Announcements

- Guest lecture today: Aseem Agarwala
- Final project out today
  - you and your partner must submit a proposal by this Friday

Today's Reading


Markov Chains

Markov Chain

- a sequence of random variables \( X_1, X_2, \ldots, X_n \)
- \( X_t \) is the state of the model at time \( t \)

\[
\begin{align*}
X_1 & \rightarrow X_2 & \rightarrow X_3 & \rightarrow X_4 & \rightarrow X_5 \\
\end{align*}
\]

Markov assumption: each state is dependent only on the previous one
  - dependency given by a conditional probability:
    \[
p(X_t | X_{t-1})
    \]
- The above is actually a first-order Markov chain
- An \( N \)th-order Markov chain:
  \[
p(X_t | X_{t-1}, \ldots, X_{t-N})
  \]

Modeling Texture

What is texture?
How can we model it?

Markov Chain Example: Text

"A dog is a man's best friend. It's a dog eat dog world out there."

- The above is actually a first-order Markov chain
- An \( N \)th-order Markov chain:
  \[
p(X_t | X_{t-1}, \ldots, X_{t-N})
  \]
Text synthesis
Create plausible looking poetry, love letters, term papers, etc.

Most basic algorithm
1. Build probability histogram
   - find all blocks of N consecutive words/letters in training documents
   - compute probability of occurrence $p(x_i|x_{i-1}, \ldots, x_{i-(n-1)})$.
2. Given words $x_1, x_2, \ldots, x_k-1$
   - compute $x_k$ by sampling from $p(x_i|x_{i-1}, \ldots, x_{i-(n-1)})$.

Example on board...

[Scientific American, June 1989, Dewdney]
"I Spent an Interesting Evening Recently with a Grain of Salt"
- Mark V. Shaney
(computer-generated contributor to USENET News group called net.singles)

Output of 2nd order word-level Markov Chain after training on 90,000 word philosophical essay:
Perhaps only the allegory of simulation is unendurable—more cruel than Artaud’s Theatre of Cruelty, which was the first to practice deterrence, abstraction, disconnection, deterritorialization, etc.; and if it were our own past. We are witnessing the end of the negative form. But nothing separates one pole from the very swing of voting “rights” to electoral..."

Modeling Texture
What is texture?
- An image obeying some statistical properties
- Similar structures repeated over and over again
- Often has some degree of randomness

Markov Random Field
A Markov random field (MRF)
- generalization of Markov chains to two or more dimensions.

First-order MRF:
- probability that pixel $X$ takes a certain value given the values of neighbors $A$, $B$, $C$, and $D$.

$$P(X|A, B, C, D)$$

- Higher order MRF’s have larger neighborhoods
Texture Synthesis [Efros & Leung, ICCV 99]

Can apply 2D version of text synthesis

Synthesizing One Pixel

- What is \( P(x|\text{neighborhood of pixels around } x) \)?
- Find all the windows in the image that match the neighborhood
  - consider only pixels in the neighborhood that are already filled in
- To synthesize \( x \)
  - pick one matching window at random
  - assign \( x \) to be the center pixel of that window

Really Synthesizing One Pixel

- An exact neighbourhood match might not be present
- So we find the best matches using SSD error and randomly choose between them, preferring better matches with higher probability

Growing Texture

- Starting from the initial image, "grow" the texture one pixel at a time
Window Size Controls Regularity

More Synthesis Results

Increasing window size

More Results

reptile skin  aluminum wire

Failure Cases

Growing garbage  Verbatim copying
Image-Based Text Synthesis

Speed

• How fast is this?
• To synthesis a patch of n pixels, given a source image of k pixels, how many pixel window lookups does this algorithm require?

\[ O(nk) \]

Speedup?

Efros & Leung '99 extended

Observation: neighbor pixels are highly correlated

**Idea:** unit of synthesis = block

• Exactly the same but now we want \( P(B | N(B)) \)
• Much faster: synthesize all pixels in a block at once

Random placement of blocks

Neighboring blocks constrained by overlap

Minimal error boundary cut
Minimal error boundary

overlapping blocks  \rightarrow\n\begin{array}{c}
\text{vertical boundary} \\
\end{array}

\begin{array}{c}
\text{overlap error} \\
\end{array} = \begin{array}{c}
\text{min. error boundary} \\
\end{array}

Their Philosophy

The “Corrupt Professor’s Algorithm”:
- Plagiarize as much of the source image as you can
- Then try to cover up the evidence

Rationale:
- Texture blocks are by definition correct samples of texture so
  problem only connecting them together

Texture Transfer

Take the texture from one object and “paint” it onto another object
- This requires separating texture and shape
- That’s HARD, but we can cheat
- Assume we can capture shape by boundary and rough shading

Then, just add another constraint when sampling: similarity to luminance of underlying image at that spot
Issues

• Imposes artificial grid of overlapping blocks on synthesized image, and greedily chooses blocks in left-right, top-bottom order.
• Dynamic programming limits applicability to related problems.
• Solution: use graph cuts instead.
• Let’s explore two examples, first.

Combining two images
Graph cut setup

Spatio-temporal texture synthesis

Source:

Destination:

$\|A(x,y) - B(x,y)\|^2 + \|A(x+1,y) - B(x+1,y)\|^2$

Graphcut textures (Kwatra '03)

Progressive Refinement
Comparison

More results & details

Sample  Image Quilting  Graphcut

Image Analogies (Hertzmann '01)

A  A'  B  B'

Artistic Filters

A  A'  B  B'
Other applications of Image Analogies

- Texture synthesis
- Super-resolution
- Texture transfer
- Image colorization
- Simple filters (blur, emboss)

More details

http://mrl.nyu.edu/projects/image-analogy/