Pattern Recognition

Pattern recognition is:

1. The name of the journal of the Pattern Recognition Society.

2. A research area in which patterns in data are found, recognized, discovered, …whatever.

3. A catchall phrase that includes
   - classification
   - clustering
   - data mining
   - …

Two Schools of Thought

1. Statistical Pattern Recognition
   
The data is reduced to vectors of numbers and statistical techniques are used for the tasks to be performed.

2. Structural Pattern Recognition
   
The data is converted to a discrete structure (such as a grammar or a graph) and the techniques are related to computer science subjects (such as parsing and graph matching).

In this course

1. How should objects to be classified be represented?

2. What algorithms can be used for recognition (or matching)?

3. How should learning (training) be done?

Classification in Statistical PR

- A class is a set of objects having some important properties in common
- A feature extractor is a program that inputs the data (image) and extracts features that can be used in classification.
- A classifier is a program that inputs the feature vector and assigns it to one of a set of designated classes or to the “reject” class.

   With what kinds of classes do you work?
Feature Vector Representation

♦ $X = [x_1, x_2, \ldots, x_n]$, each $x_j$ a real number
♦ $x_j$ may be an object measurement
♦ $x_j$ may be count of object parts
♦ Example: object rep. [#holes, #strokes, moments, …]

Some Terminology

♦ Classes: set of $m$ known categories of objects
  (a) might have a known description for each
  (b) might have a set of samples for each
♦ Reject Class:
  a generic class for objects not in any of the designated known classes
♦ Classifier:
  Assigns object to a class based on features

Possible features for char rec.

<table>
<thead>
<tr>
<th>Class</th>
<th>Character</th>
<th>Area</th>
<th>Height</th>
<th>Width</th>
<th>Number of Holes</th>
<th>Number of Strokes</th>
<th>Center (cx, cy)</th>
<th>Aspect Ratio</th>
<th>Inertia</th>
</tr>
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<tbody>
<tr>
<td>'A'</td>
<td>medium</td>
<td>3/4</td>
<td>1</td>
<td>3</td>
<td>1/2, 2/3, 90</td>
<td>medium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>medium</td>
<td>3/4</td>
<td>1</td>
<td>1</td>
<td>1/2, 2/3, 90</td>
<td>medium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>'C'</td>
<td>medium</td>
<td>2/3</td>
<td>1</td>
<td>0</td>
<td>1/2, 2/3, 90</td>
<td>large</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>'D'</td>
<td>low</td>
<td>1/4</td>
<td>0</td>
<td>1</td>
<td>1/2, 2/3, 90</td>
<td>low</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>'E'</td>
<td>high</td>
<td>3/4</td>
<td>0</td>
<td>4</td>
<td>1/2, 2/3, 90</td>
<td>large</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>high</td>
<td>3/4</td>
<td>0</td>
<td>2</td>
<td>1/2, 2/3, 90</td>
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<td>1/2</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>'H'</td>
<td>low</td>
<td>2/3</td>
<td>0</td>
<td>1</td>
<td>1/2, 2/3, 90</td>
<td>low</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>'I'</td>
<td>low</td>
<td>2/3</td>
<td>0</td>
<td>1</td>
<td>1/2, 2/3, 90</td>
<td>low</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Discriminant functions

♦ Functions $f(x, K)$ perform some computation on feature vector $x$
♦ Knowledge $K$ from training or programming is used
♦ Final stage determines class
Classification using nearest class mean

- Compute the Euclidean distance between feature vector $X$ and the mean of each class.
- Choose closest class, if close enough (reject otherwise)

Nearest mean might yield poor results with complex structure

- Class 2 has two modes; where is its mean?
- But if modes are detected, two subclass mean vectors can be used

Scaling coordinates by std dev

We can compute a modified distance from feature vector $x$ to class mean vector $x_c$ by scaling by the spread, or standard deviation, $\sigma_i$ of class $c$ along each dimension $i$.

\[
\text{scaled Euclidean distance from } x \text{ to class mean } x_c = \sqrt{\sum_{i=1}^{d} (x[i] - x_c[i]/\sigma_i)^2}
\]

In the previous 3 class problem, an observed $X$ near the top of the Class 3 distribution will scale to be closer to the mean of Class 3 than to the mean of Class 2. Without scaling, $X$ would be closer to the mean of Class 2.

Nearest Neighbor Classification

- Keep all the training samples in some efficient look-up structure.
- Find the nearest neighbor of the feature vector to be classified and assign the class of the neighbor.
- Can be extended to $K$ nearest neighbors.
Receiver Operating Curve ROC

- Plots correct detection rate versus false alarm rate
- Generally, false alarms go up with attempts to detect higher percentages of known objects

Confusion matrix shows empirical performance

<table>
<thead>
<tr>
<th>Class j output by the pattern recognition system</th>
<th>true object 1</th>
<th>true object 2</th>
<th>true object 3</th>
<th>true object 4</th>
<th>true object 5</th>
<th>true object 6</th>
<th>true object 7</th>
<th>true object 8</th>
<th>true object 9</th>
</tr>
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<tbody>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>98</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>'3'</td>
<td>0</td>
<td>0</td>
<td>98</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>'4'</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>98</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>'5'</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>98</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>'6'</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>98</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
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<td>0</td>
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<td>0</td>
<td>98</td>
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<td>0</td>
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<tr>
<td>'8'</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>98</td>
<td>0</td>
</tr>
<tr>
<td>'9'</td>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>98</td>
</tr>
</tbody>
</table>

Confusion may be unrecognizable between some classes for example, between 9's and 4's, or between u's and j's for handprinted characters.

Bayesian decision-making

- Classify into class $w_i$ that is most likely based on observations $X$
- In order to compute the likelihoods given the measurement $X$, the following distributions are needed.

  - Class conditional distribution: $p(x|w_i)$ for each class $w_i$(1)
  - Prior probability: $P(w_i)$ for each class $w_i$ (2)
  - Unconditional distribution: $p(x)$ (3)

- Use Bayes rule if all of the classes $w_i$ are disjoint

  $$P(w_i|x) = \frac{P(x|w_i)P(w_i)}{p(x)} = \frac{p(x|w_i)P(w_i)}{\sum_{i=1}^{m}p(x|w_i)P(w_i)}$$

Classifiers often used in CV

- Decision Tree Classifiers
- Artificial Neural Net Classifiers
- Bayesian Classifiers and Bayesian Networks (Graphical Models)
- Support Vector Machines
Decision Trees

- Decision Tree Characteristics
  1. Training
     How do you construct one from training data?
     Entropy-based Methods
  2. Strengths
     Easy to Understand
  3. Weaknesses
     Overtraining

Entropy-Based Automatic Decision Tree Construction

Quinlan suggested information gain in his ID3 system and later the gain ratio, both based on entropy.

Entropy

Given a set of training vectors $S$, if there are $c$ classes,

$$\text{Entropy}(S) = \sum_{i=1}^{c} -p_i \log_2 (p_i)$$

Where $p_i$ is the proportion of category $i$ examples in $S$.

If all examples belong to the same category, the entropy is 0.

If the examples are equally mixed ($1/c$ examples of each class), the entropy is a maximum at 1.0.

e.g. for $c=2$, $-0.5 \log_2 0.5 - 0.5 \log_2 0.5 = -0.5(-1) - 0.5(-1) = 1$
Information Gain

The information gain of an attribute $A$ is the expected reduction in entropy caused by partitioning on this attribute.

$$\text{Gain}(S,A) = \text{Entropy}(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} \cdot \text{Entropy}(S_v)$$

where $S_v$ is the subset of $S$ for which attribute $A$ has value $v$.

Choose the attribute $A$ that gives the maximum information gain.

Gain Ratio

Gain ratio is an alternative metric from Quinlan’s 1986 paper and used in the popular C4.5 package (free!).

$$\text{GainRatio}(S,A) = \frac{\text{Gain}(S,A)}{\text{SplitInfo}(S,A)}$$

$$\text{SplitInfo}(S,A) = \sum_{i=1}^{n} \frac{|S_i|}{|S|} \cdot \log_2 \left( \frac{|S_i|}{|S|} \right)$$

where $S_i$ is the subset of $S$ in which attribute $A$ has its $i$th value.

SplitInfo measures the amount of information provided by an attribute that is not specific to the category.

Information Gain (cont)

Set $S$

Attribute $A$

\[ v_1 \quad v_2 \quad v_k \]

Set $S'$

$S' = \{ s \in S \mid \text{value}(A) = v_1 \}$

repeat recursively

Information gain has the disadvantage that it prefers attributes with large number of values that split the data into small, pure subsets.

Information Content

Note:

A related method of decision tree construction using a measure called Information Content is given in the text, with full numeric example of its use.
Artificial Neural Nets

Artificial Neural Nets (ANNs) are networks of artificial neuron nodes, each of which computes a simple function.

An ANN has an input layer, an output layer, and “hidden” layers of nodes.

Node Functions

\[ \text{output} = g \left( \sum a_j \cdot w(j,i) \right) \]

Function g is commonly a step function, sign function, or sigmoid function (see text).

Neural Net Learning

That’s beyond the scope of this text; only simple feed-forward learning is covered.

The most common method is called back propagation.

We’ve been using a free package called NevProp.

What do you use?

Support Vector Machines (SVM)

Support vector machines are learning algorithms that try to find a hyperplane that separates the differently classified data the most. They are based on two key ideas:

- Maximum margin hyperplanes
- A kernel ‘trick’
Maximal Margin

Find the hyperplane with maximal margin for all the points. This originates an optimization problem Which has a unique solution (convex problem).

Non-separable data

What can be done if data cannot be separated with a hyperplane?

The kernel trick

The SVM algorithm implicitly maps the original data to a feature space of possibly infinite dimension in which data (which is not separable in the original space) becomes separable in the feature space.

Our Current Application

- Sal Ruiz is using support vector machines in his work on 3D object recognition.
- He is training classifiers on data representing deformations of a 3D model of a class of objects.
- The classifiers are starting to learn what kinds of surface patches are related to key parts of the model (i.e., a snowman’s face)
Snowman with Patches