
  – metrics: Hausdorff, shape context, ...
- 3D: surfaces, spin images
  – metrics: chamfer (ICP)
- ...

## Issues: feature selection

If all you have is one image:
non-maximum suppression, etc.

If you have a training set of images:
AdaBoost, etc.

## Issues: data modeling

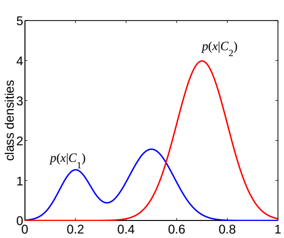
Generative methods

- model the “shape” of each class
  - histograms, PCA, mixtures of Gaussians
  - graphical models (HMM’s, belief networks, etc.)
  - ...

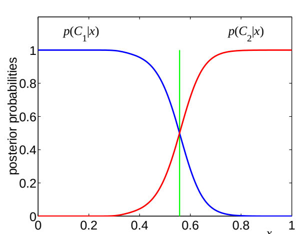
Discriminative methods

- model boundaries between classes
  - perceptrons, neural networks
  - support vector machines (SVM’s)

## Generative vs. Discriminative




**Generative Approach**
model individual classes, priors



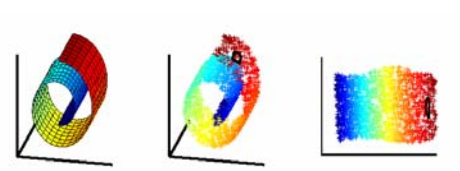

**Discriminative Approach**
model posterior directly

from Chris Bishop

## Issues: dimensionality

What if your space isn’t *flat*?

- PCA may not help



**Nonlinear methods**
LLE, MDS, etc.

# Other Issues

Some other factors
- Prior information, context
- Classification vs. inference
- Representation
- Other recognition problems
  - individuals
  - classes
  - activities
  - low-level properties
    - materials, super-resolution, edges, circles, etc...

# Issues: speed

Case study: Viola Jones face detector

Exploits three key strategies:
- simple, super-efficient features
- image pyramids
- pruning (cascaded classifiers)

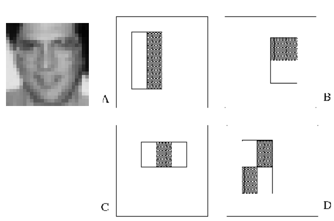
# Viola/Jones: features

"Rectangle filters"

Similar to Haar wavelets

Papageorgiou, et al.

Differences between sums of pixels in adjacent rectangles



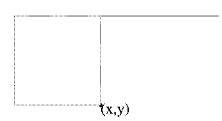

$$h_t(x) = \begin{cases} +1 & \text{if } f_t(x) > \theta_t \\ -1 & \text{otherwise} \end{cases}$$

$60{,}000 \times 100 = 6{,}000{,}000$
Unique Features

# Integral Image (aka. summed area table)

Define the Integral Image

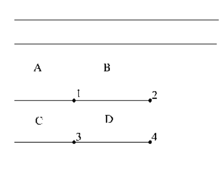
$$I'(x,y) = \sum_{\substack{x' \le x \\ y' \le y}} I(x',y')$$




Any rectangular sum can be computed in constant time:

$$D = 1 + 4 - (2 + 3)$$
$$= A + (A + B + C + D) - (A + C + A + B)$$
$$= D$$

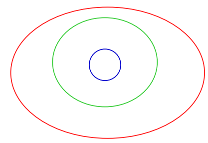
Rectangle features can be computed as differences between rectangles

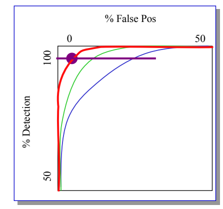


## Viola/Jones: handling scale


## Viola/Jones: cascaded classifiers

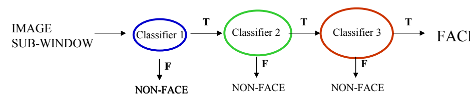
Given a nested set of classifier hypothesis classes



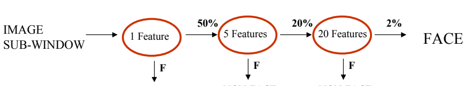



**ROC curves**

Computational Risk Minimization




## Cascaded Classifier

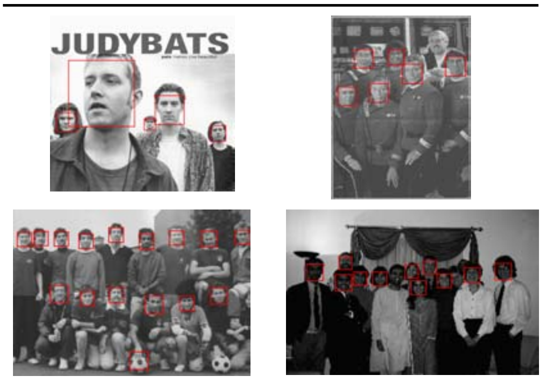



first classifier: 100% detection, 50% false positives.
second classifier: 100% detection, 40% false positives
(20% cumulative)
- using data from previous stage.

third classifier: 100% detection,10% false positive rate
(2% cumulative)

Put cheaper classifiers up front

## Viola/Jones results:




Run-time: 15fps (384x288 pixel image on a 700 Mhz Pentium III)