CSE 573: Artificial Intelligence

Constraint Satisfaction Problems

Factored (aka Structured) Search

[With many slides by Dan Klein and Pieter Abbeel (UC Berkeley) available at http://ai.berkeley.edu.]

Final Presentations

- 21 groups / 40 people / 110 min
  - Minus transfers & tournament replay
- Presentations (with questions)
  - One person groups 2.5 min
  - Two person groups 4.5 min
  - Three person groups 6.5 min
- Everyone should speak (unless OOT)
- Rehearse
- Add URL for slides to g-doc
  - https://docs.google.com/spreadsheets/d/1Qt5BW0DkSAg6Q4MOM98jSSwjR2wTZpi5i01XdT0X-fs/edit#gid=0

Final report

- Default project ~2 pages
- Other projects ~6 pages
  - Experiments
  - Lessons learned
  - http://courses.cs.washington.edu/courses/cse573/17wi/reports.html
- Everyone
  - See note on appendices – dynamics & external code

AI Topics

- Search
  - Problem spaces
  - BFS, DFS, UCS, A* (tree and graph), local search
  - Completeness and Optimality
  - Heuristics: admissibility and consistency; pattern DBs
- CSPs
  - Constraint graphs, backtracking search
  - Forward checking, AC3 constraint propagation, ordering heuristics
- Games
  - Minimax, Alpha-beta pruning,
  - Expectimax
  - Evaluation Functions
- MDPs
  - Bellman equations
  - Value iteration, policy iteration
- Reinforcement Learning
  - Exploration vs Exploitation
  - Model-based vs. model-free
  - Q-learning
  - Linear value function approx.
- Hidden Markov Models
  - Markov chains, DBNs
  - Forward algorithm
  - Particle Filters
- POMDPs
  - Belief space
  - Piecewise linear approximation to value fun
- Beneficial AI
- Bayesian Networks
  - Basic definition, independence (d-sep)
  - Variable elimination
  - Sampling (rejection, importance)
- Learning
  - BN parameters with complete data
  - Search thru space of BN structures
  - Expectation maximization

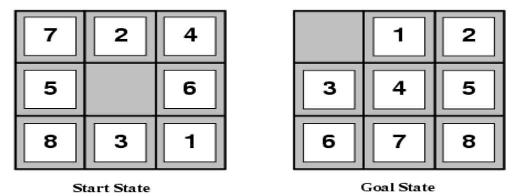
What is intelligence?

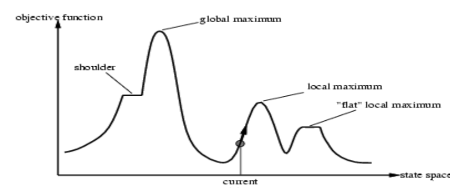
- (bounded) Rationality
  - Agent has a performance measure to optimize
  - Given its state of knowledge
  - Choose optimal action
  - With limited computational resources

- Human-like intelligence/behavior

State-Space Search

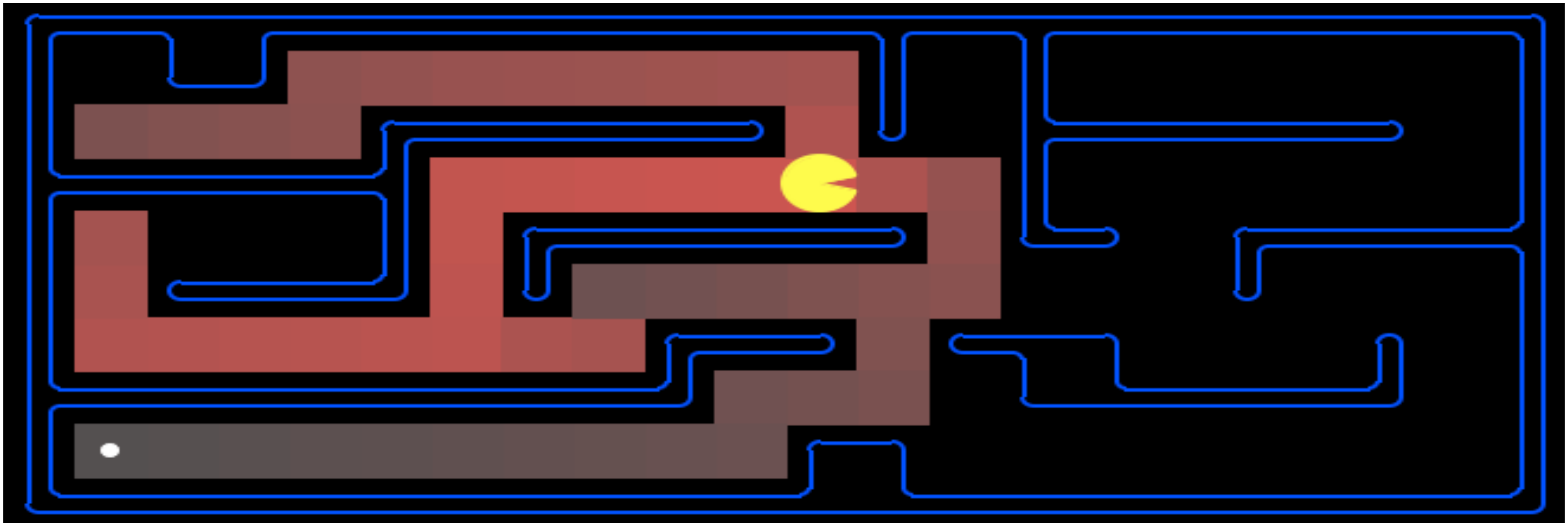
- X as a search problem
  - states, actions, transitions, cost, goal-test
- Types of search
  - uninformed systematic: often slow
    - DFS, BFS, uniform-cost, iterative deepening
  - Heuristic-guided: better
    - Greedy best first, A*
    - Relaxation leads to heuristics
  - Local: fast, fewer guarantees; often local optimal
    - Hill climbing and variations
    - Simulated Annealing: global optimal
  - (Local) Beam Search



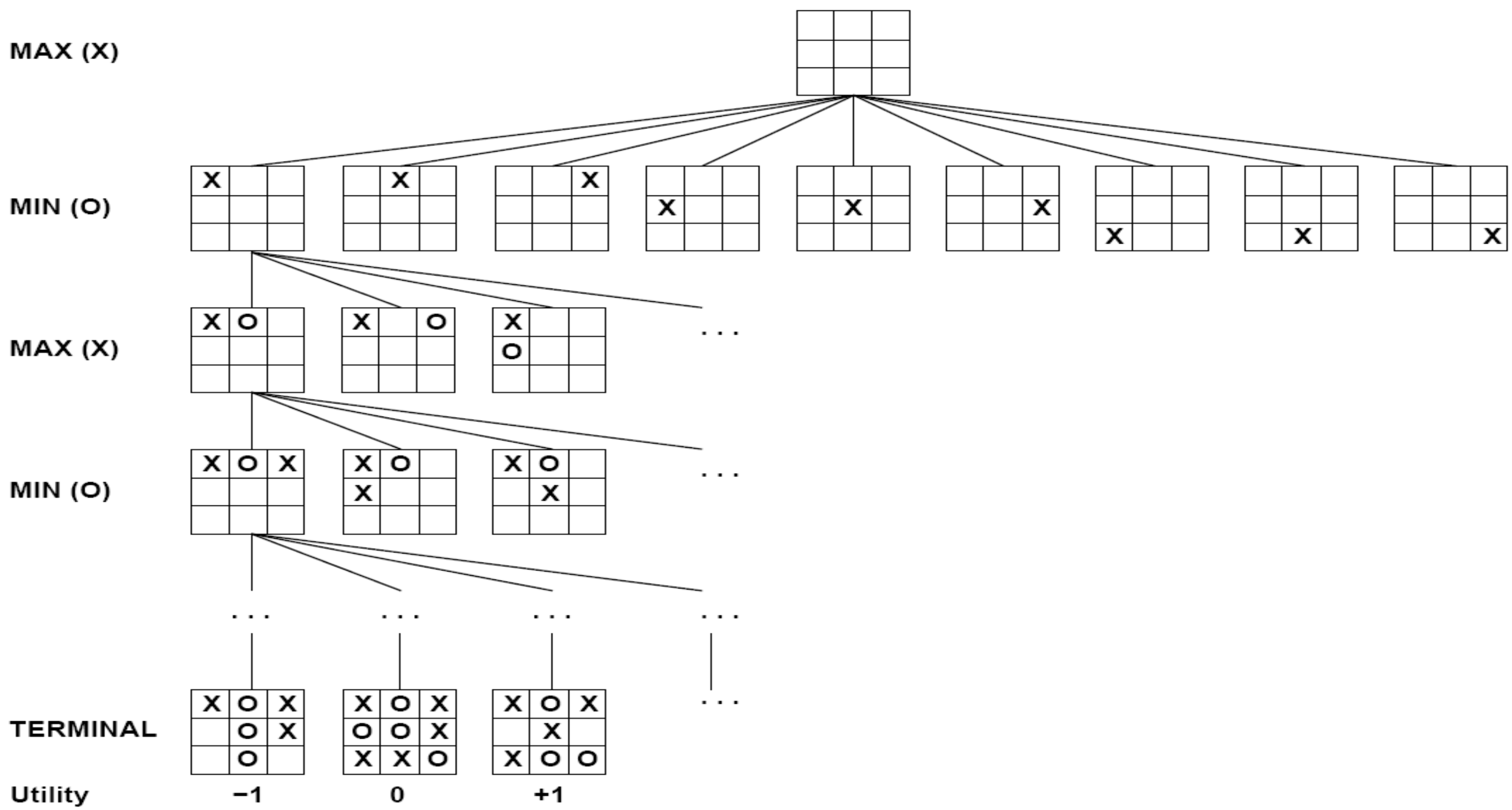


Which Algorithm?

- A*, Manhattan Heuristic:

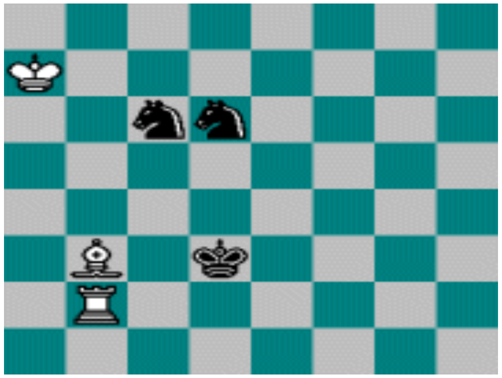


Adversarial Search



Adversarial Search

- AND/OR search space (max, min)
- minimax objective function
- minimax algorithm (~dfs)
  - alpha-beta pruning
- Utility function for partial search
  - Learning utility functions by playing with itself
- Openings/Endgame databases



Policy Iteration

- Let i =0
- Initialize $\pi_i(s)$ to random actions
- Repeat
  - Step 1: Policy evaluation:
    - Initialize k=0;   Forall s, $V_0^\pi(s) = 0$
    - Repeat until V^π converges
      - For each state s, $V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s,\pi_i(s),s') \left[R(s,\pi_i(s),s') + \gamma\, V_k^{\pi_i}(s')\right]$
      - Let k += 1
  - Step 2: Policy improvement:
    - For each state, s, $\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^{\pi_i}(s')\right]$
    - If $\pi_i == \pi_{i+1}$ then it's optimal; return it.
    - Else let i += 1

Example

Initialize π_0 to "always go right"

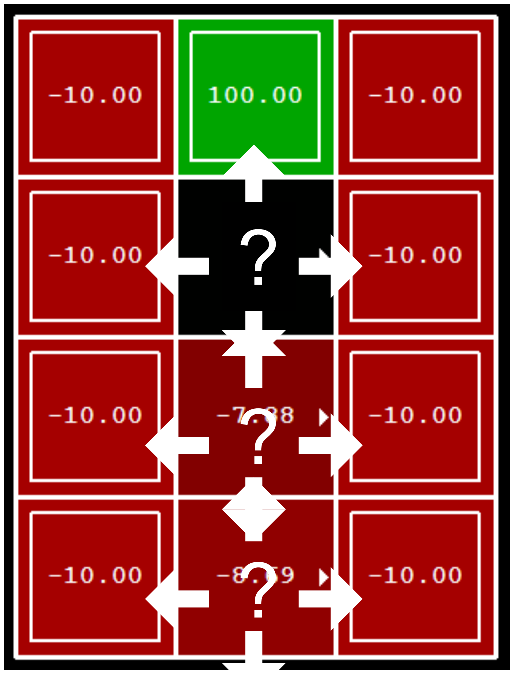
Perform policy evaluation

Perform policy improvement

Iterate through states

Has policy changed?

Yes! i += 1



Example

π_1 says "always go up"

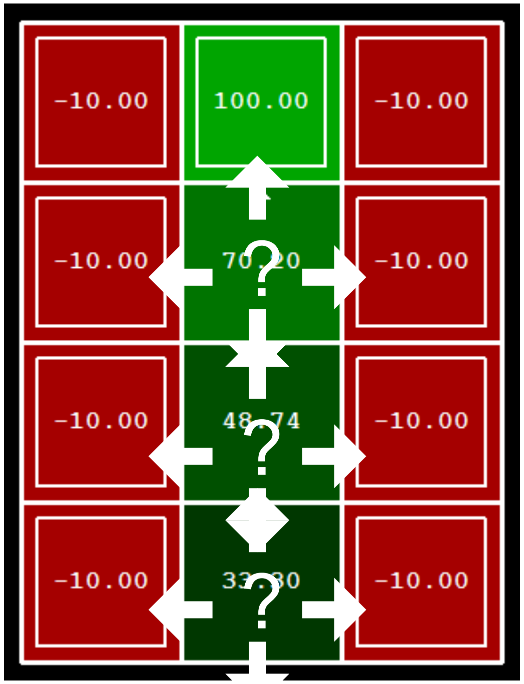
Perform policy evaluation

Perform policy improvement

Iterate through states

Has policy changed?

No! We have the optimal policy



Reinforcement Learning

- **Forall s, a**
  - **Initialize Q(s, a) = 0**
- **Repeat Forever**

  Where are you? s.

  Choose some action a

  Execute it in real world: transition =*(s, a, r, s')*

  Do update:

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s, a)$$

$$Q(s, a) \leftarrow Q(s, a) + \alpha\,[\text{difference}]$$

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- **Forall s, a**
  - **Initialize $w_i = 0$**
- **Repeat Forever**

  Where are you? s.
  Choose some action a
  Execute it in real world: transition =*(s, a, r, s')*
  Do updates:

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s,a)$$

$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a)$$

- Interpretation as search
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the active features

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear Q-functions:

$$\text{transition } = (s, a, r, s')$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\, [\text{difference}]$$

Old way: Exact Q's

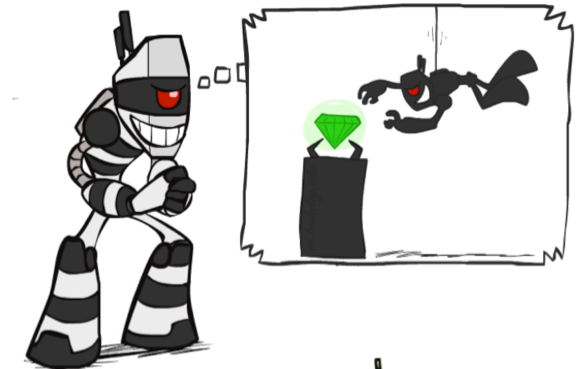
$$w_i \leftarrow w_i + \alpha\, [\text{difference}]\, f_i(s,a)$$

Now: Approximate Q's

- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were ***active***: ***disprefer all states with that state's features***

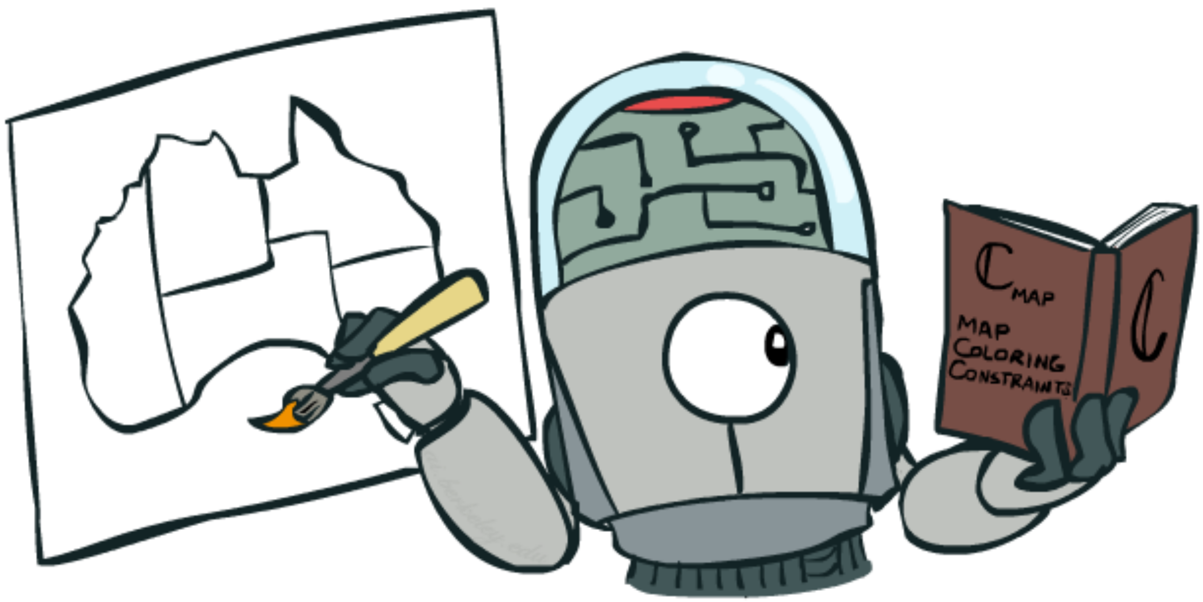
What is Search For?

- **Planning:** sequences of actions
  - The ***path to the goal*** is the important thing
  - Paths have various costs, depths
  - Assume little about problem structure



- **Identification:** assignments to variables
  - The ***goal itself*** is important, ***not the path***
  - All paths at the same depth (for some formulations)

Constraint Satisfaction Problems



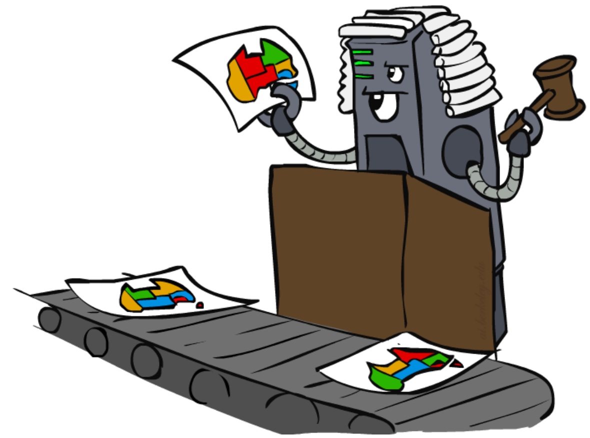
CSPs are *structured* (factored) identification problems

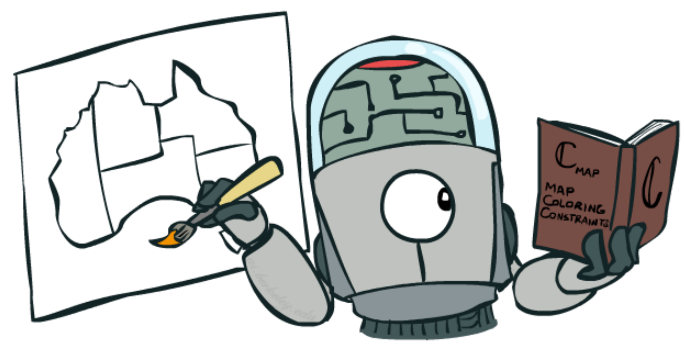
Constraint Satisfaction Problems

- Standard search problems:
  - State is a “black box”: arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything

- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables

- Making use of CSP formulation allows for optimized algorithms
  - Typical example of trading generality for utility (in this case, speed)

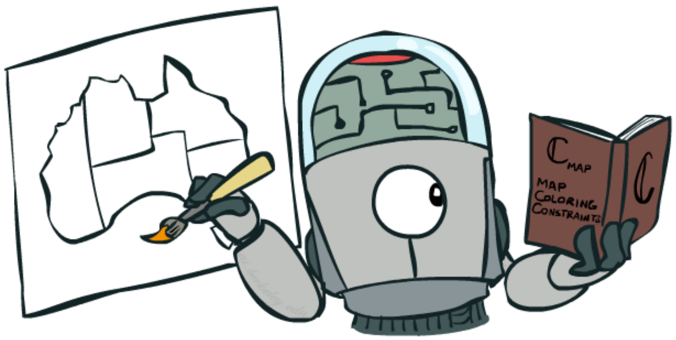




Constraint Satisfaction Problems

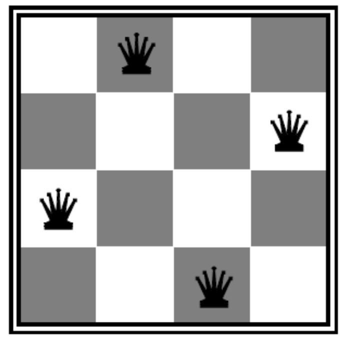
- "Factoring" the state space
- Representing the state space in a knowledge representation

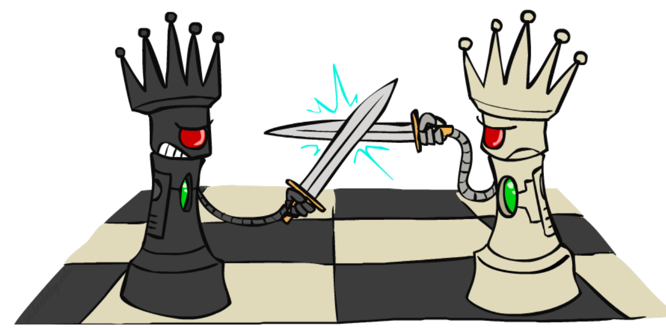
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables



CSP Example: N-Queens

Is there a queen at X_{ij}?





- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0, 1\}$
  - Constraints

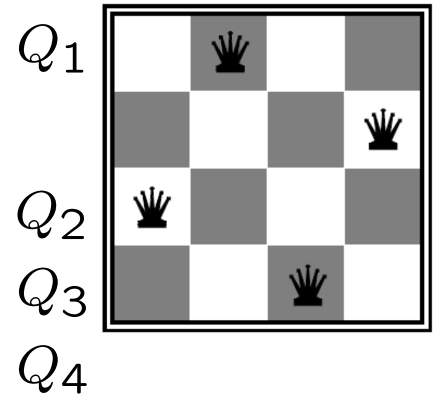
$$\forall i, j, k \;\; (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

CSP Example: N-Queens

What column is the queen on for row k?

- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots N\}$
  - Constraints:

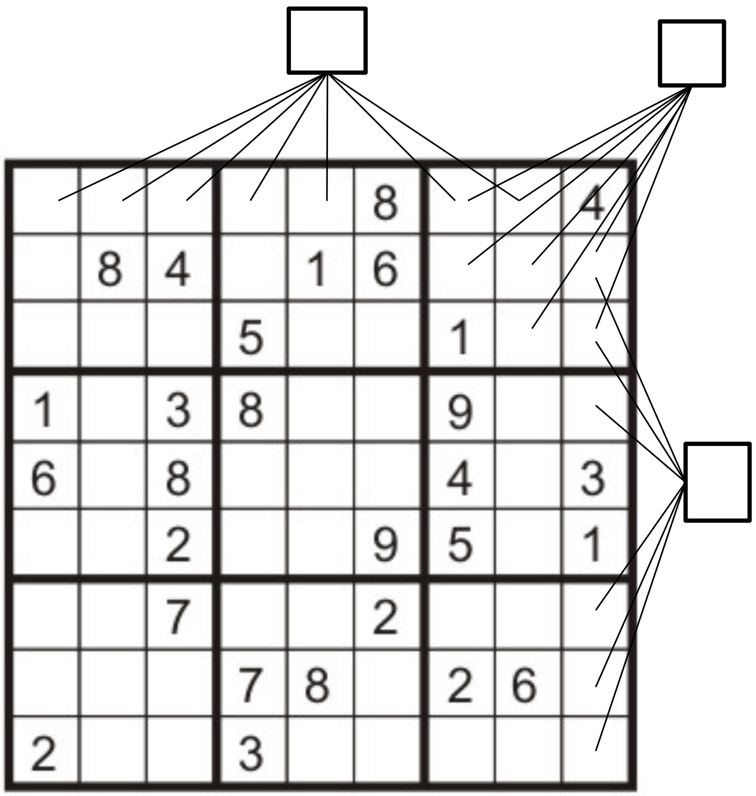


Implicit: $\forall i, j \;\; \text{non-threatening}(Q_i, Q_j)$

Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

$\ldots$

CSP Example: Sudoku



- Variables:
  - Each (open) square
- Domains:
  - {1,2,…,9}
- Constraints:

9-way alldiff for each column

9-way alldiff for each row

9-way alldiff for each region

(or can have a bunch
of pairwise inequality
constraints)

Propositional Logic

$$((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$$

- Variables: propositional variables
- Domains: {T, F}
- Constraints: logical formula

CSP Example: Map Coloring

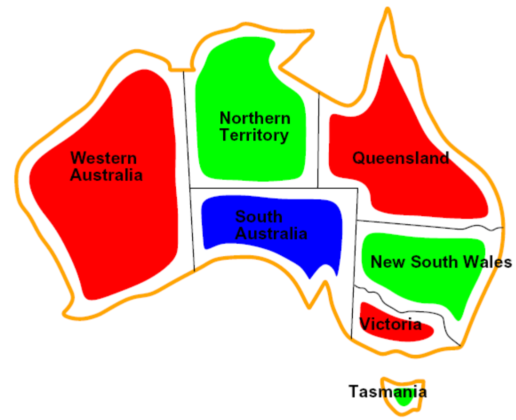
- Variables: $WA,\ NT,\ Q,\ NSW,\ V,\ SA,\ T$
- Domains: $D = \{\text{red}, \text{green}, \text{blue}\}$
- Constraints: adjacent regions must have different colors

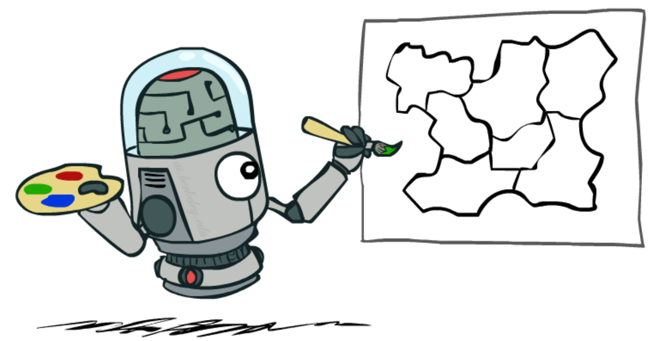
  Implicit: $WA \neq NT$

  Explicit: $(WA, NT) \in \{(\text{red}, \text{green}), (\text{red}, \text{blue}), \ldots\}$

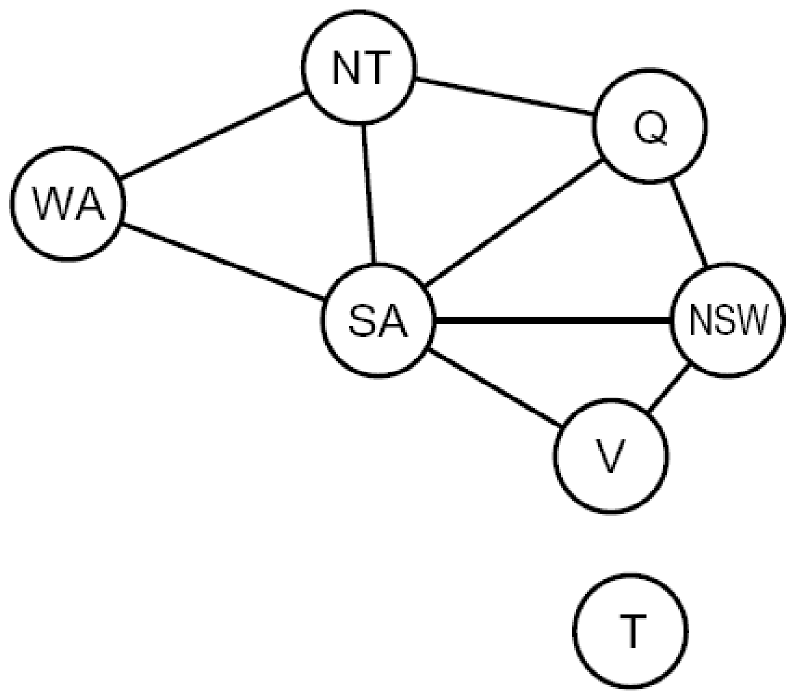
- Solutions are assignments satisfying all constraints, e.g.:

  $\{WA{=}\text{red},\ NT{=}\text{green},\ Q{=}\text{red},\ NSW{=}\text{green},\ V{=}\text{red},\ SA{=}\text{blue},\ T{=}\text{green}\}$



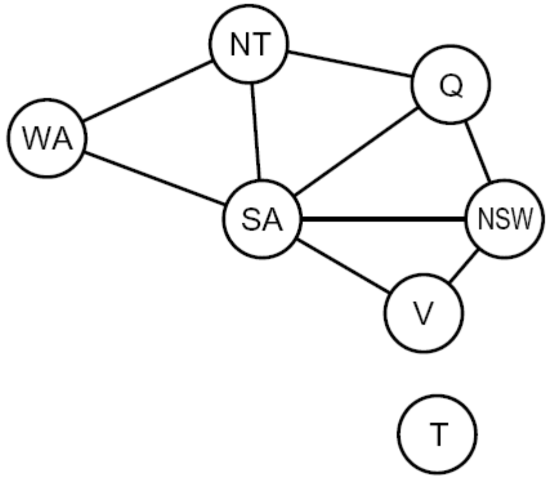


Constraint Graphs



Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



Example: Cryptarithmetic

- Variables:

  $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$

- Domains:

  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

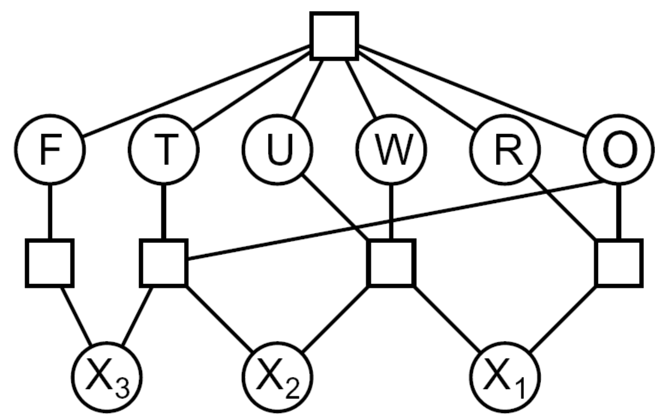
- Constraints:

  $\text{alldiff}(F, T, U, W, R, O)$

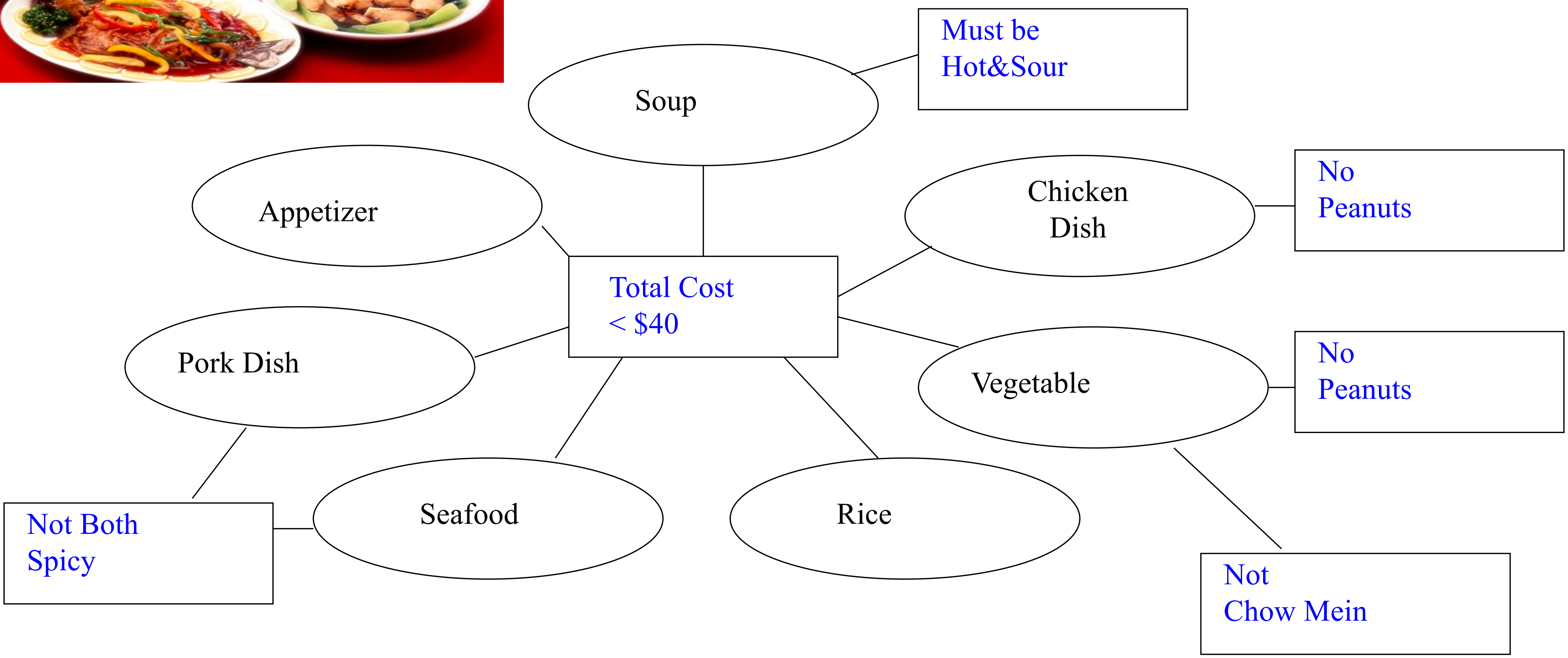
  $O + O = R + 10 \cdot X_1$

  $\cdots$

$$
\begin{array}{r}
T\ W\ O \\
+\ T\ W\ O \\
\hline
F\ O\ U\ R
\end{array}
$$

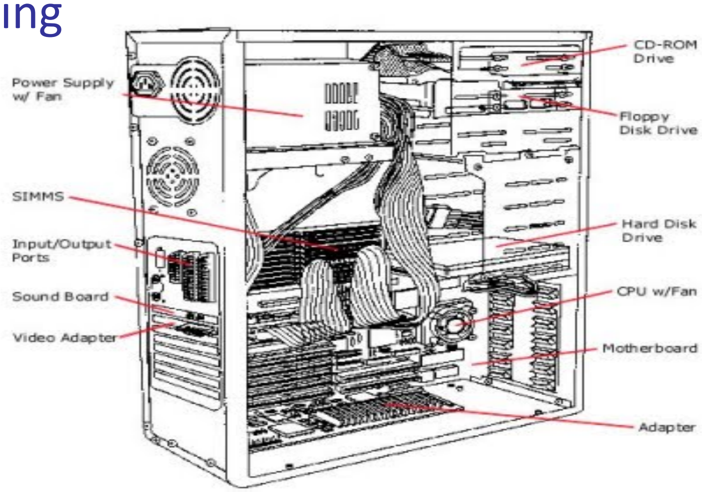


Chinese Constraint Network



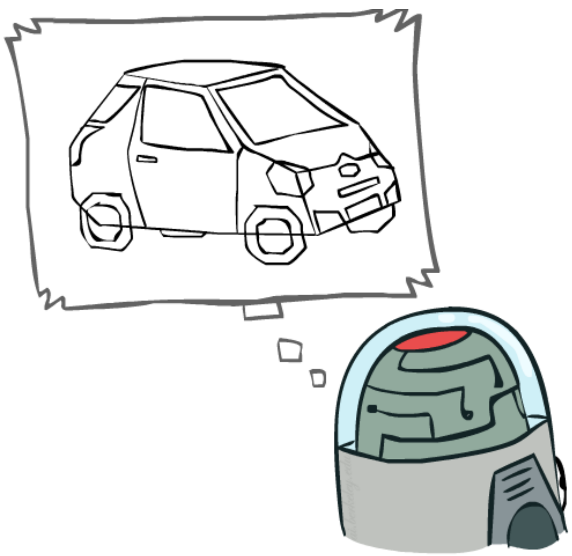
Real-World CSPs

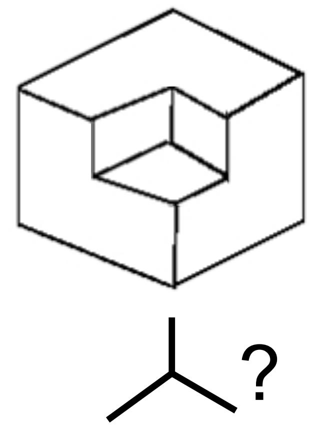
- Assignment problems: e.g., who teaches what class
- Timetabling problems: e.g., which class is offered when and where?
- Hardware configuration
- Gate assignment in airports
- Space Shuttle Repair
- Transportation scheduling
- Factory scheduling
- … lots more!



Example: The Waltz Algorithm

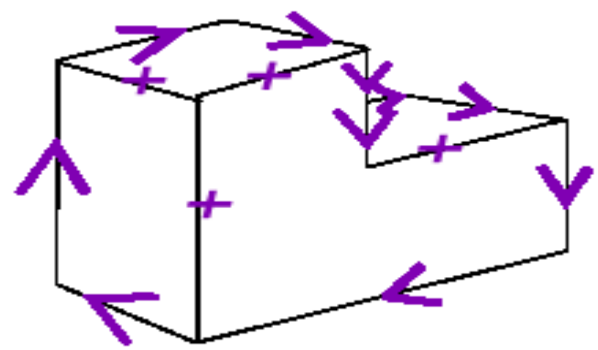
- The Waltz algorithm is for interpreting line drawings of solid polyhedra as 3D objects
- An early example of an AI computation posed as a CSP





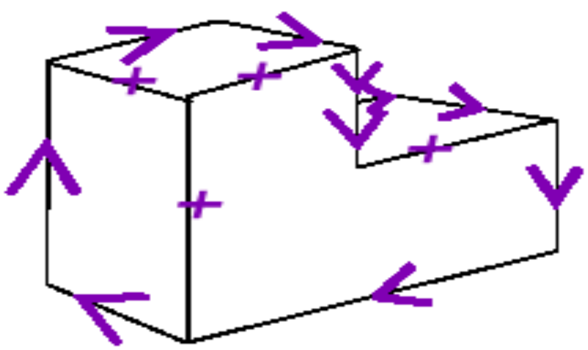
Waltz on Simple Scenes

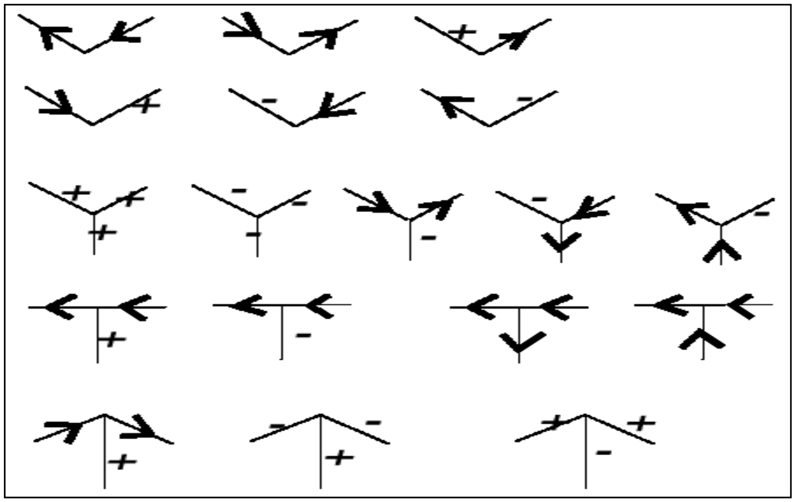
- Assume all objects:
  - Have no shadows or cracks
  - Three-faced vertices
  - "General position": no junctions change with small movements of the eye.
- Then each line on image is one of the following:
  - Boundary line (edge of an object) (>) with right hand of arrow denoting "solid" and left hand denoting "space"
  - Interior convex edge (+)
  - Interior concave edge (-)



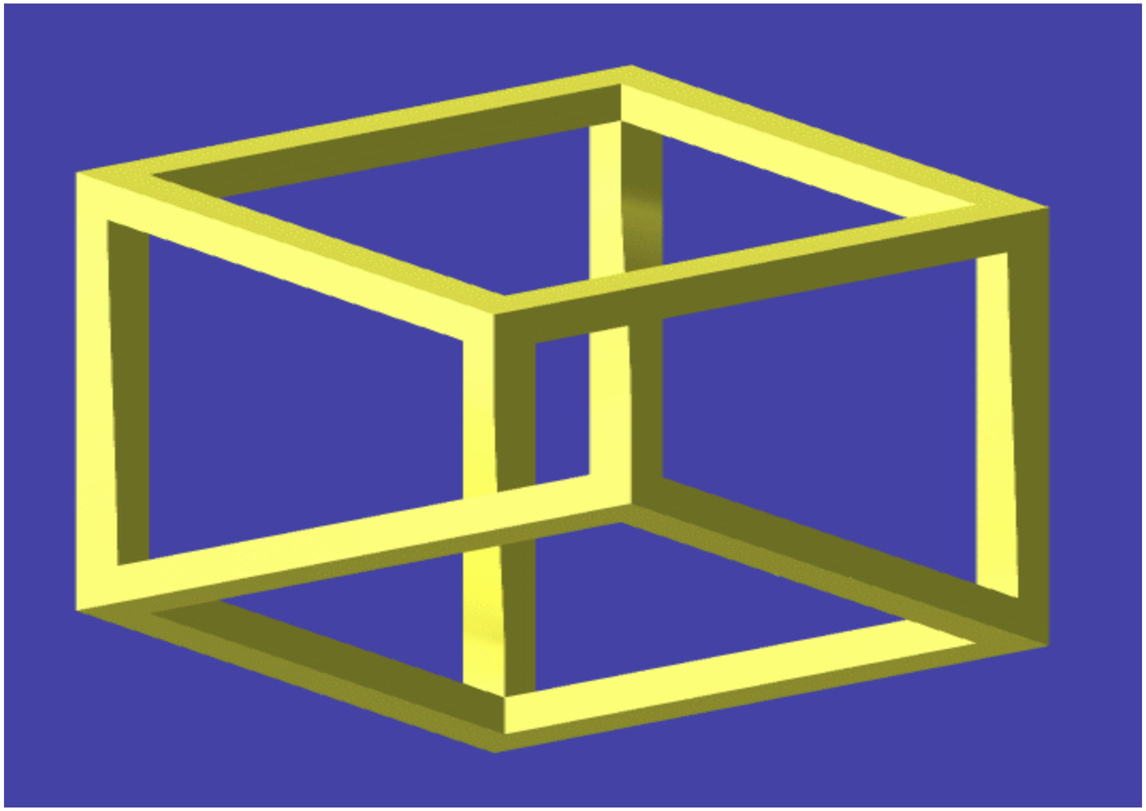
Legal Junctions

- Only certain junctions are physically possible
- How can we formulate a CSP to label an image?
- **Variables:** edges
- **Domains:** >, <, +, -
- **Constraints:** legal junction types





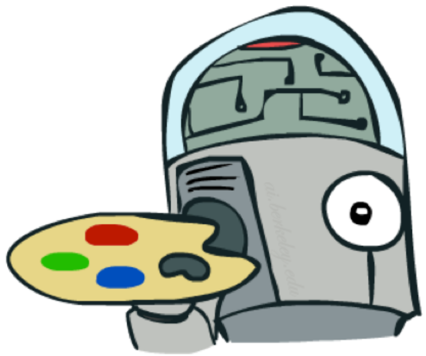
Slight Problem: Local *vs* Global Consistency



Varieties of CSPs

Varieties of CSP Variables

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable



- Continuous variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by linear program methods (see CSE 521 for a bit of LP theory)

Varieties of CSP Constraints

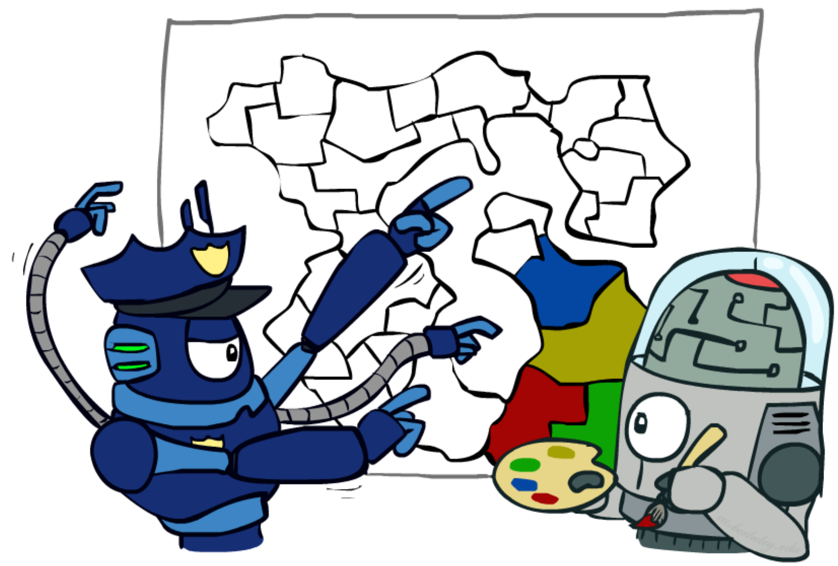
- Varieties of Constraints
  - Unary constraints involve a single variable (equivalent to reducing domains), e.g.:

    $$SA \neq green$$

  - Binary constraints involve pairs of variables, e.g.:

    $$SA \neq WA$$

  - Higher-order constraints involve 3 or more variables:
    e.g., cryptarithmetic column constraints



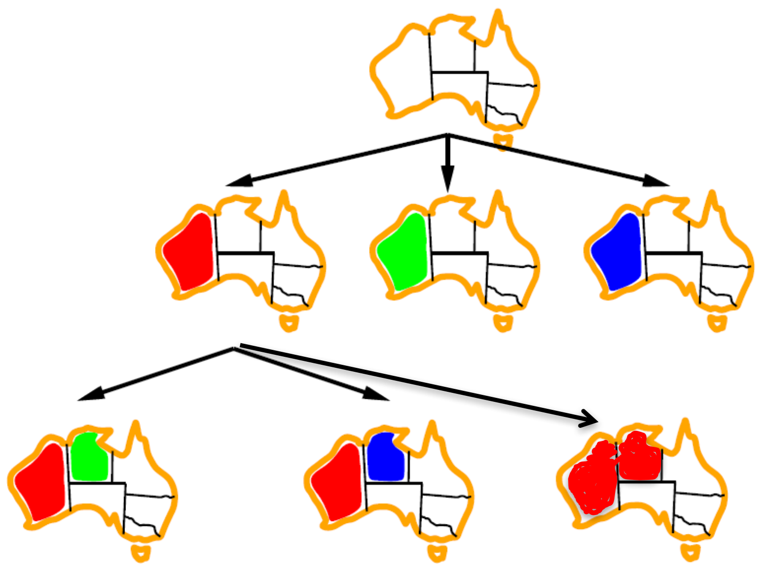
- Preferences (soft constraints):
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

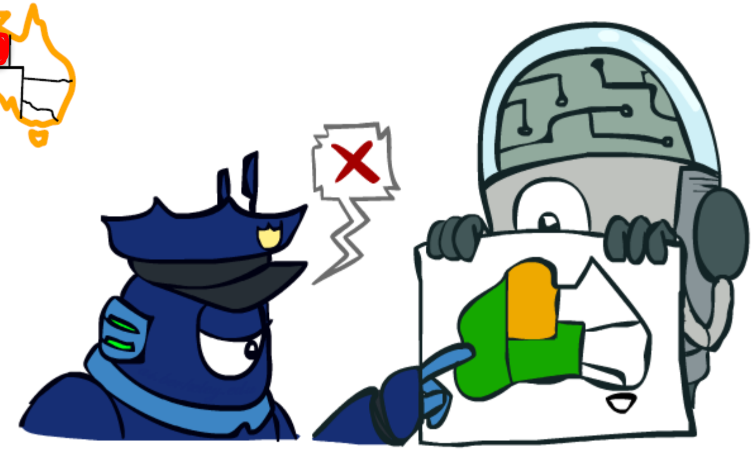
Solving CSPs

CSP as Search

- States
- Operators
- Initial State
- Goal State

Standard Depth First Search



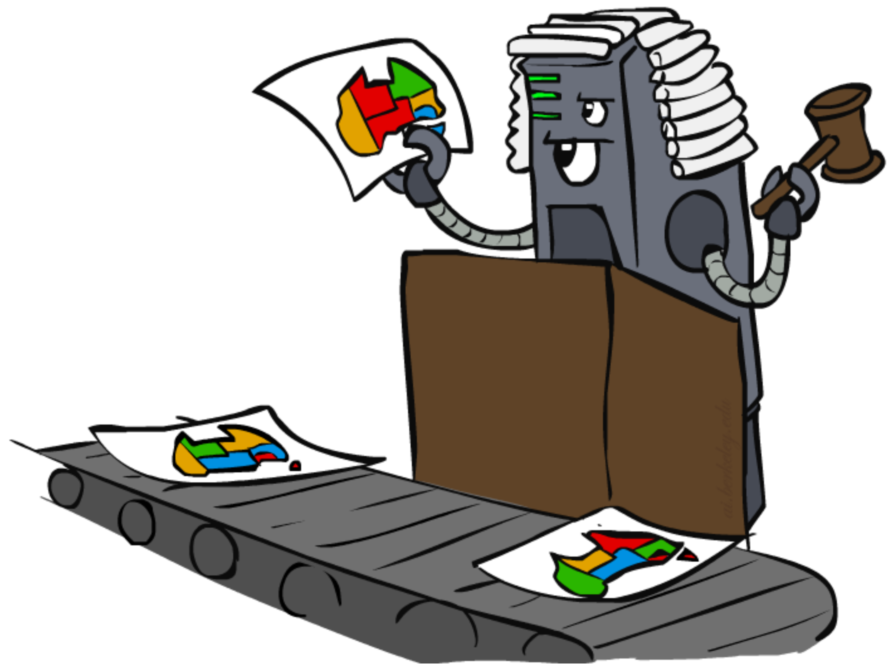


Standard Search Formulation

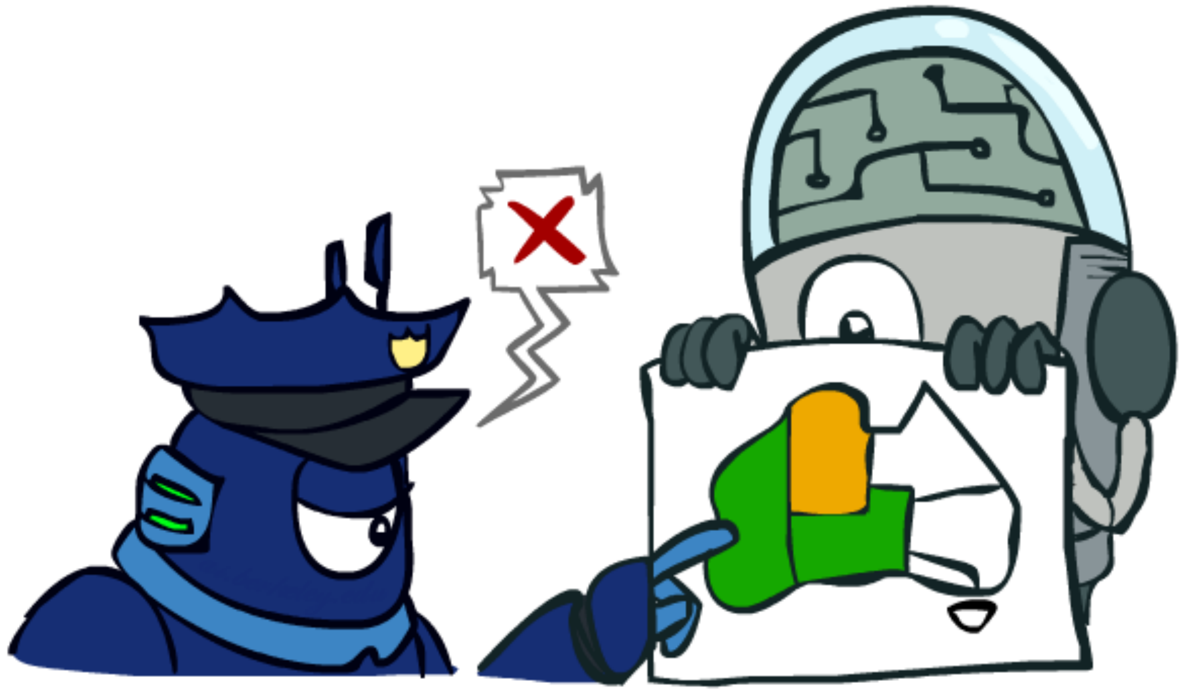
- Standard search formulation of CSPs

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - **Goal test: the current assignment is complete and satisfies all constraints**

- We'll start with the straightforward, naïve approach, then improve it

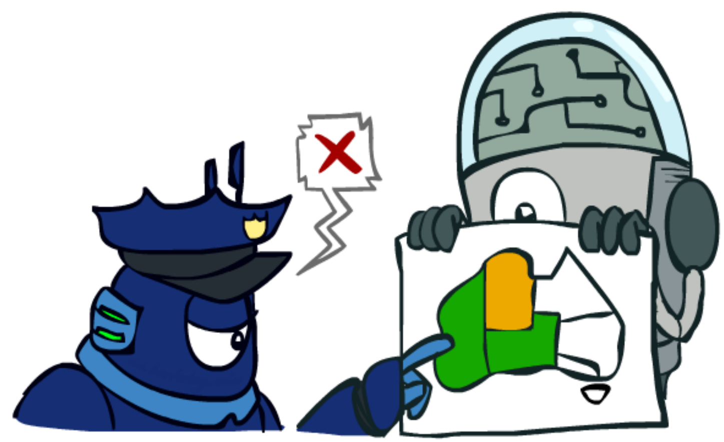


Backtracking Search

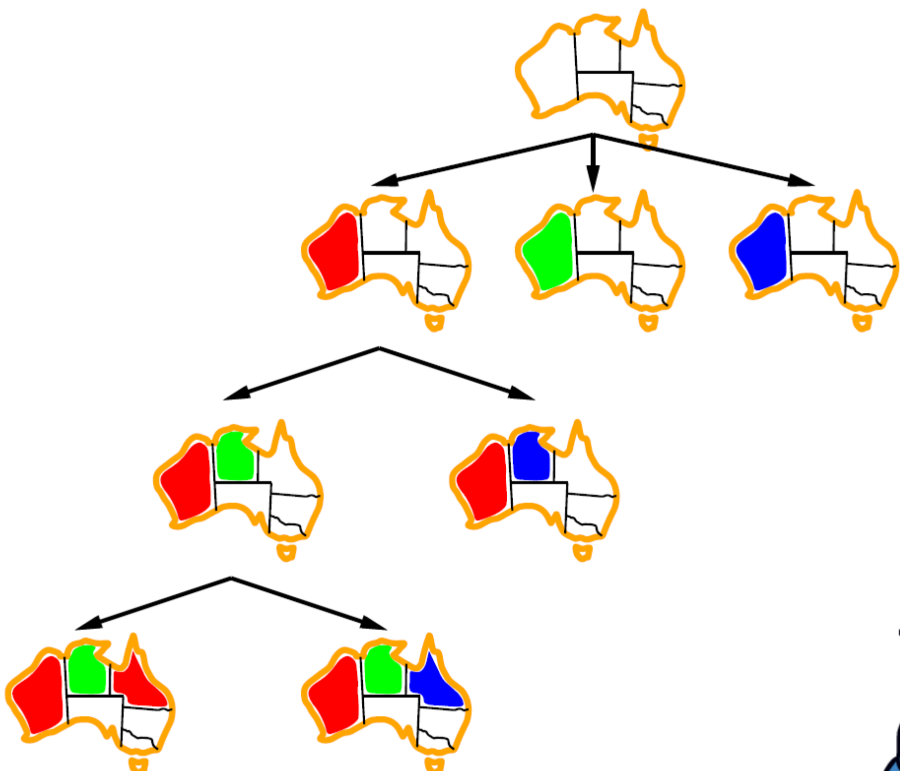


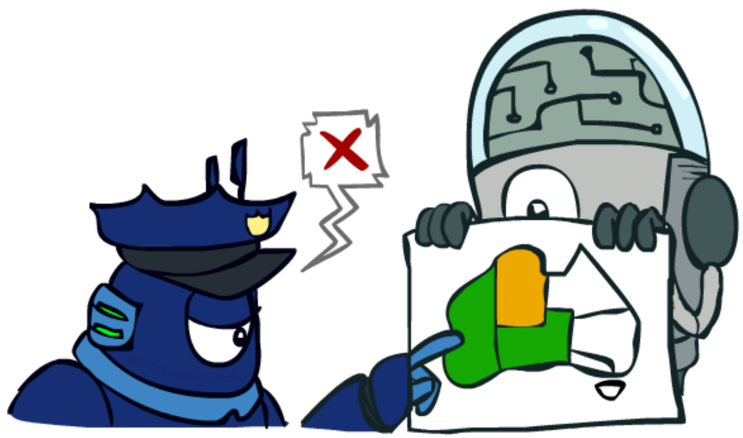
Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs
- Idea 1: One variable at a time
  - Variable assignments are commutative, so fix ordering
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step
- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the constraints
  - “Incremental goal test”
- Depth-first search with these two improvements
  is called *backtracking search*
- Can solve n-queens for $n \approx 25$



Backtracking Example





Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

- What are the choice points?

[Demo: coloring -- backtracking]

Backtracking Search

- Kind of depth first search
- Is it ***complete***?

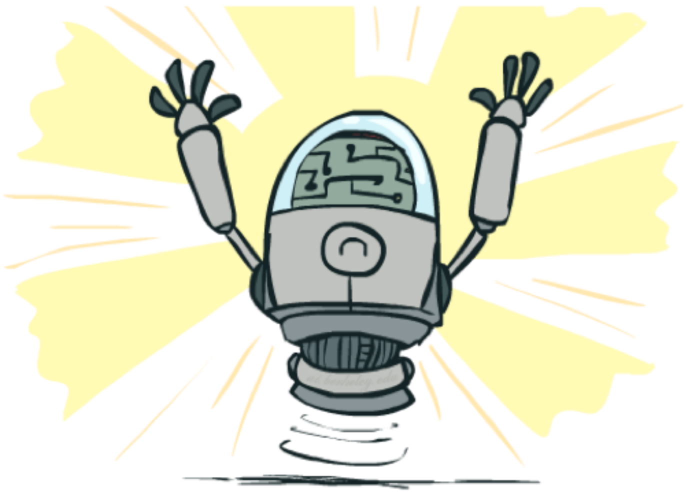
Improving Backtracking

- General-purpose ideas give huge gains in speed

- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?

- Filtering: Can we detect inevitable failure early?

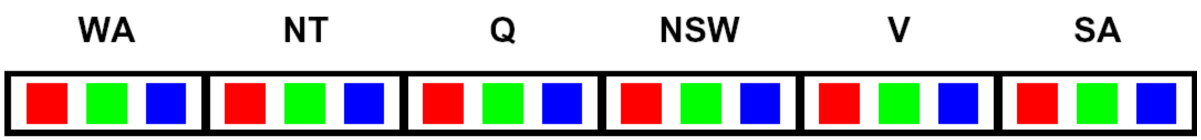
- Structure: Can we exploit the problem structure?



Filtering

Filtering: Forward Checking

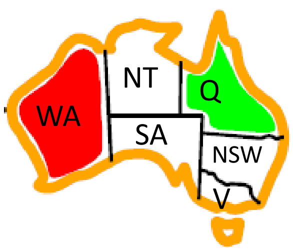
- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment

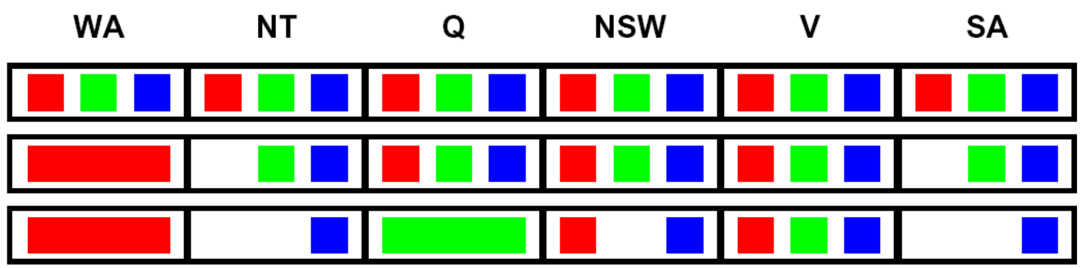


[Demo: coloring -- forward checking]

Filtering: Constraint Propagation

- Forward checking only propagates information from assigned to unassigned
- It doesn't catch when ***two unassigned variables*** have no consistent assignment:

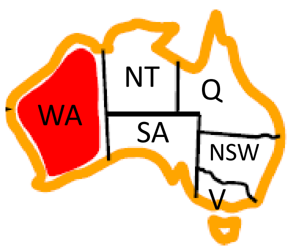


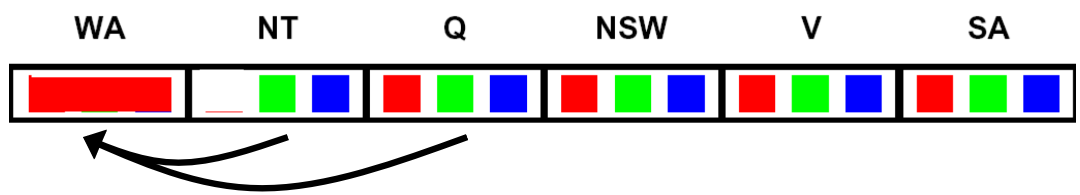


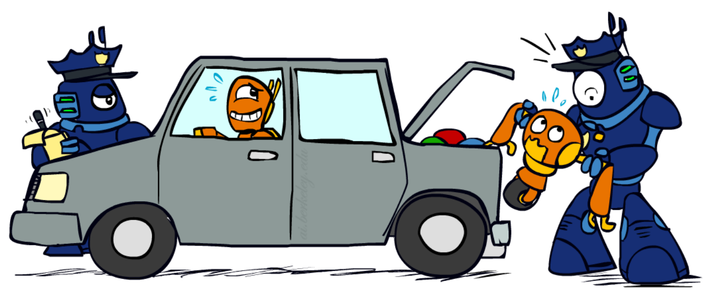
- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of a Single Arc

- An arc X $\rightarrow$ Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint





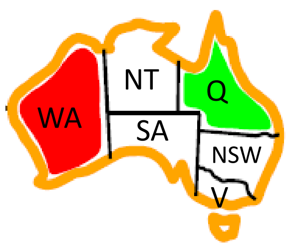


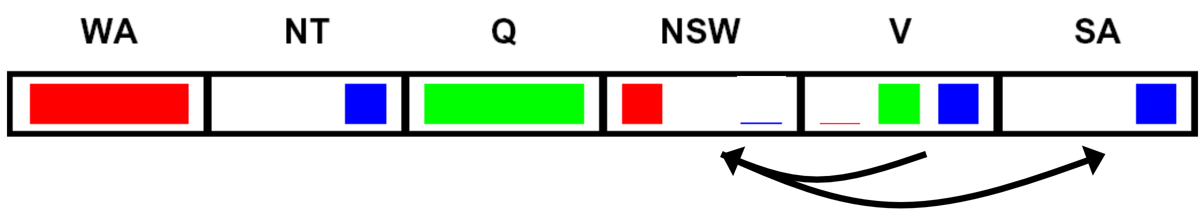
Delete from the tail!

- Forward checking: Enforcing consistency ***of arcs pointing to each new assignment***

Arc Consistency of an **Entire CSP**

- A simple form of propagation makes sure all arcs are consistent:





- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure ***earlier*** than forward checking
- Can be run as a preprocessor ***or*** after each assignment
- What's the ***downside*** of enforcing arc consistency?

Remember: Delete from the tail!

AC-3 algorithm for Arc Consistency

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

  while *queue* is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REMOVE-INCONSISTENT-VALUES(X_i, X_j) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add (X_k, X_i) to *queue*

---

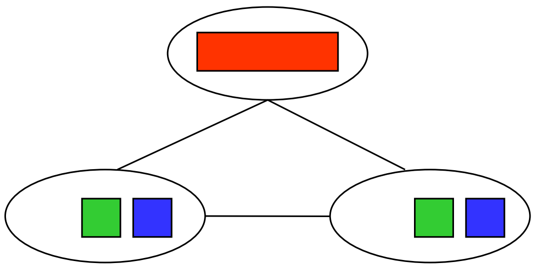
function REMOVE-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff succeeds
  removed $\leftarrow$ *false*
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; *removed* $\leftarrow$ *true*
  return *removed*

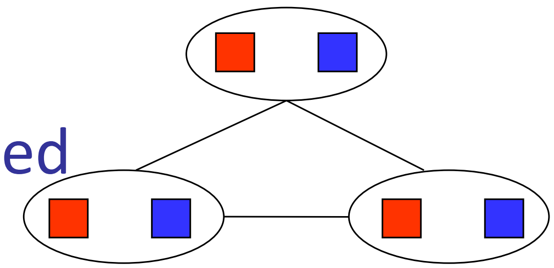
- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting ***all*** possible future problems is NP-hard – why?

[Demo: CSP applet (made available by aispace.org) -- n-queens]

Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)





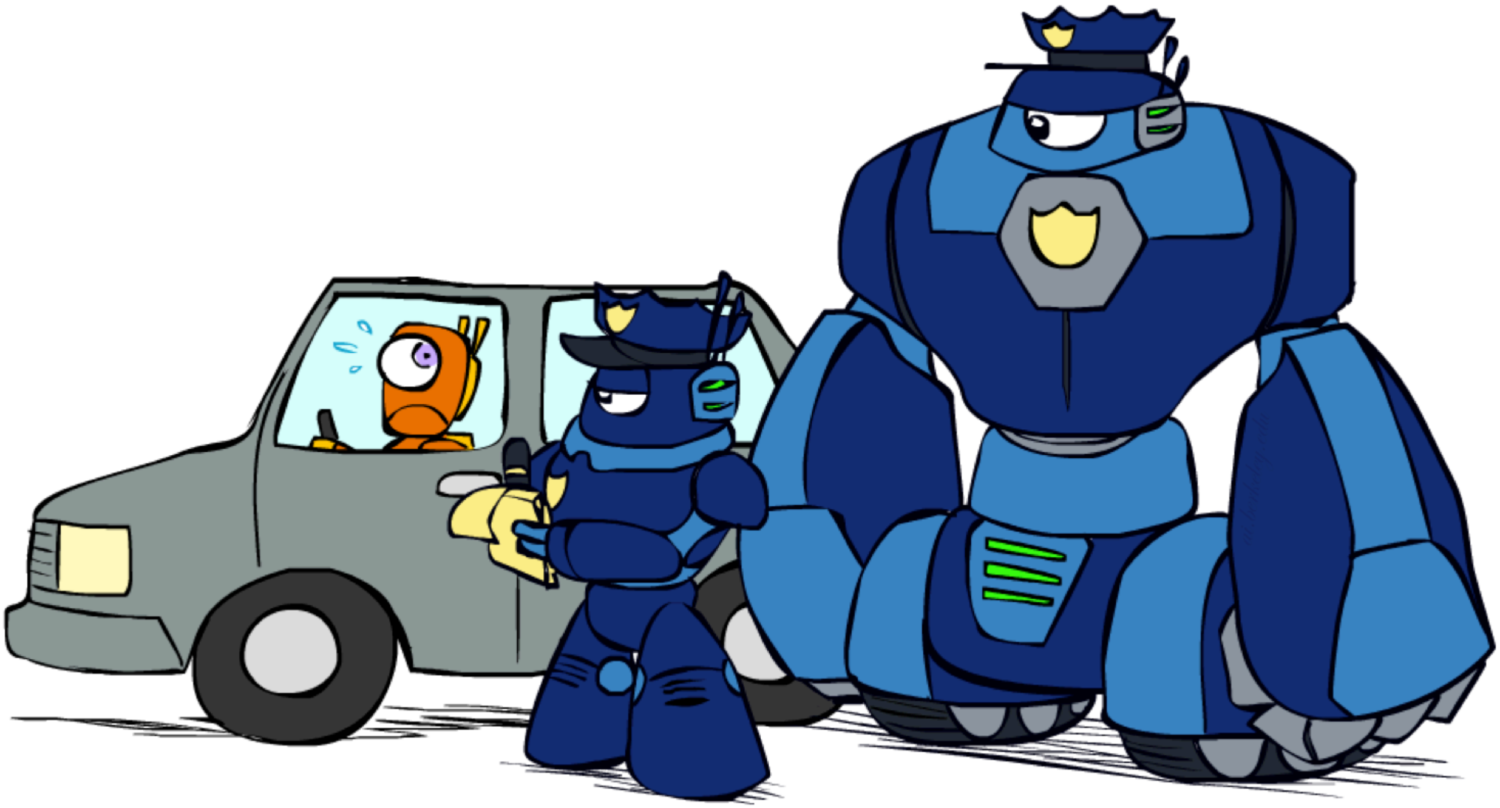
What went wrong here?

- Even with Arc Consistency you still need backtracking search!
  - Could run at even step of that search
  - Usually better to run it ***once, before search***

Video of Demo Coloring – Backtracking with Forward Checking – Complex Graph

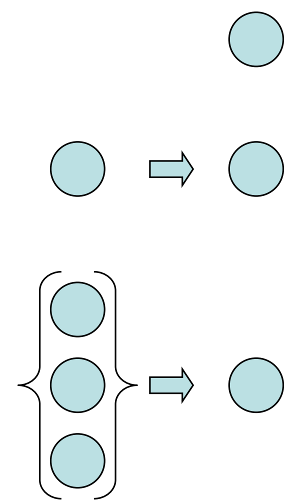
Video of Demo Coloring – Backtracking with Arc Consistency – Complex Graph

K-Consistency

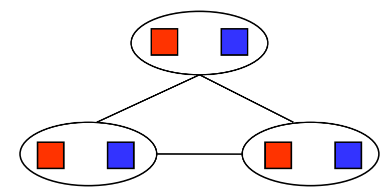


K-Consistency

- Increasing degrees of consistency
  - 1-Consistency (Node Consistency): Each single variable's domain has a value which meets that variables unary constraints
  - 2-Consistency (Arc Consistency): For each pair of variables, any consistent assignment to one can be extended to the other
  - 3-Consistency (Path Consistency): For every set of 3 vars, any consistent assignment to 2 of the variables can be extended to the third var
  - K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the k^{th} node.



- Higher k more expensive to compute
- (You need to know the algorithm for k=2 case: arc consistency)



Strong K-Consistency

- Strong k-consistency: also k-1, k-2, … 1 consistent
- Claim: strong n-consistency means we can solve without backtracking!
- Why?
  - Choose any assignment to any variable
  - Choose a new variable
  - By 2-consistency, there is a choice consistent with the first
  - Choose a new variable
  - By 3-consistency, there is a choice consistent with the first 2
  - …

Ordering

Backtracking Search

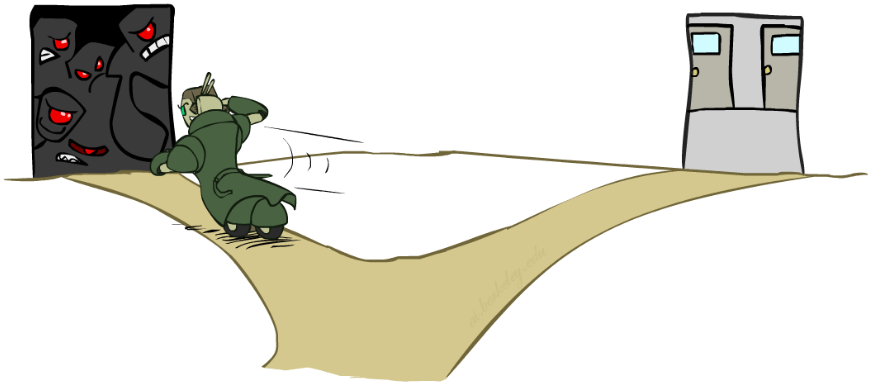
```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

Ordering: Minimum Remaining Values

- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

- Why min rather than max?
- Also called "most constrained variable"
- "Fail-fast" ordering



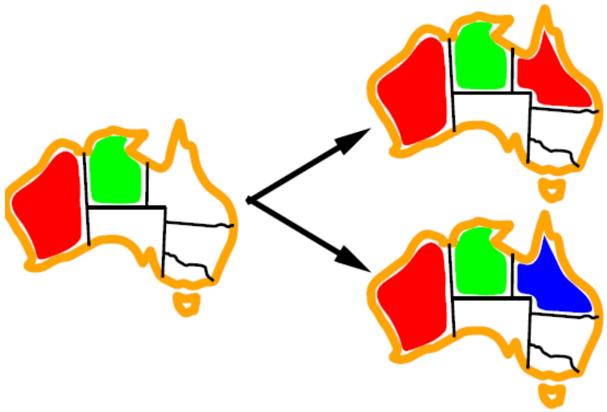
Ordering: Maximum Degree

- Tie-breaker among MRV variables
  - What is the very first state to color? (All have 3 values remaining.)
- Maximum degree heuristic:
  - Choose the variable participating in the most constraints on remaining variables

- Why most rather than fewest constraints?

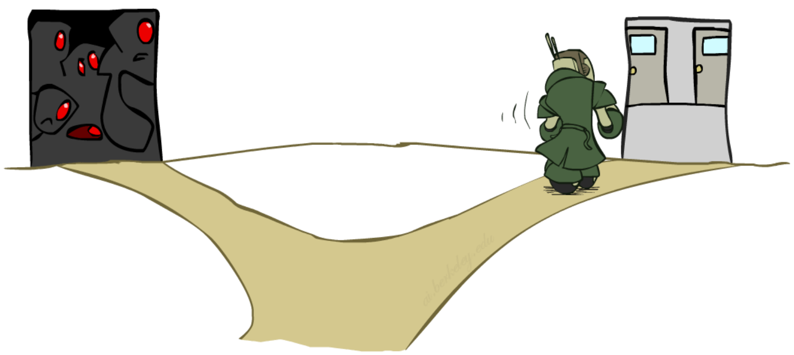
Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



- Why least rather than most?

- Combining these ordering ideas makes 1000 queens feasible

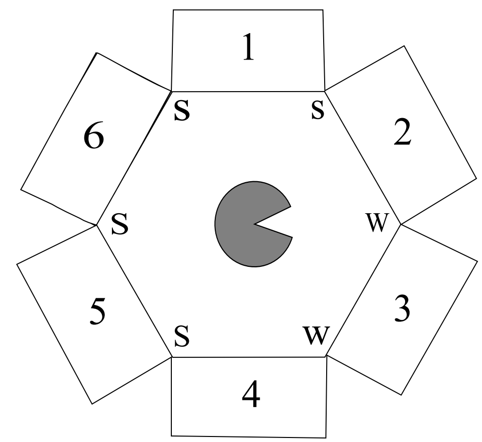


Rationale for MRV, MD, LCV

- We want to enter the most promising branch, but we also want to detect failure quickly
- MRV+MD:
  - Choose the variable that is most likely to cause failure
  - It must be assigned at some point, so if it is doomed to fail, better to find out soon
- LCV:
  - We hope our early value choices do not doom us to failure
  - Choose the value that is most likely to succeed

Trapped

- Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost(G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.
- The one sign of what lies behind the corridors is the wind: a pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all. Unfortunately, Pacman cannot measure the strength of the breeze at a specific corridor. Instead, he can stand between two adjacent corridors and feel the max of the two breezes. For example, if he stands between a pit and an exit he will sense a strong (S) breeze, while if he stands between an exit and a ghost, he will sense a weak (W) breeze. The measurements for all intersections are shown in the figure below.
- Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will not both be exits.



Variables?

Trapped

- Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost(G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.
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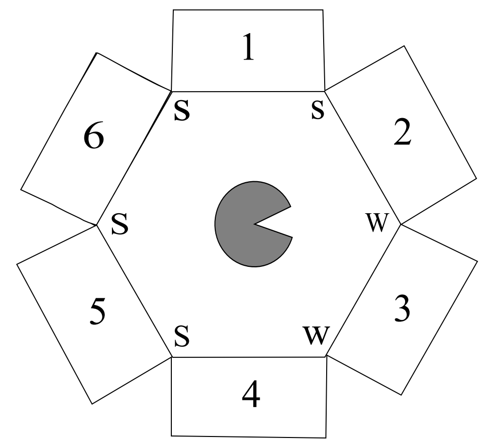

Variables? X_1, … X_6
Domains {P, G, E}

Trapped

- A pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all.
- Pacman feels the max of the two breezes.
- the total number of exits might be zero, one, or more,
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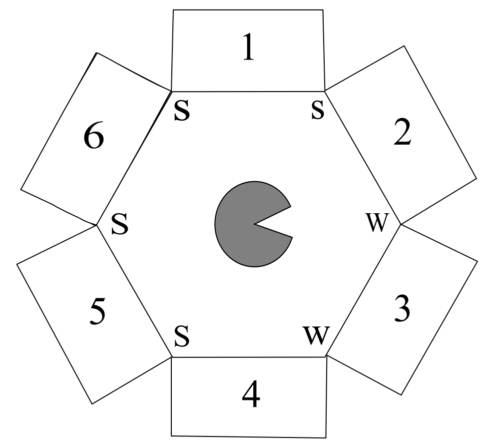

Constraints?

Variables? X_1, … X_6

Domains {P, G, E}

Trapped

- A pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all.
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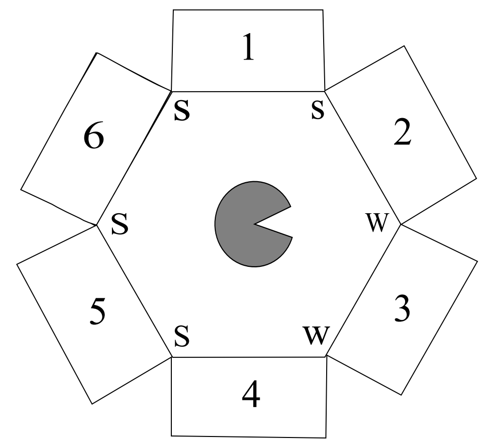

Constraints?

$X_1 = P$ or $X_2 = P$ $\quad$ $X_4 = P$ or $X_5 = P$

$X_2 = E$ or $X_3 = E$ $\quad$ $X_5 = P$ or $X_6 = P$

$X_3 = E$ or $X_4 = E$ $\quad$ $X_6 = P$ or $X_1 = P$

$X_i = E$ nand $X_{i+1|7} = E$

Also! $X_2 =/= P$
$X_3 =/= P$
$X_4 =/= P$

X_1	P	G	E
X_2	P	G	E
X_3	P	G	E
X_4	P	G	E
X_5	P	G	E
X_6	P	G	E

Trapped

- A pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all.
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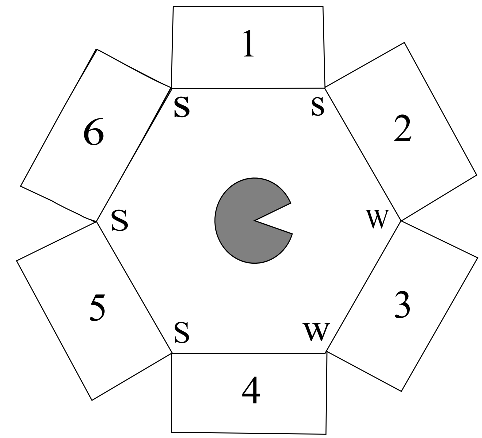

Constraints?

$X_1 = P$ or $X_2 = P$
$X_2 = E$ or $X_3 = E$
$X_3 = E$ or $X_4 = E$

$X_4 = P$ or $X_5 = P$
$X_5 = P$ or $X_6 = P$
$X_6 = P$ or $X_1 = P$

$X_i = E$ nand $X_{i+1|7} = E$

Also! X_2 =/= P
X_3 =/= P
X_4 =/= P

Arc consistent?

X_1	P	G	E
X_2	P	G	E
X_3	P	G	E
X_4	P	G	E
X_5	P	G	E
X_6	P	G	E

Trapped

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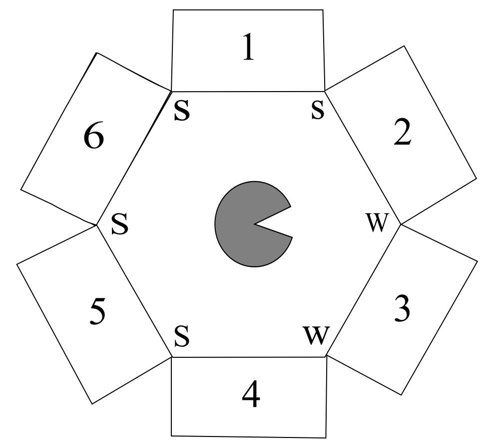


Constraints?

X_1 = P or X_2= P

X_2 = E or X_3= E

X_3 = E or X_4= E

X_4 = P or X_5= P

X_5 = P or X_6= P

X_6 = P or X_1= P

X_i = E nand $X_{i+1|7}$ = E

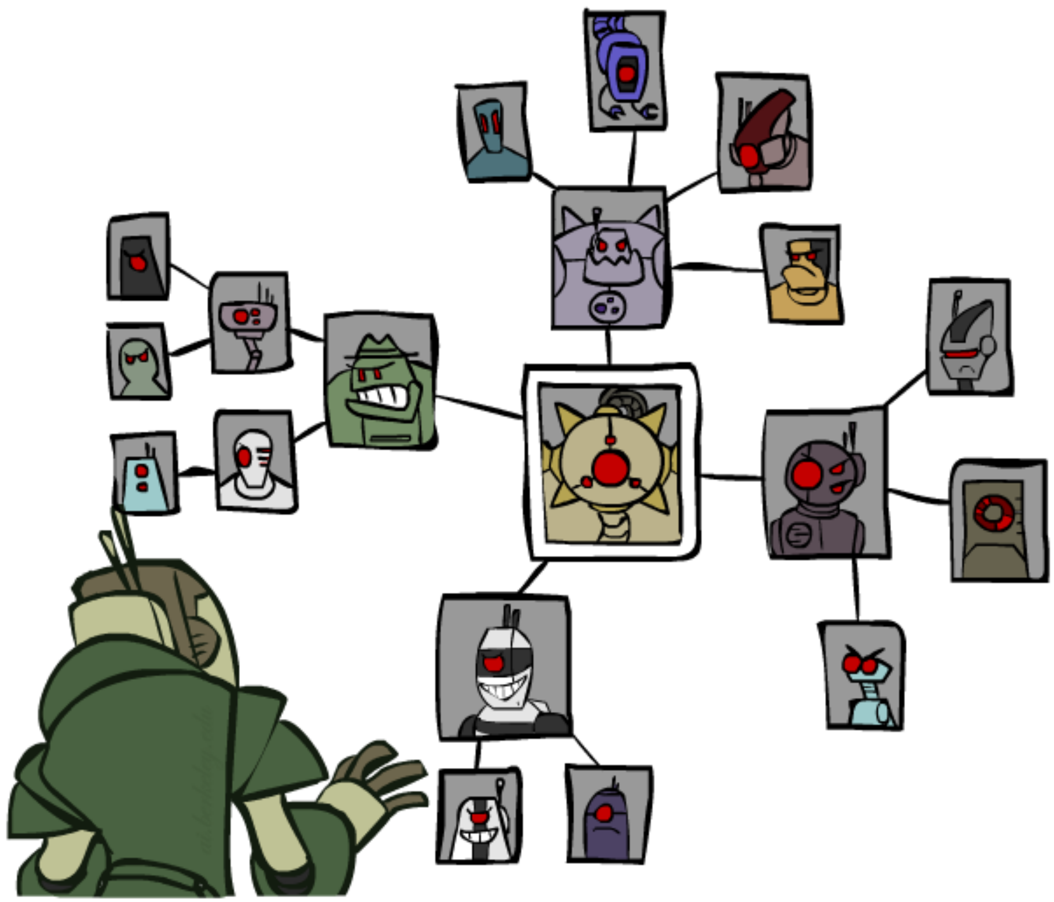
Also! X_2 =/= P
X_3 =/= P
X_4 =/= P

MRV heuristic?

Arc consistent?

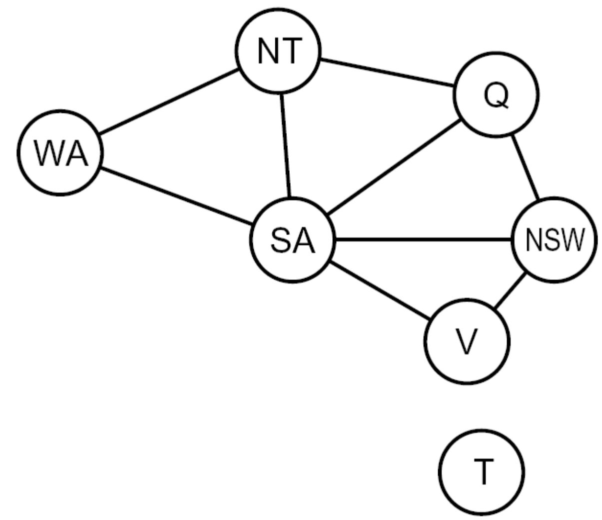
X_1	P	~~G~~	~~E~~
X_2	~~P~~	G	E
X_3	~~P~~	G	E
X_4	~~P~~	G	E
X_5	P	~~G~~	~~E~~
X_6	P	G	E

Structure

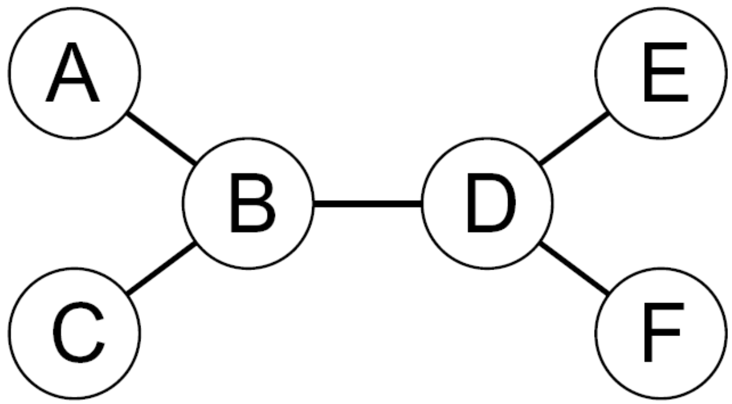


Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



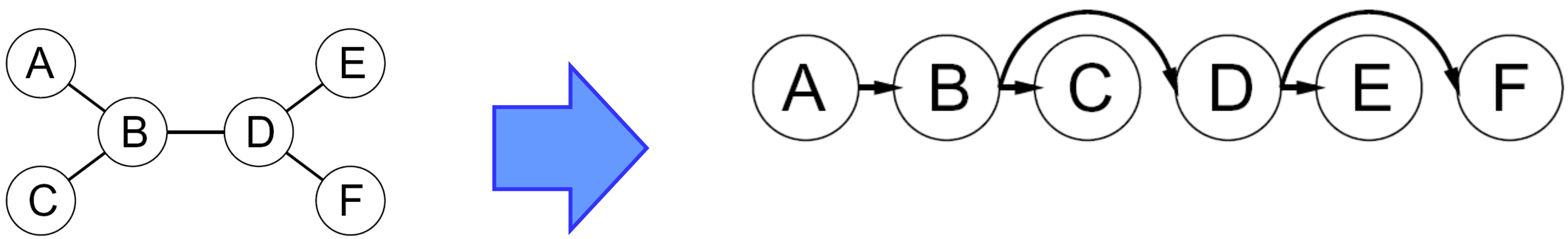
Tree-Structured CSPs



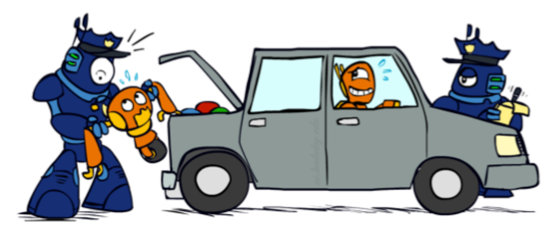
- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$
- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children

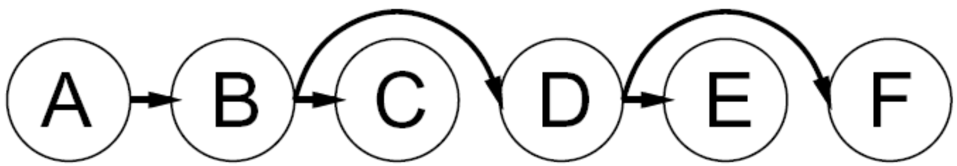


  - Remove backward: For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
  - Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)
- Runtime: O(n d^2) (why?)



Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent
- Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)



- Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
- Proof: Induction on position

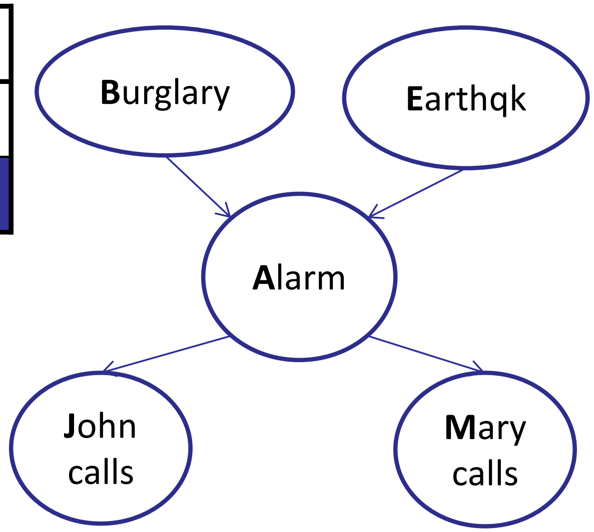
- Why doesn't this algorithm work with cycles in the constraint graph?

- Note: we'll see this basic idea again with Bayes' nets

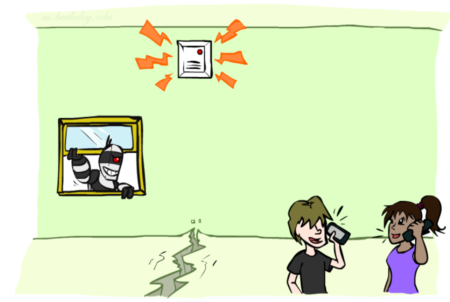
Connection to Bayes Nets

Bayes Net Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999



E	P(E)
+e	0.002
-e	0.998

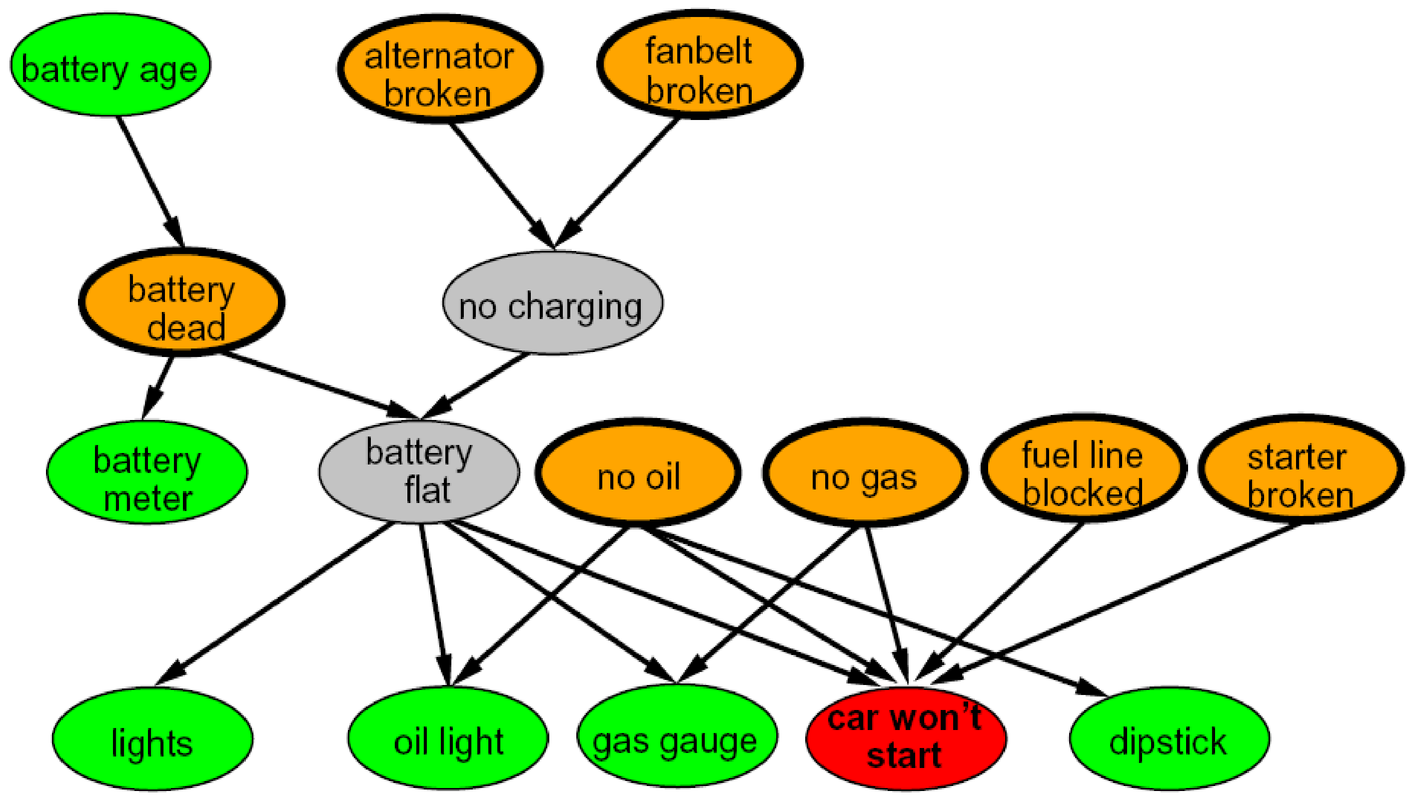


A	J	P(J|A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

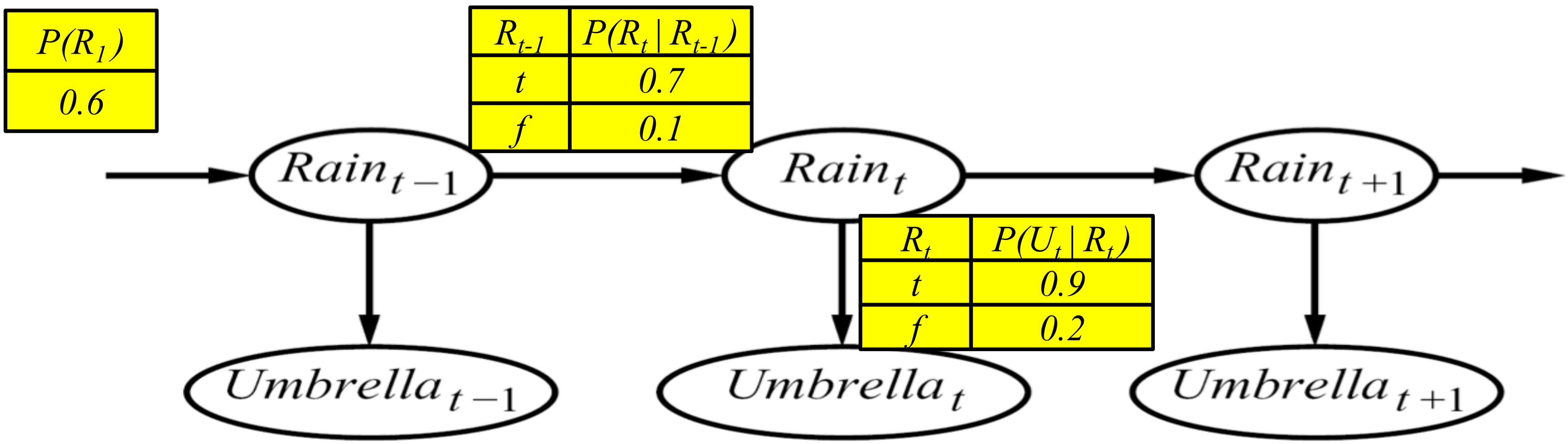
A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

More Complex Bayes' Net: Auto Diagnosis



Hidden Markov Model (Tree Structured)



- An HMM is defined by:
  - Initial distribution: $P(X_1)$
  - Transitions: $P(X_t|X_{t-1})$
  - Emissions: $P(E|X)$

Forward Algorithm


$B'(x_1=r) = 0.7$

$B'(x_2=r) = P(x_2=r \mid x_1=r)*0.875 + P(x_2=r \mid x_1=s)*0.125$
$= 0.8*0.875 + 0.6*0.125$
$= 0.775$

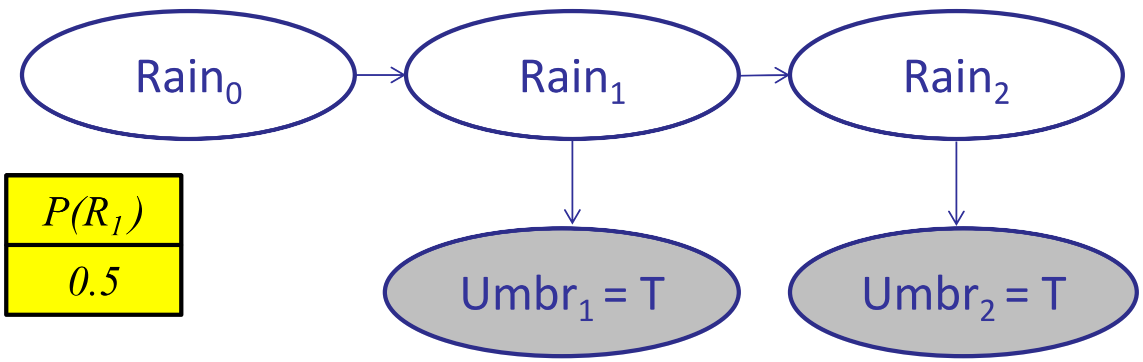

$B(x_1=r) \propto 0.9 * 0.775 = 0.6975$
$B(x_1=s) \propto 0.3 * 0.225 = 0.0675$

Divide by 0.765 to normalize

$B(x_0=r) = 0.5$ $B(x_1=r) = 0.875$ $B(x_1=r) = 0.912$

$$B'(X_{t+1}) = \sum_{x_t} P(X'|x_t)B(x_t)$$

$$B(X_{t+1}) \propto_{X_{t+1}} P(e_{t+1}|X_{t+1})B'(X_{t+1})$$



$P(R_1)$
0.5

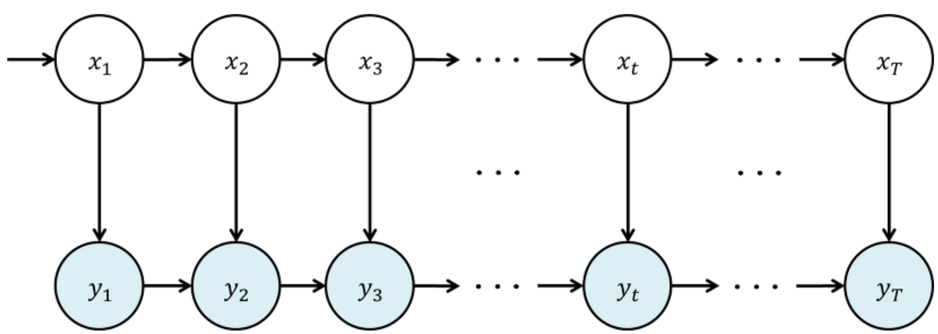
R_{t-1}	$P(R_t \mid R_{t-1})$
t	0.8
f	0.6

R_t	$P(U_t \mid R_t)$
t	0.9
f	0.3

More Complex HMM Inference

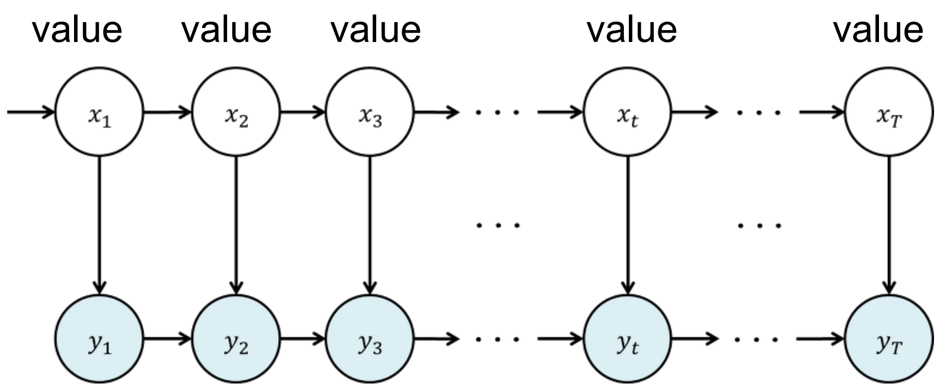
- Forward Backward
  - Computes marginal probabilities of ***all*** hidden states given sequence of observations

P(x_t = value)

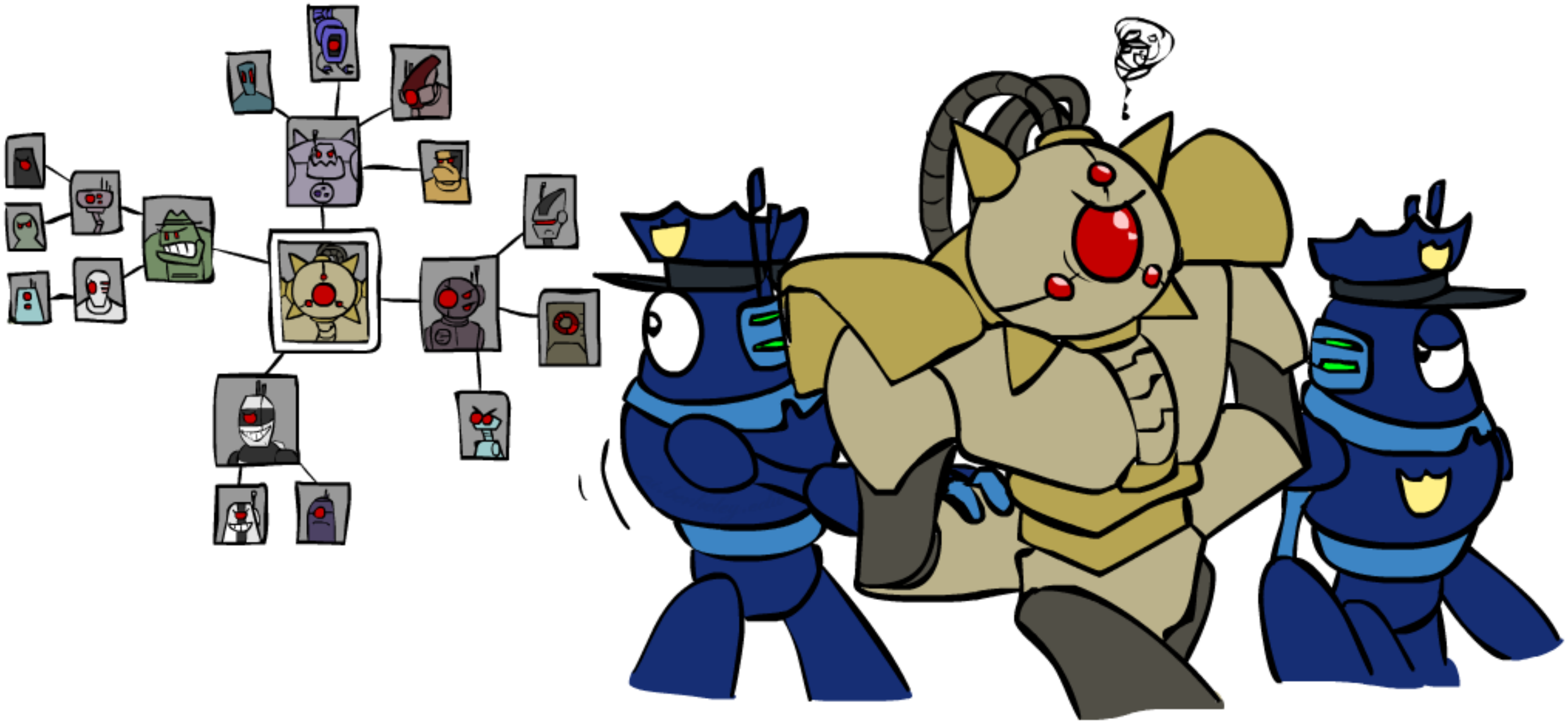


More Complex HMM Inference

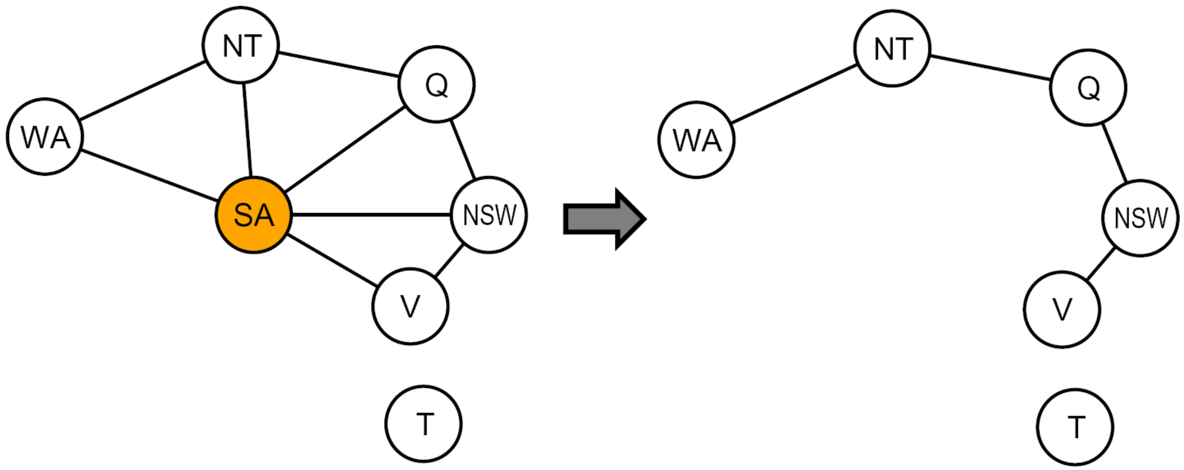
- Forward Backward
  - Computes marginal ***probabilities*** of all hidden states given sequence of observations
- Viterbi
  - Computes most likely ***sequence of states***



Improving Structure



Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c) (n-c) d^2)$, very fast for small c

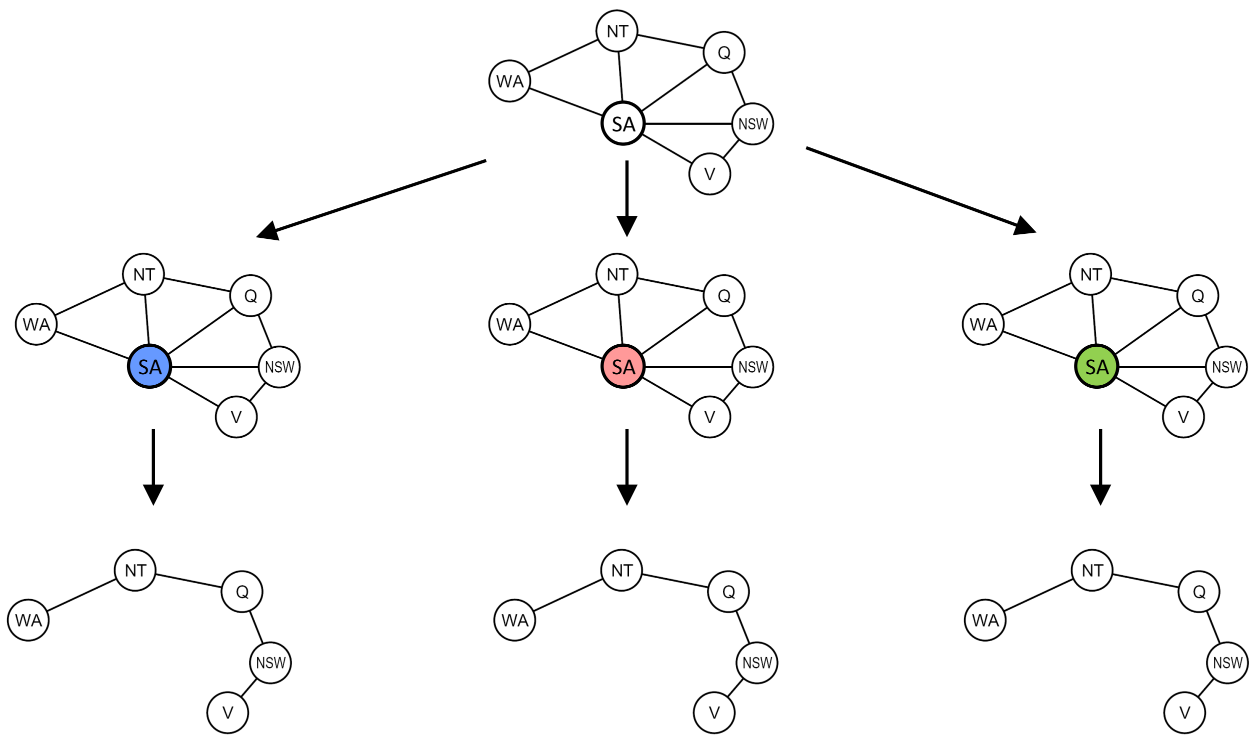
Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

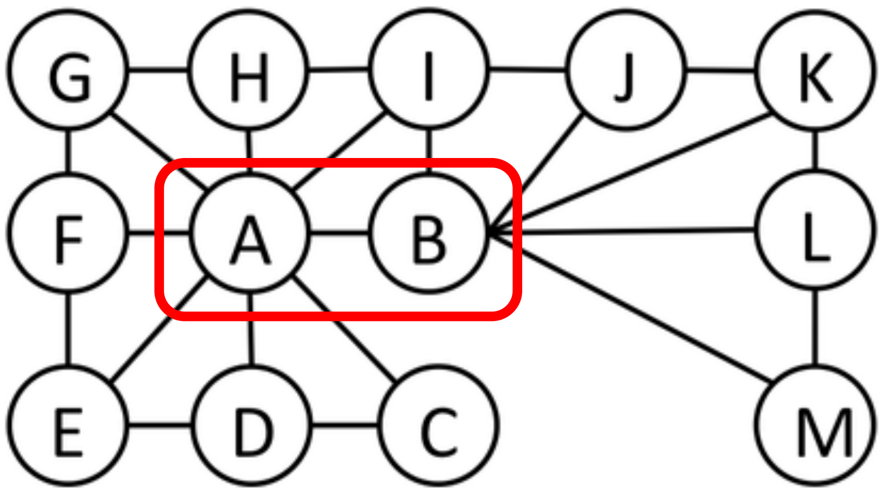
Compute residual CSP for each assignment

Solve the residual CSPs (tree structured)

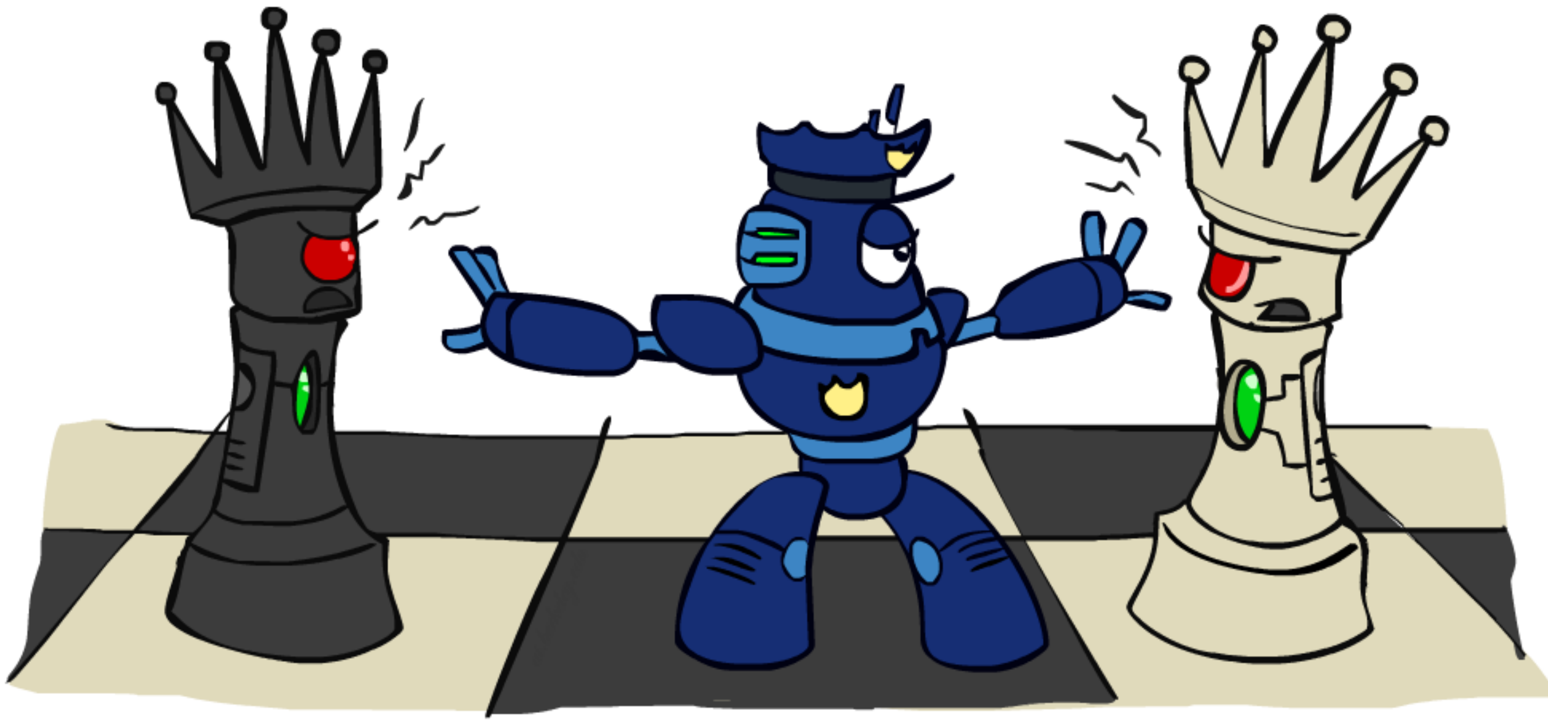


Cutset Quiz

- Find the smallest cutset for the graph below.

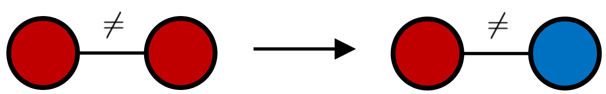


Local Search for CSPs



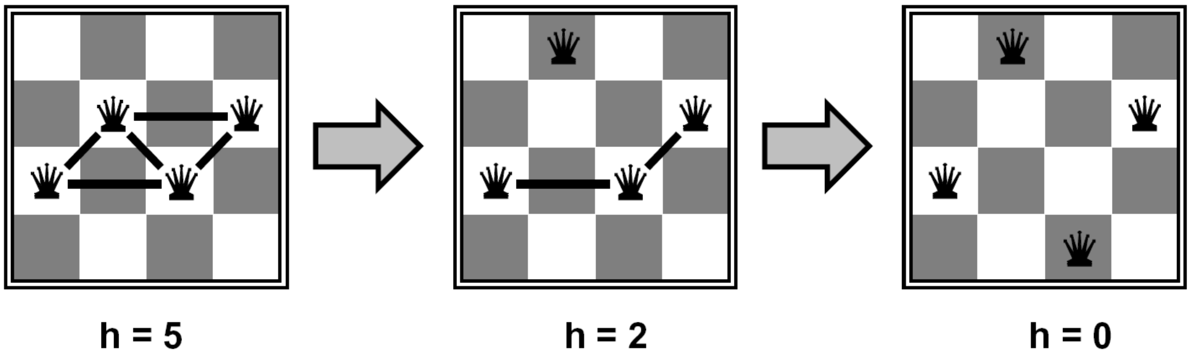
Iterative Algorithms for CSPs

- Local search methods typically work with "complete" states, i.e., all variables assigned
- To apply to CSPs:
  - Take an assignment with unsatisfied constraints
  - Operators *reassign* variable values
  - No fringe! Live on the edge.



- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - I.e., hill climb with h(n) = total number of violated constraints

Example: 4-Queens



- States: 4 queens in 4 columns (4^4 = 256 states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: c(n) = number of attacks

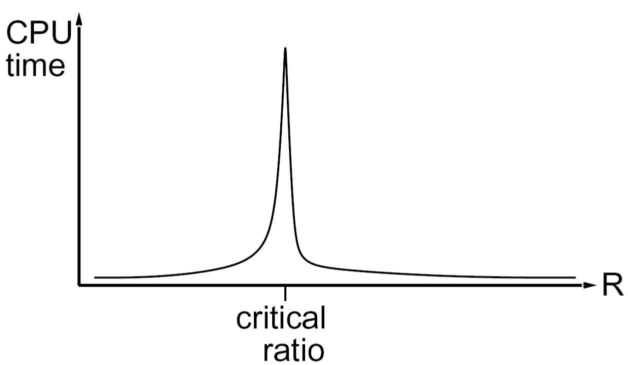
[Demo: n-queens – iterative improvement (L5D1)]
[Demo: coloring – iterative improvement]

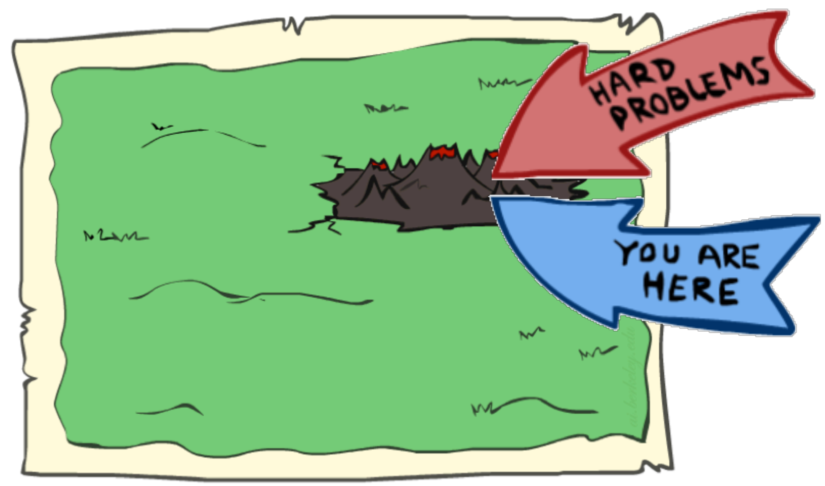
Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!

- The same appears to be true for any ***randomly-generated*** CSP *except* in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$





Summary: CSPs

- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constraints
- Basic solution: backtracking search
- Speed-ups:
  - Ordering
  - Filtering
  - Structure (cutset conditioning)
- Iterative min-conflicts is often effective in practice