CSE 573: Artificial Intelligence

Adversarial Search

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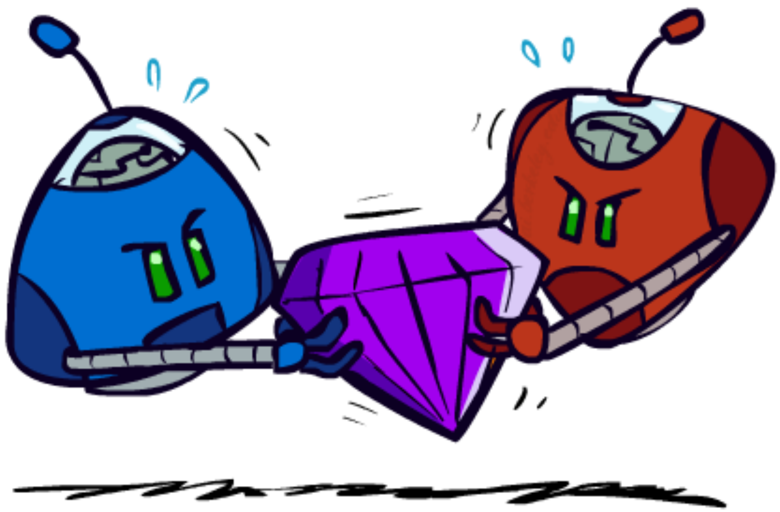


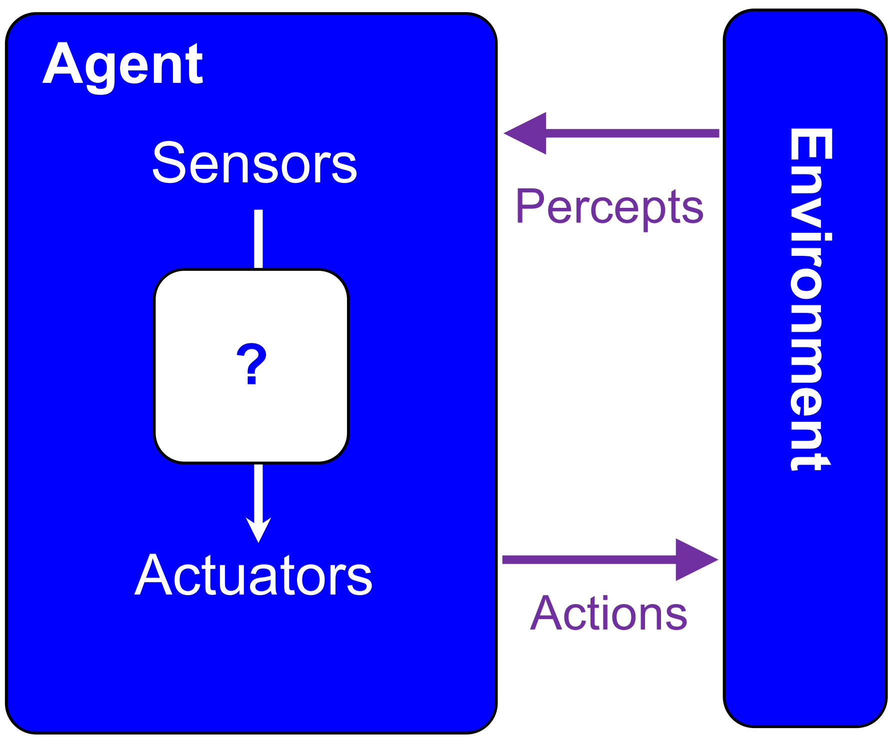
Based on slides from

Dan Klein, Stuart Russell, Pieter Abbeel, Andrew Moore and Luke Zettlemoyer

(best illustrations from ai.berkeley.edu)

Types of Environments

- Fully observable *vs.* partially observable
- Single agent *vs.* ***multi-agent***
- Deterministic *vs.* ***stochastic***
- Episodic *vs.* sequential
- Discrete *vs.* continuous

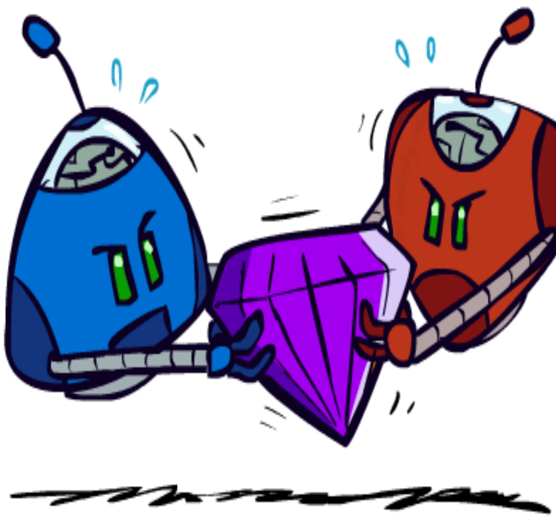


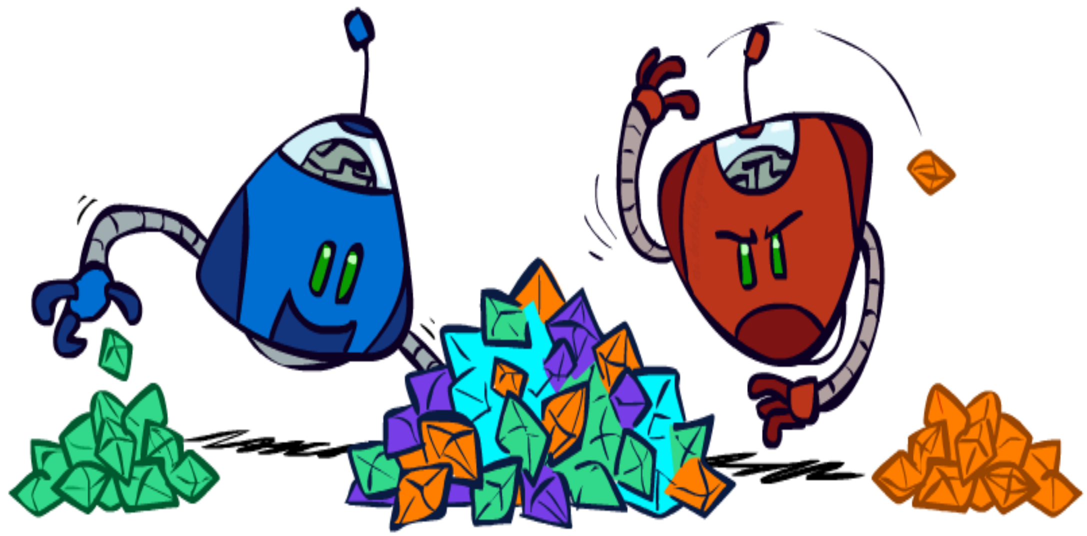
Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon, monopoly
imperfect information	stratego	bridge, poker, scrabble, nuclear war

Number of Players? 1, 2, …?

Zero-Sum Games





- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, & more are possible
  - More later on non-zero-sum games

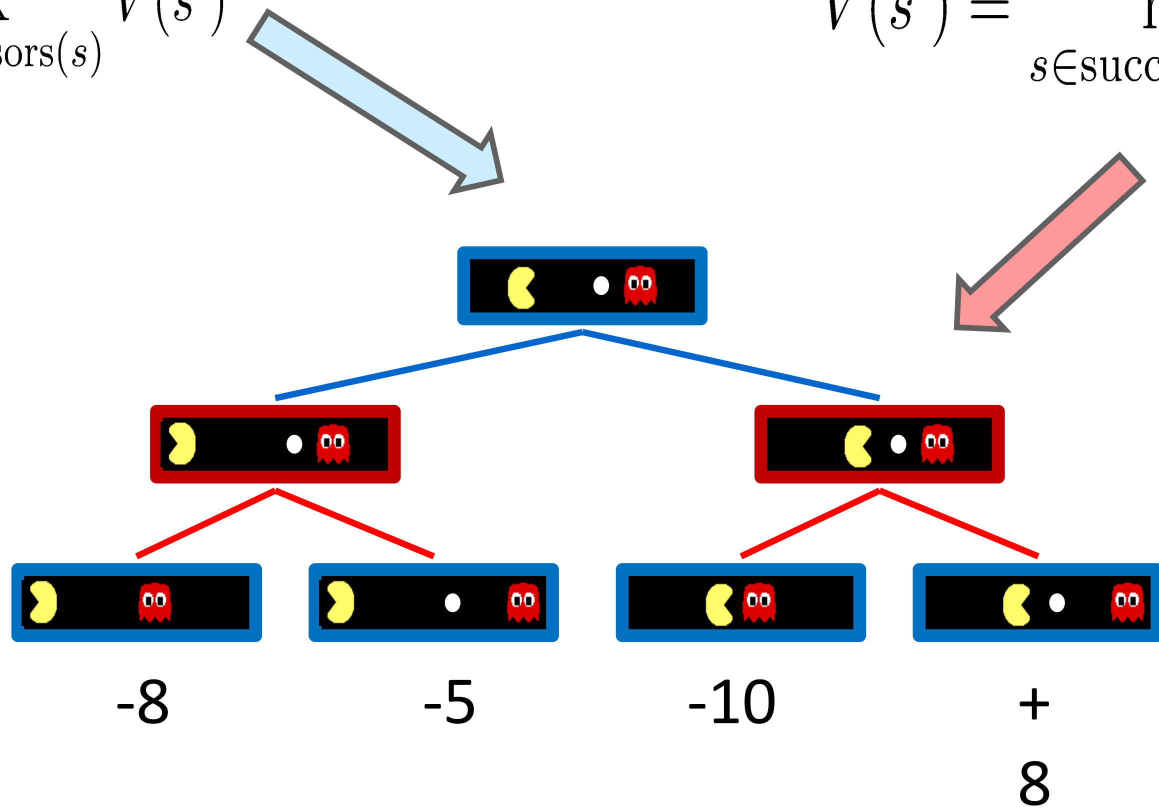
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

$$V(s) = \text{known}$$

Minimax Implementation

Need ***Base case*** for recursion

```
def max-value(state):
    if leaf?(state), return U(state)
    initialize v = -∞
    for each c in children(state)
        v = max(v, min-value(c))
    return v
```

```
def min-value(state):
    if leaf?(state), return U(state)
    initialize v = +∞
    for each c in children(state)
        v = min(v, max-value(c))
    return v
```

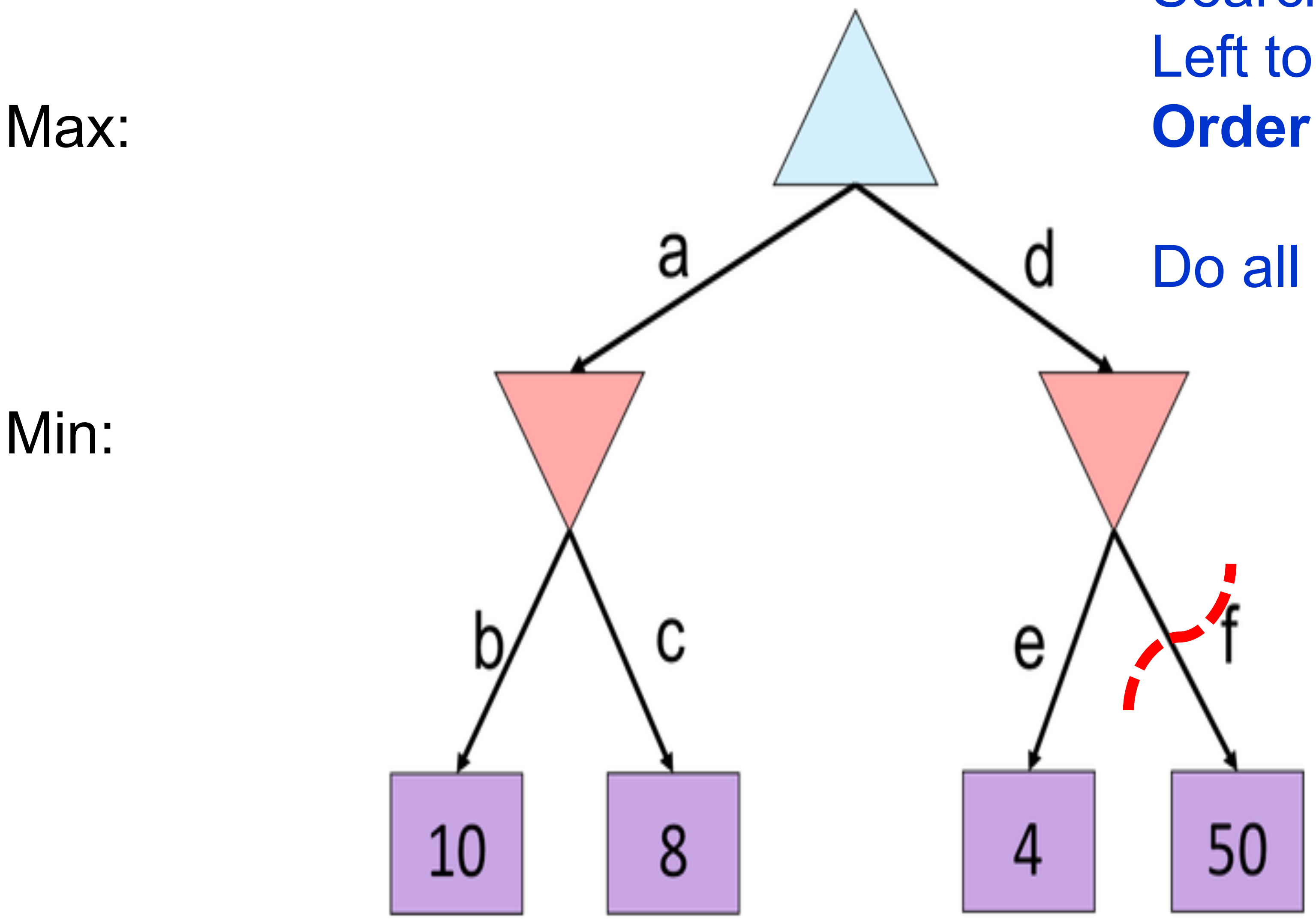
$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Alpha-Beta Quiz

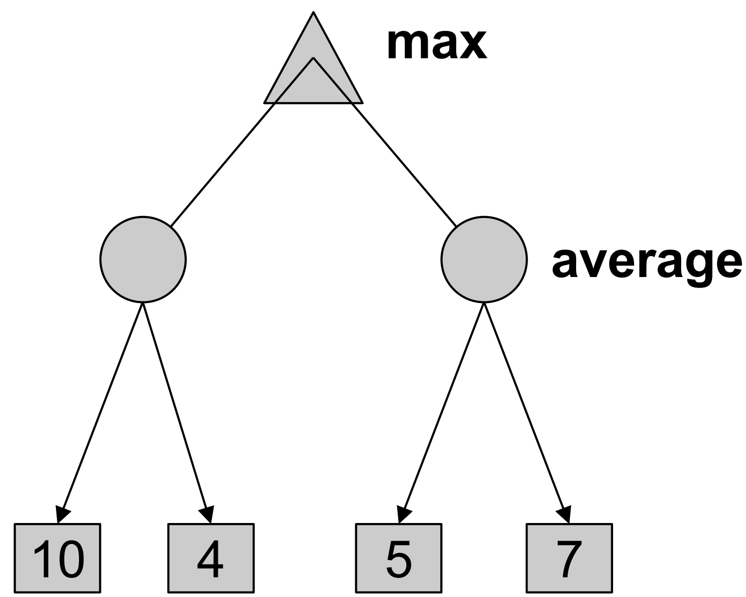
Search depth-first
Left to right
Order is important

Do all nodes matter?



Stochastic Single-Player

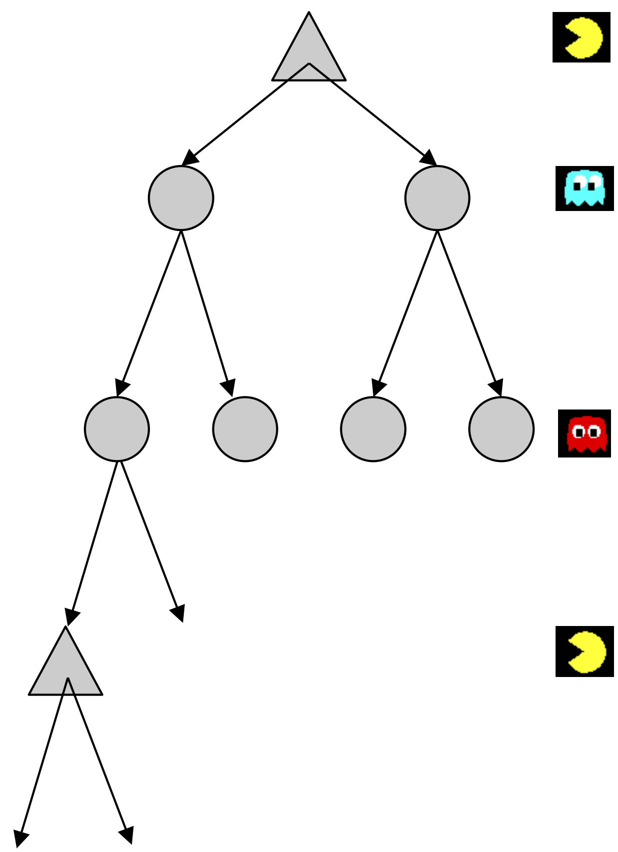
- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, shuffle is unknown
  - In minesweeper, mine locations
- Can do **expectimax search**
  - Chance nodes, like actions except the environment controls the action chosen
  - Max nodes as before
  - Chance nodes take average (expectation) of value of children



ExpectiMax Search

In ExpectiMax search, we have a probabilistic model of how the opponent (or environment) will behave in any state

- Model could be a simple uniform distribution (roll a die)…
  or more complex
- We have a node for every outcome out of our control: opponent or environment



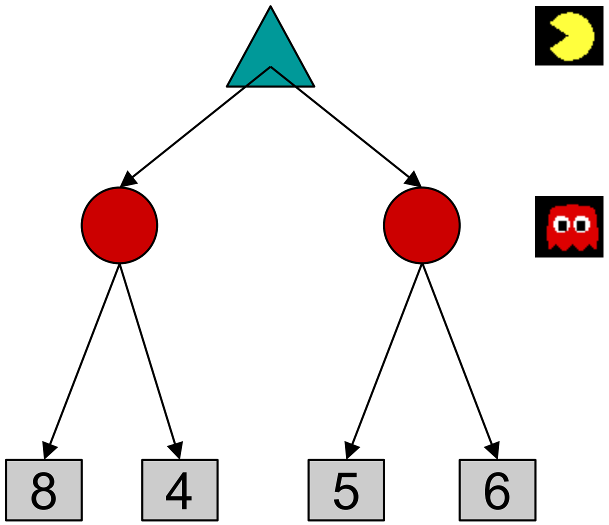
For now, assume ∀ states we magically have a distribution to assign probabilities to enemy-actions / environment outcomes

Expectimax Pseudocode

```
def value(s)
    if s is a max node return maxValue(s)
    if s is an exp node return expValue(s)
    if s is a terminal node return evaluation(s)

def maxValue(s)
    values = [value(s’) for s’ in successors(s)]
    return max(values)

def expValue(s)
    values = [value(s’) for s’ in successors(s)]
    weights = [probability(s, s’) for s’ in successors(s)]
    return expectation(values, weights)
```



ExpectiMax for Pacman

- Note: that we've gotten away from thinking that the ghosts are trying to minimize pacman's score
- Instead, they are now a part of the environment
- Pacman has a belief (distribution) over how they will act

- Quiz: Can we see minimax as a special case of expectimax?
- Quiz: what would pacman's computation look like if we assumed that the ghosts were doing 1-ply minimax and taking the result 80% of the time, otherwise moving randomly?

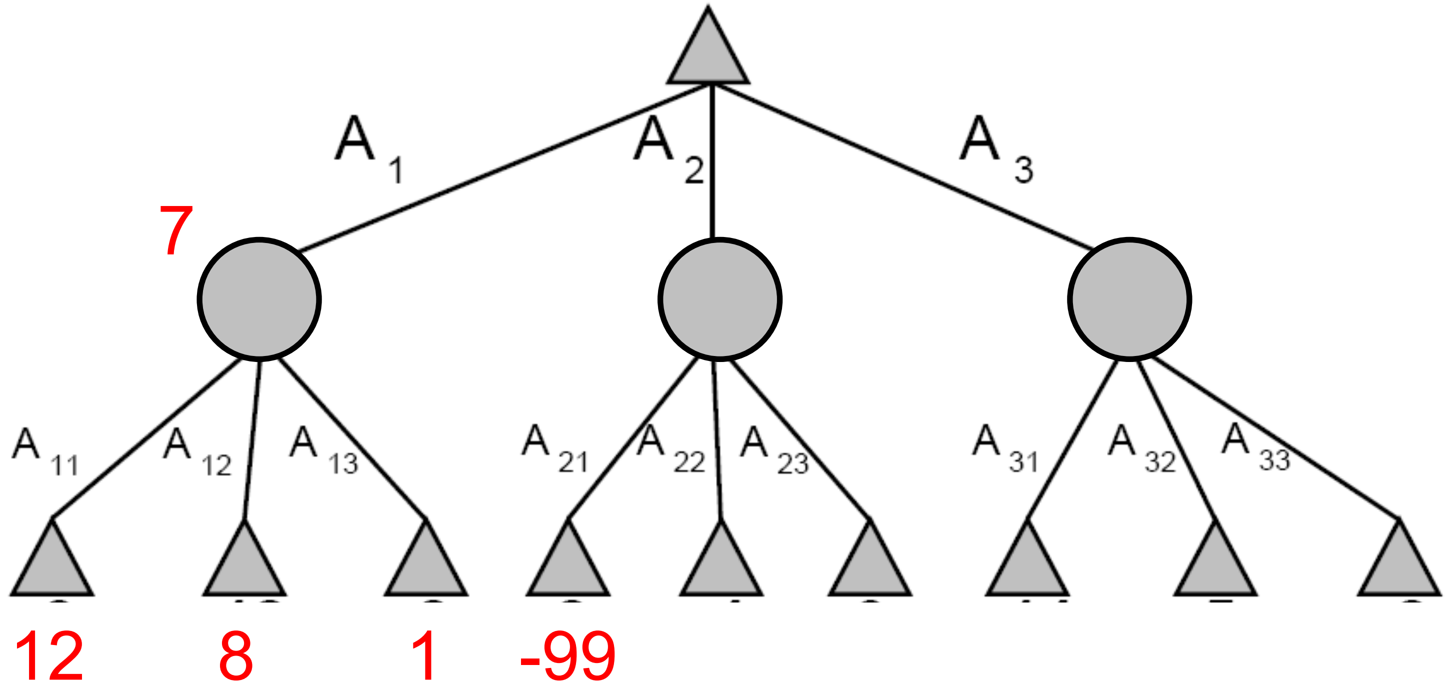
Expectimax for Pacman

Results from playing 5 games

	Minimizing Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 493	Won 5/5 Avg. Score: 483
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Pacman does depth 4 search with an eval function that avoids trouble
Minimizing ghost does depth 2 search with an eval function that seeks Pacman

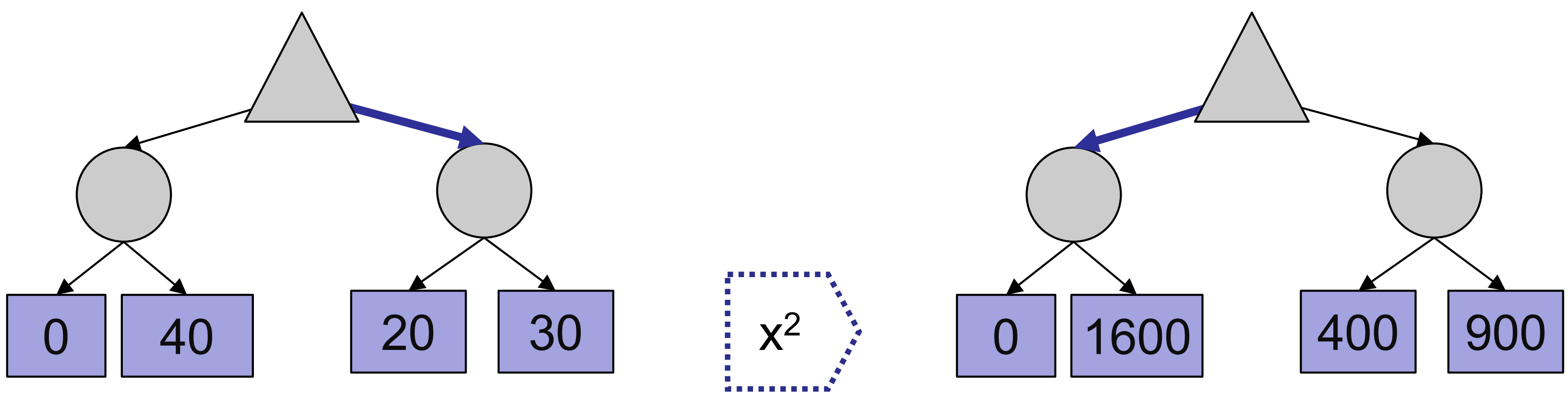
ExpectiMax Pruning?



- Not easy
  - exact: need bounds on possible values
  - approximate: sample high-probability branches

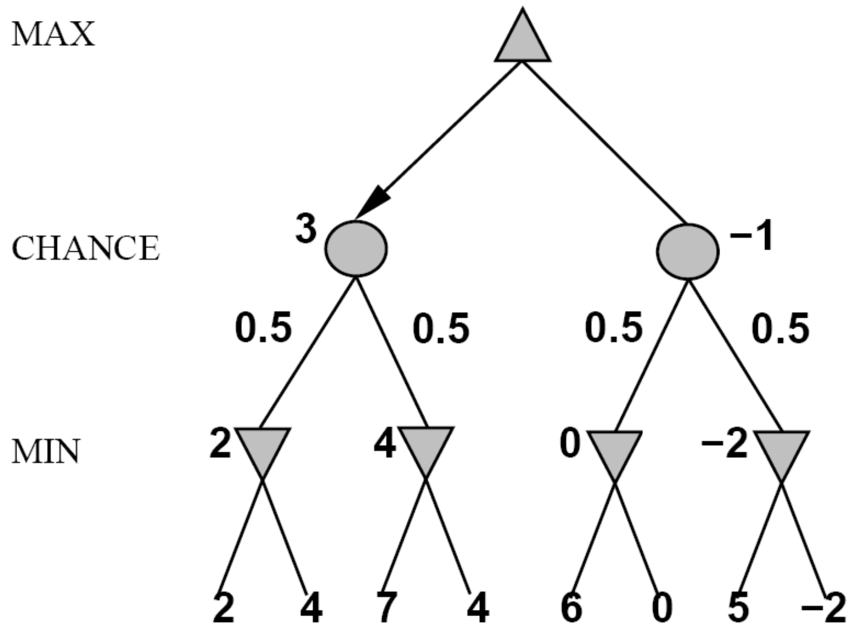
ExpectiMax Evaluation

- Evaluation functions quickly return an estimate for a node's true value (which value, expectimax or minimax?)
- For minimax, evaluation function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For expectimax, we need *magnitudes* to be meaningful



Mixed Layer Types

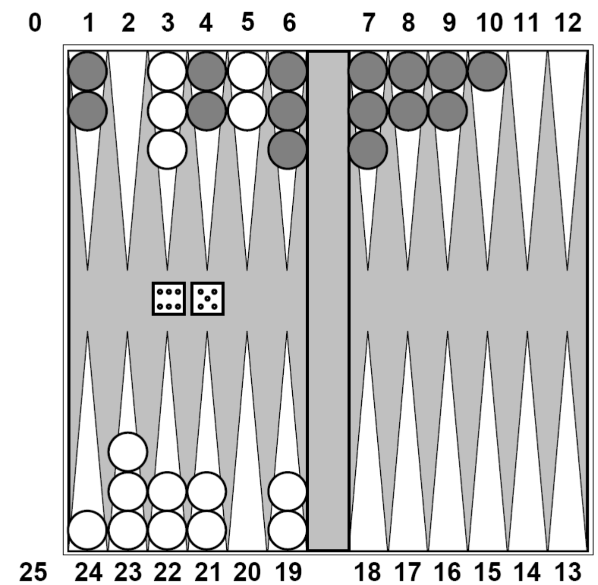
- E.g. Backgammon
- Expecti-Mini-Max
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



if *state* is a MAX node **then**
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 = 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…

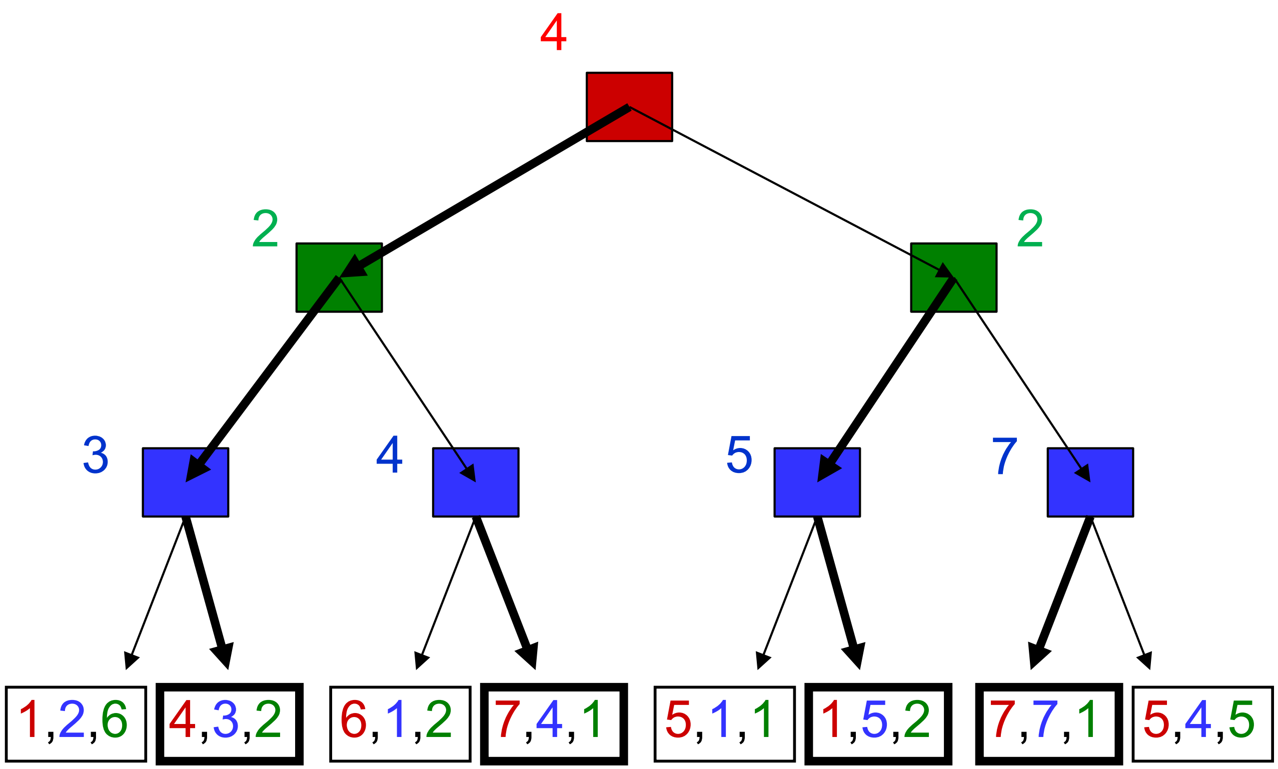


- TDGammon used depth-2 search + very good eval function
  - Learned via NN & reinforcement learning
  - World-champion level play (1992, Gerald Tesauro)

Multi-player Non-Zero-Sum Games

Similar to minimax:

- Utilities are now tuples
- Each player maximizes their own entry at each node
- Propagate (or back up) nodes from children
- Can give rise to cooperation and competition dynamically…



In this example… three agents