CSE 573: Artificial Intelligence

Problem Spaces & Search

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With slides from
Dan Klein, Stuart Russell, Andrew Moore, Luke Zettlemoyer, Dana Nau…

Outline

- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Iterative Deepening Search
  - Uniform-Cost Search
- Heuristic Search Methods
  - Uniform Cost
  - Greedy
  - A*
  - IDA*
- Heuristic Generation

Search thru a
Problem Space (aka State Space)

- Input:
  - Set of states
  - Operators *[and costs]*
  - Start state
  - Goal state *[or test]*

Functions: States → States

Aka "Successor Function"

- Output:
  - Path: start ⇒ a state satisfying goal test

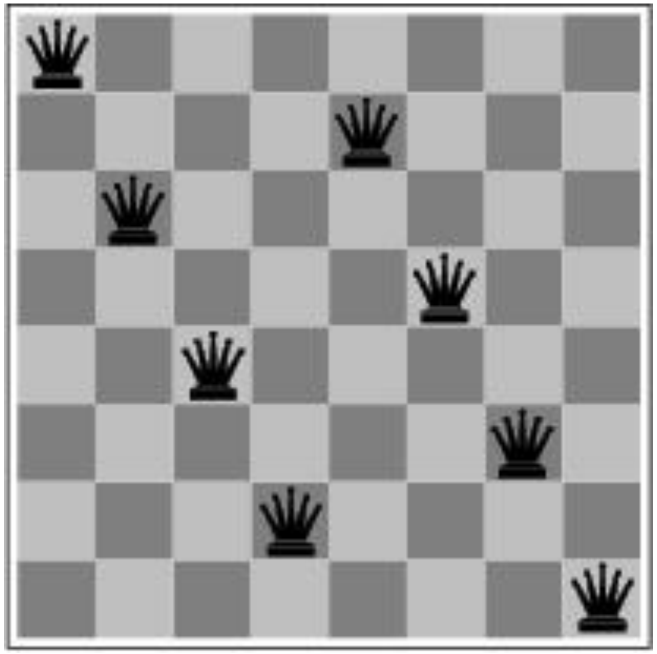
    [May require shortest path]

    [Sometimes just need a state that passes test]

N Queens Problem

Place N queens so they don't attack each other (same row, same col, same diagonal)

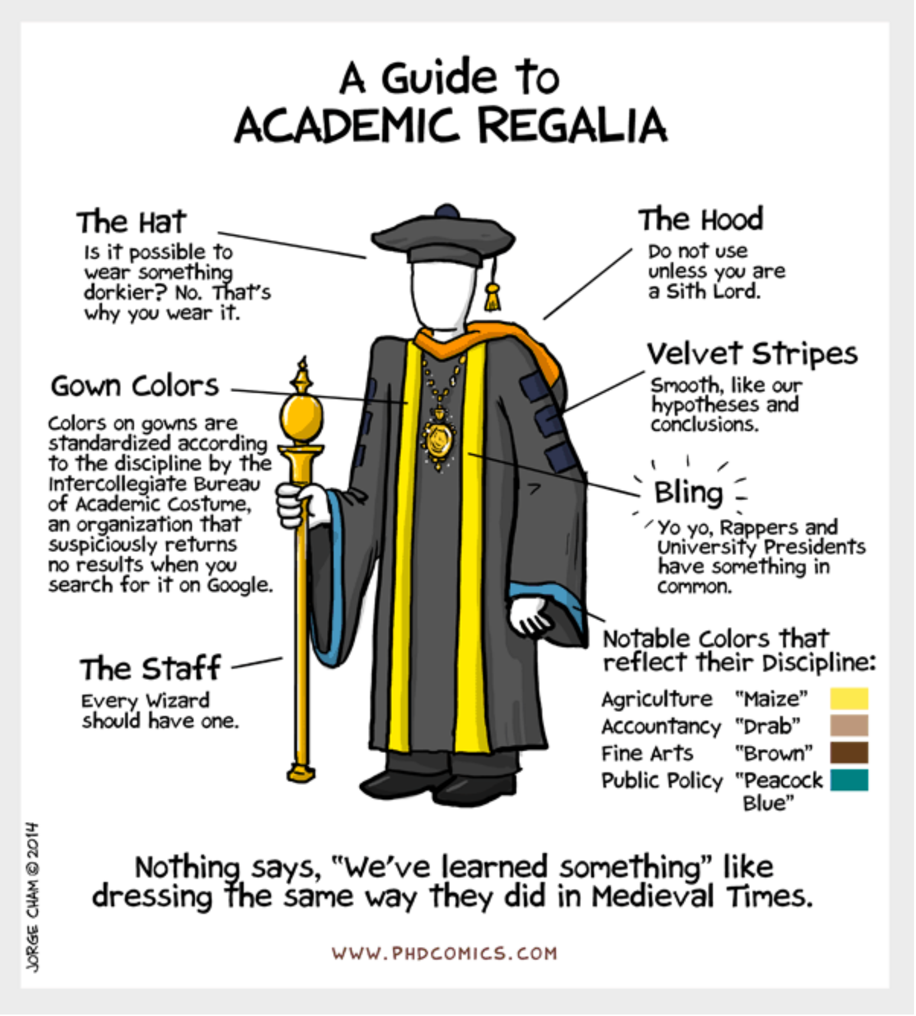
- States
  Chess board with
  0 or more queens
- Operators
  Add a queen
- Initial
  No queens
- Goal
  N queens



Getting a PhD In CSE

Input:

- Set of states
- Operators [costs]
- Start state
- Goal state (test)



Best-First Search

- Generalization of breadth-first search
- Fringe = ***Priority*** queue of nodes to be explored
- Ranking function ***f(n)*** applied to each node

```
Add initial state to priority queue
While queue not empty
        Node = head(queue)
        If goal?(node) then return node
        Add new children of node to queue, sorted
```

"expanding the node"

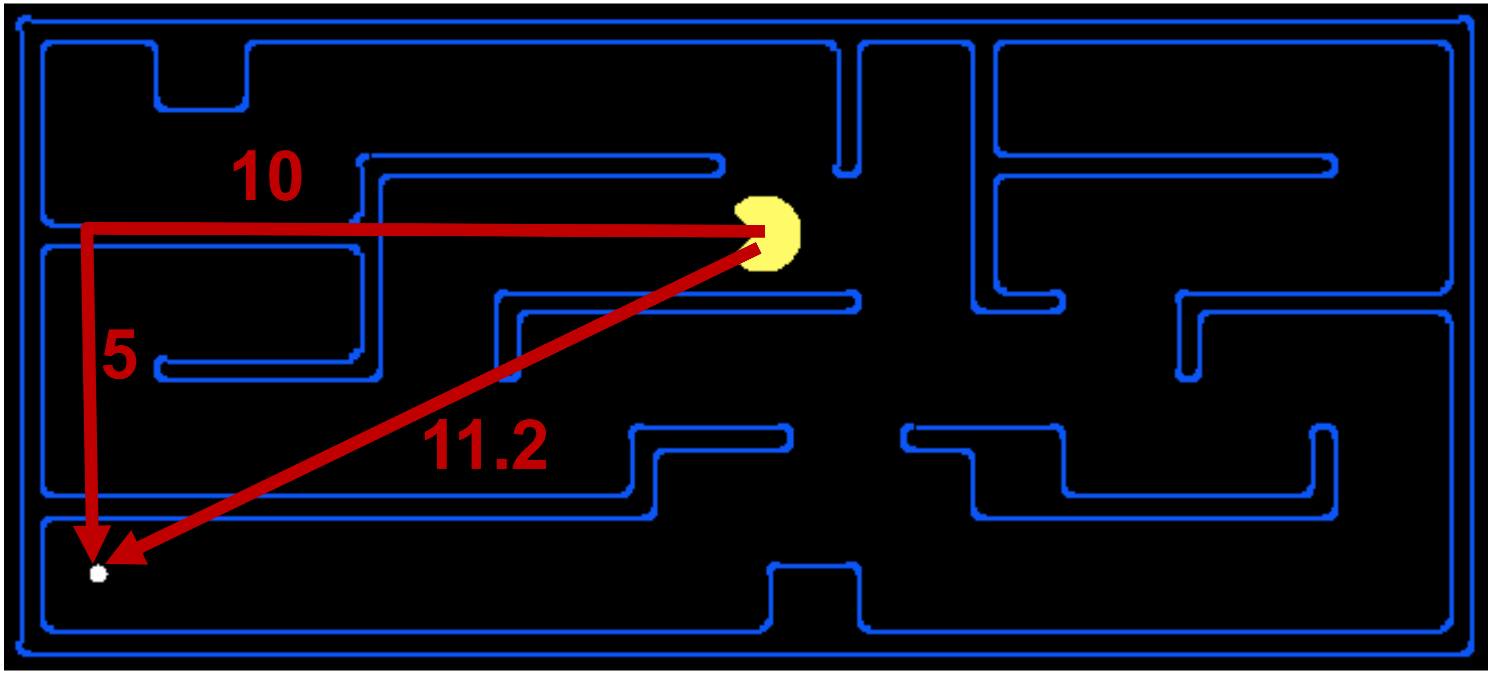
Old Friends

- Breadth First =
  - Best First
  - with f(n) = depth(n)

- Dijkstra's Algorithm (Uniform cost) =
  - Best First
  - with f(n) = the sum of edge costs from start to n

What is a ***Heuristic***?

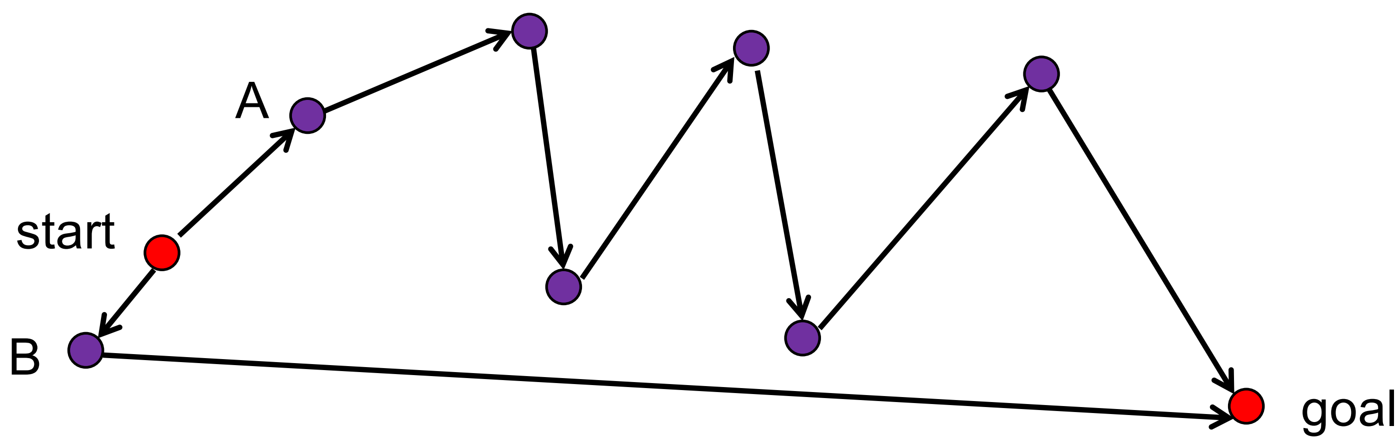
- An ***estimate*** of how close a state is to a goal
- Designed for a particular search problem



- Examples: Manhattan distance: 10+5 = 15
  Euclidean distance: 11.2

Greedy Search

Expand the node that seems closest…



What can go wrong?

A* Search

Hart, Nilsson & Rafael 1968

Best first search with $f(n) = g(n) + h(n)$

- $g(n)$ = sum of costs from start to n
- $h(n)$ = estimate of lowest cost path n $\rightarrow$ goal

  h(goal) = 0

Can view as cross-breed:

g(n) ~ uniform cost search

h(n) ~ greedy search

Best of both worlds…

A* Search

Hart, Nilsson & Rafael 1968

Best first search with $f(n) = g(n) + h(n)$

- $g(n)$ = sum of costs from start to n
- $h(n)$ = estimate of lowest cost path n → goal

  $h(goal) = 0$

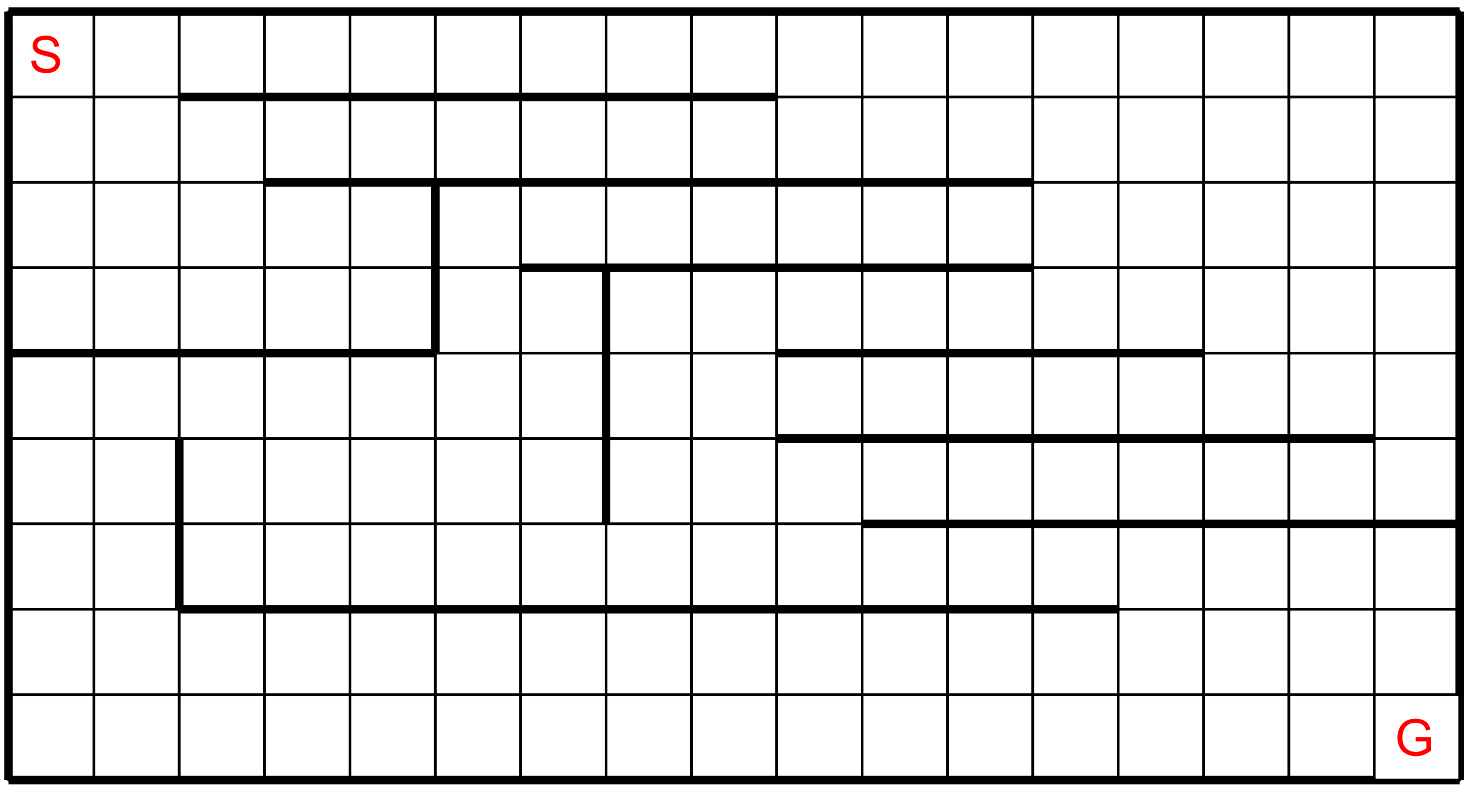
If h(n) is admissible and monotonic
then A* is ***optimal***

Underestimates (≤) cost of reaching goal from node

f values never decrease From node to descendants (triangle inequality)

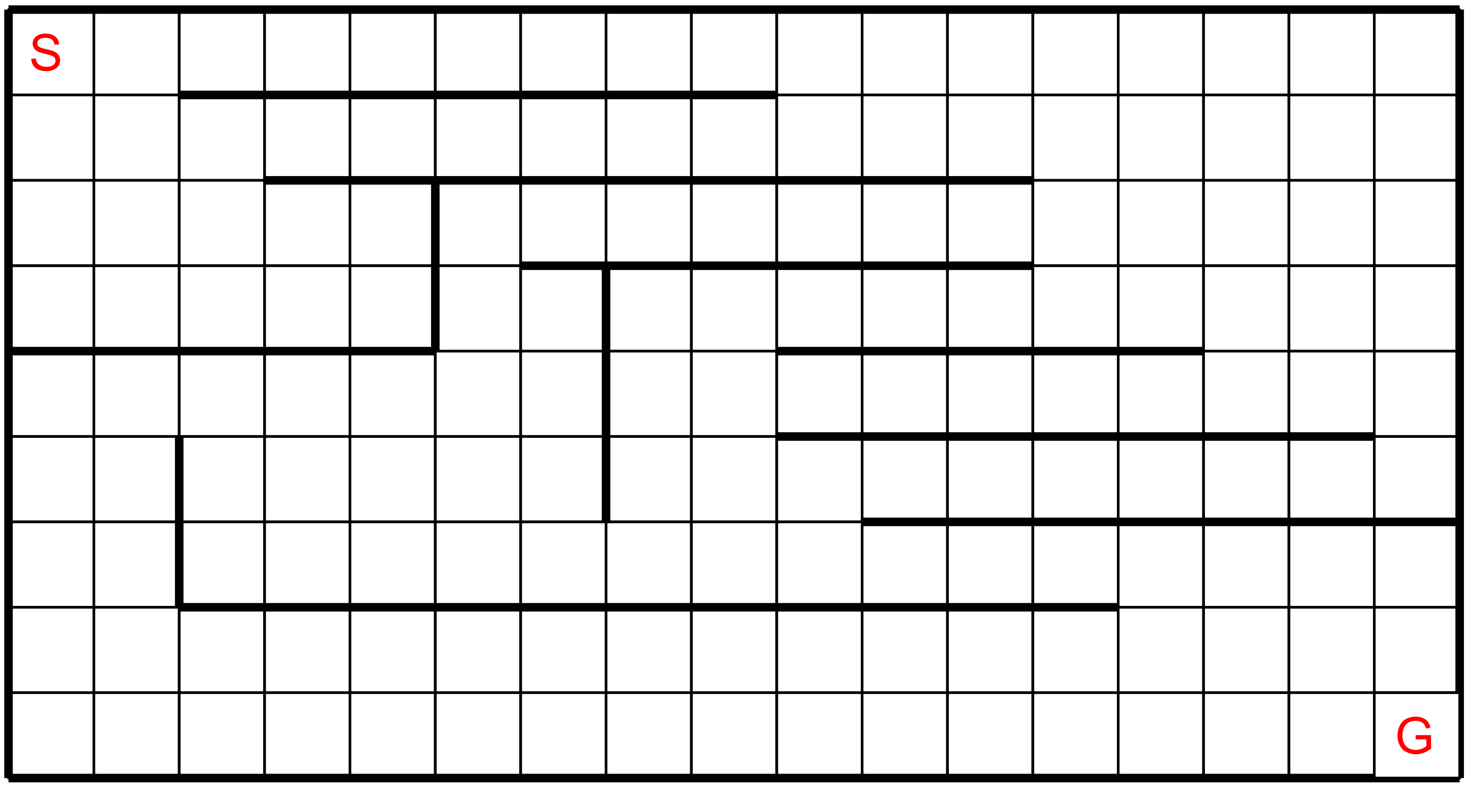
Is Manhattan distance **admissible**?

- Underestimate?

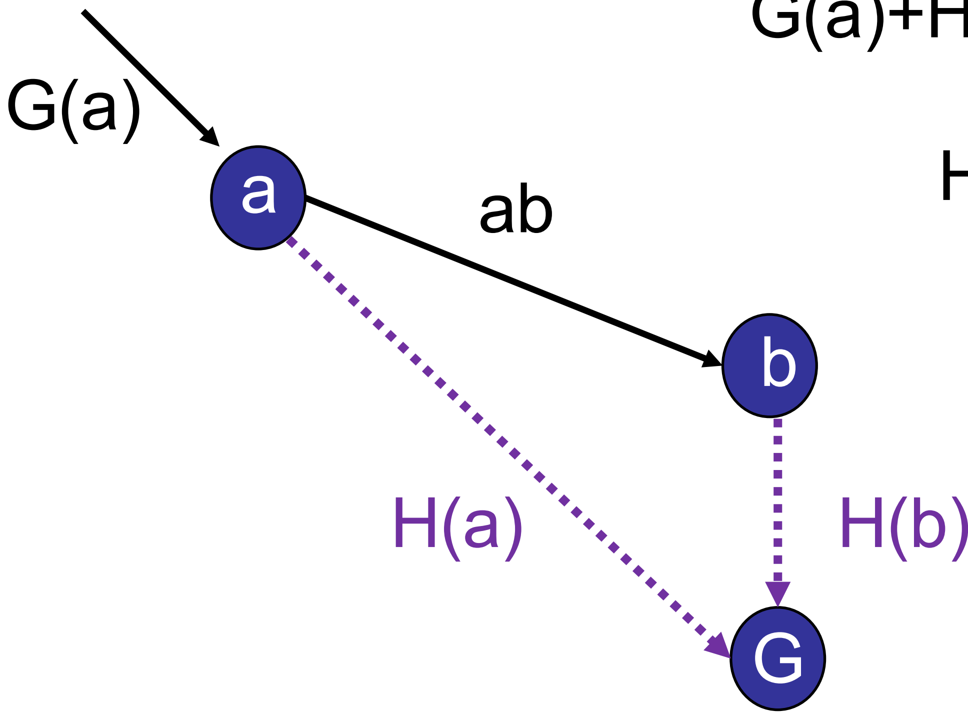


Is Manhattan distance **monotonic**?

- f values increase from node to children
- (triangle inequality)



Monotonicity (aka Consistency)



Defn monotonic:

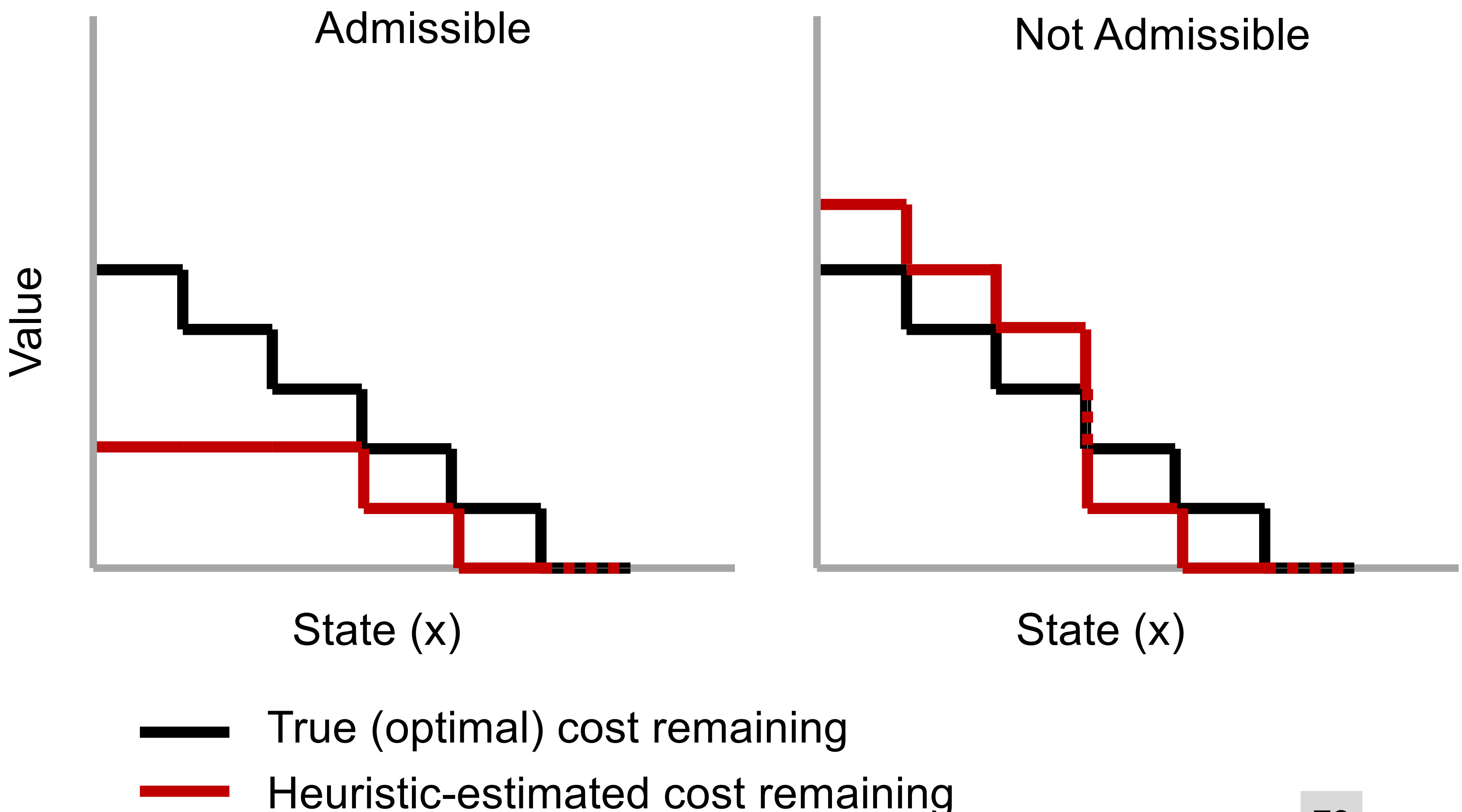
$F(a) \geq F(b)$

$G(a)+H(a) \geq G(b)+H(b)$

$\geq G(a)+ab + H(b)$

$H(a) \geq ab + H(b)$

Admissible Heuristics



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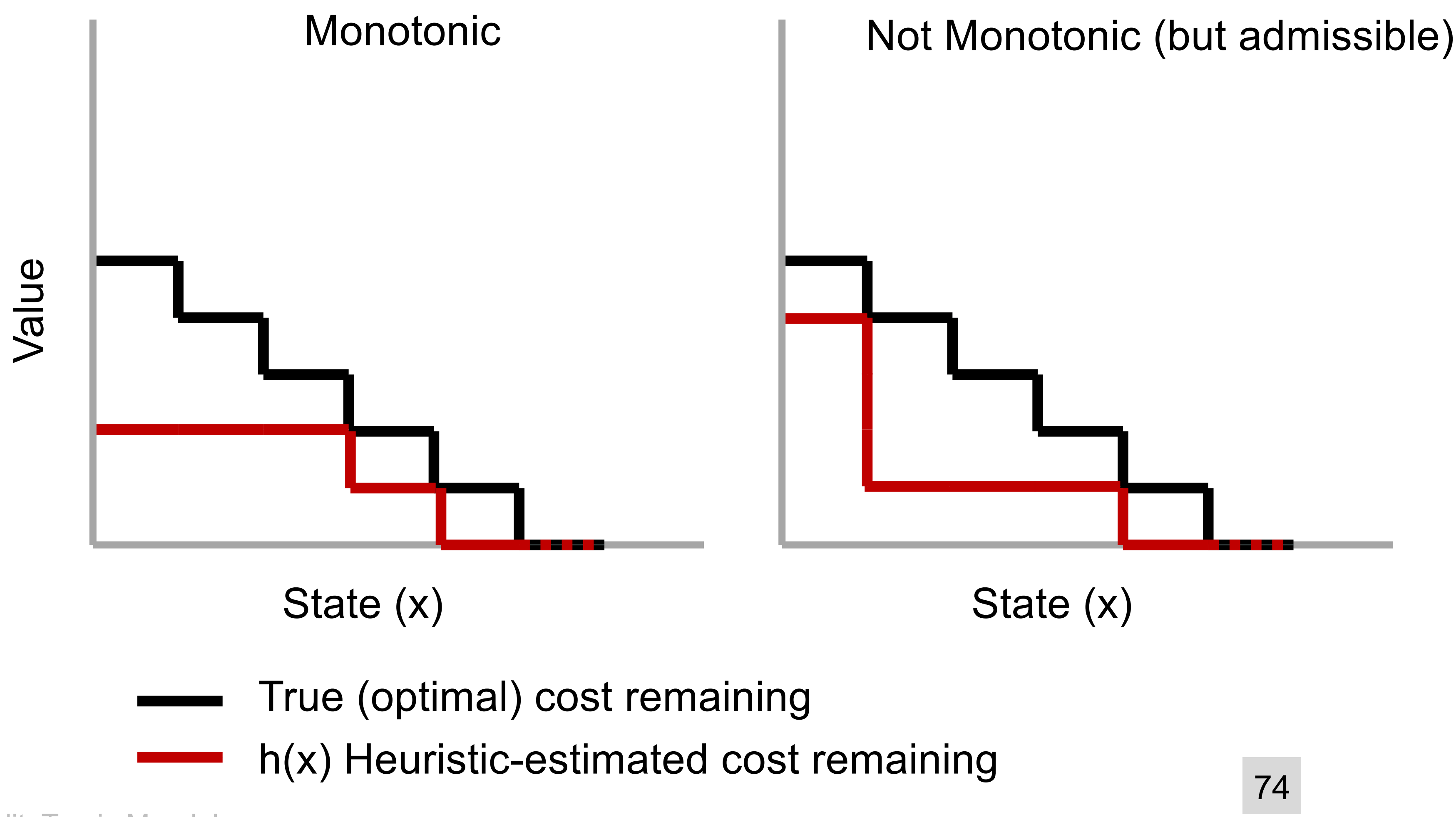
Monotonic/Consistent Heuristics



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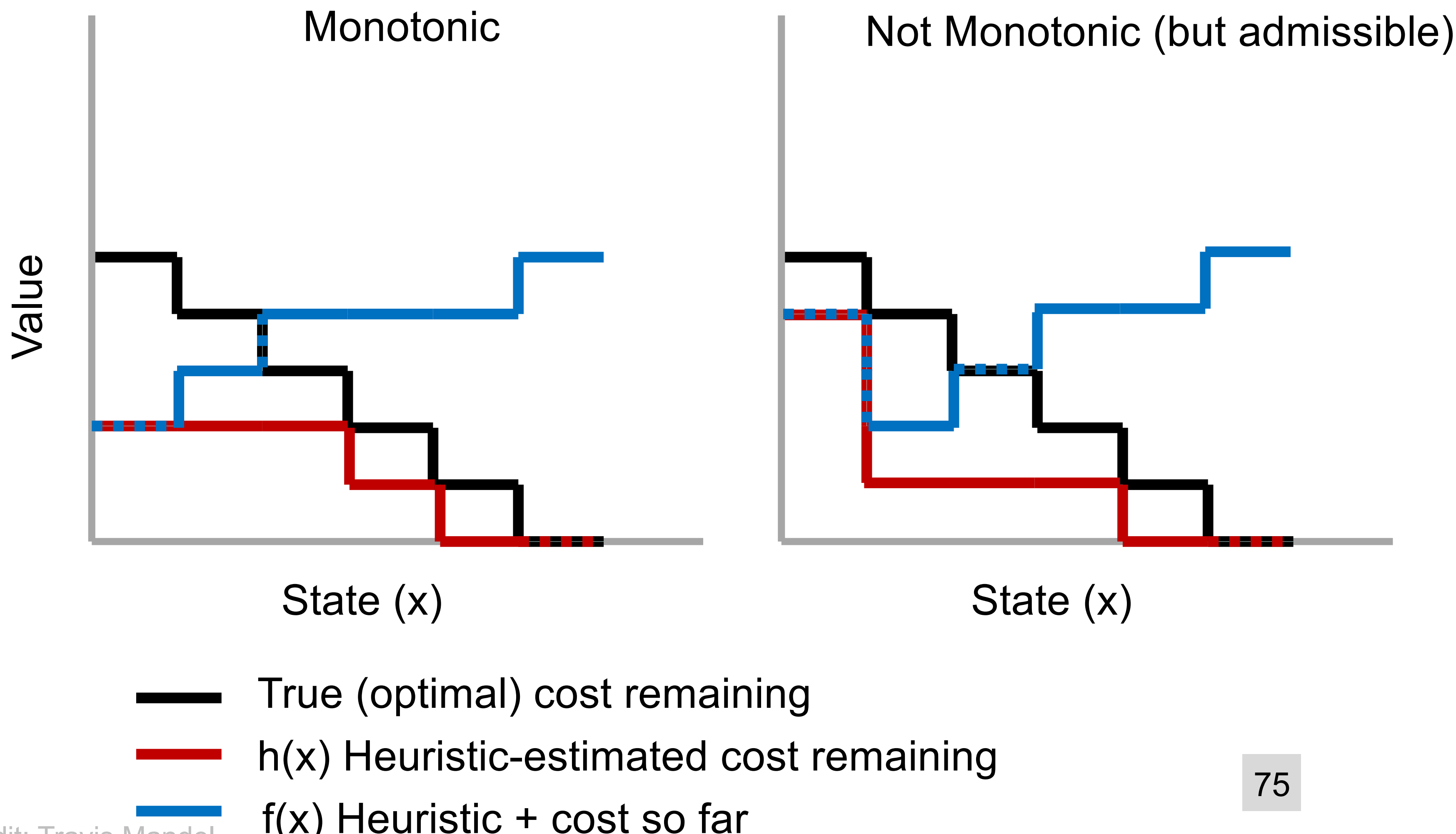
Monotonic/Consistent Heuristics



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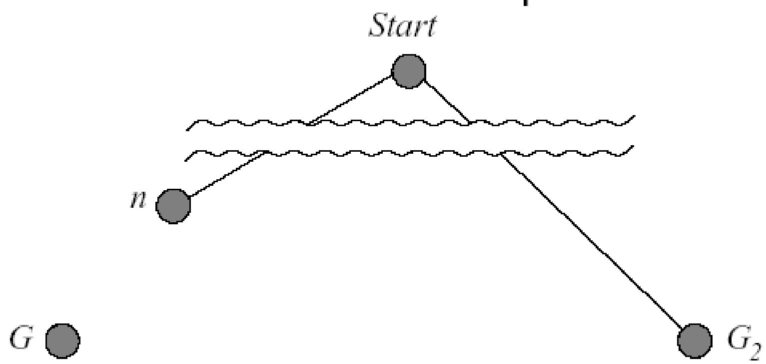
Optimality of A* (tree search)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.

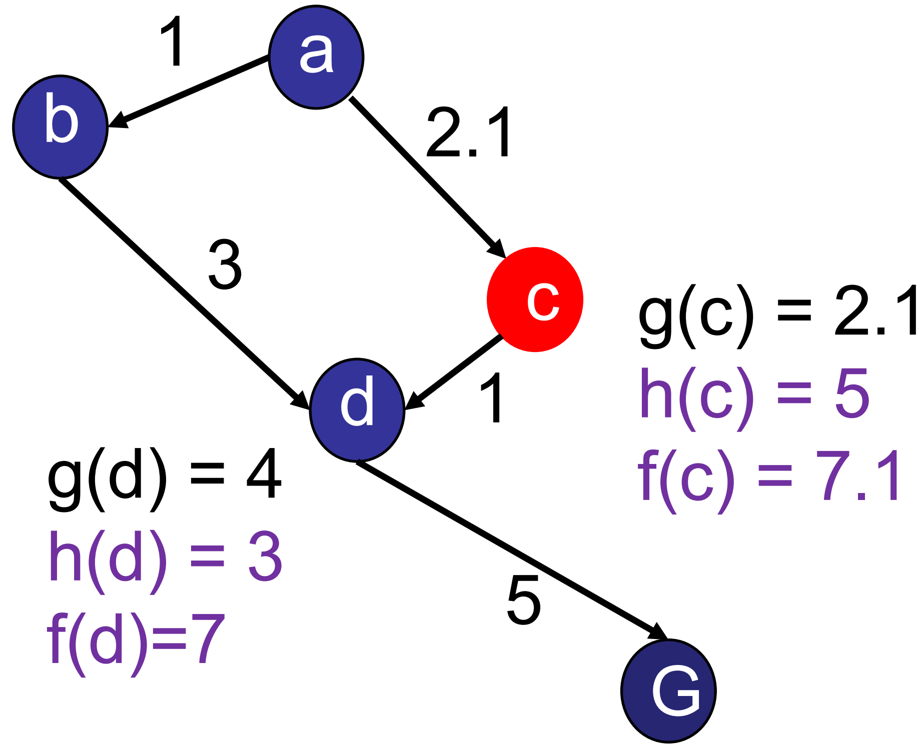


$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Monotonicity required for proof in graph search version

Monontonicity Required to Ensure A* Graph Search is Optimal



Monotonicity needed to ensure optimality

Given GS optimization of queue

Suppose node(a>b>d) has been expanded but not node(a>c>d)

It won't be, because it's state (d) is closed

Optimality of A*

- Lemma 1

  If h(n) is monotonic, then the values of f along any path are non decreasing ~ by defn.

- Lemma 2

  Whenever A* selects node n for expansion, the optimal path to that node has been found

  Assume not. Then $\exists$ node m on frontier which is on a better path to n, but by lemma 1, it would have been explored.

- Lemma 3

  A* expands nodes in order of increasing f value

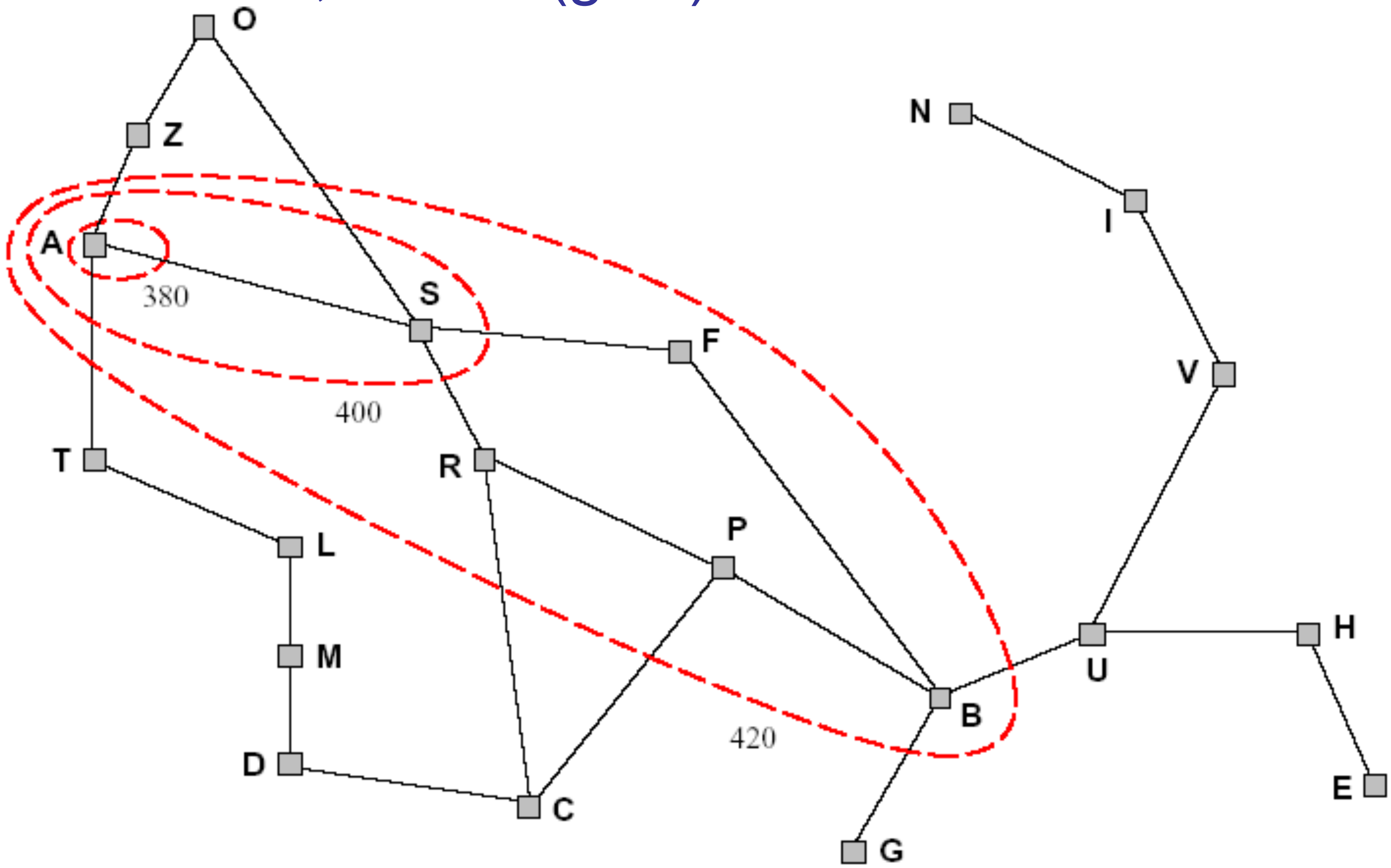
Optimality Continued

A* gradually adds “f-contours” of nodes.

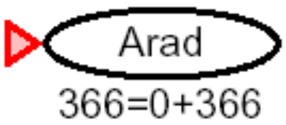
Contour i has all nodes with f = f_i, where $f_i < f_{i+1}$

First goal expanded must have lowest f-value

→ Lowest cost, since h(goal) = 0



A* Example



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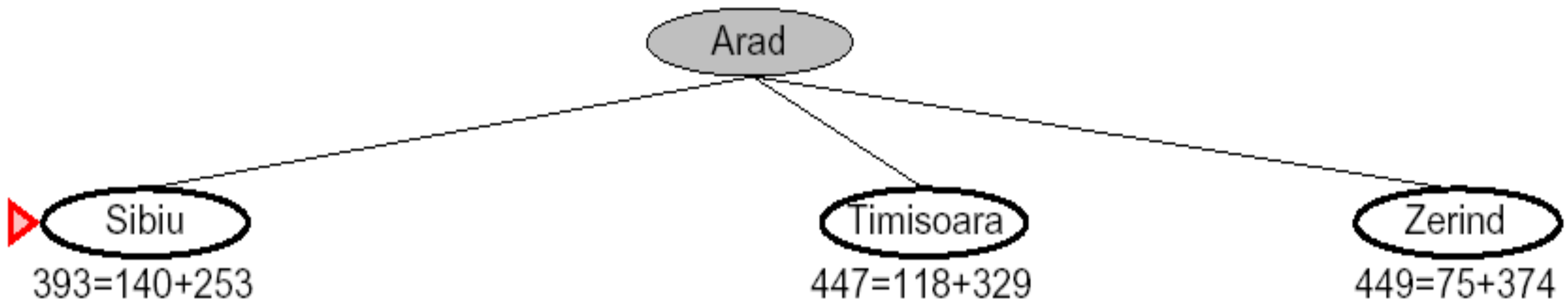
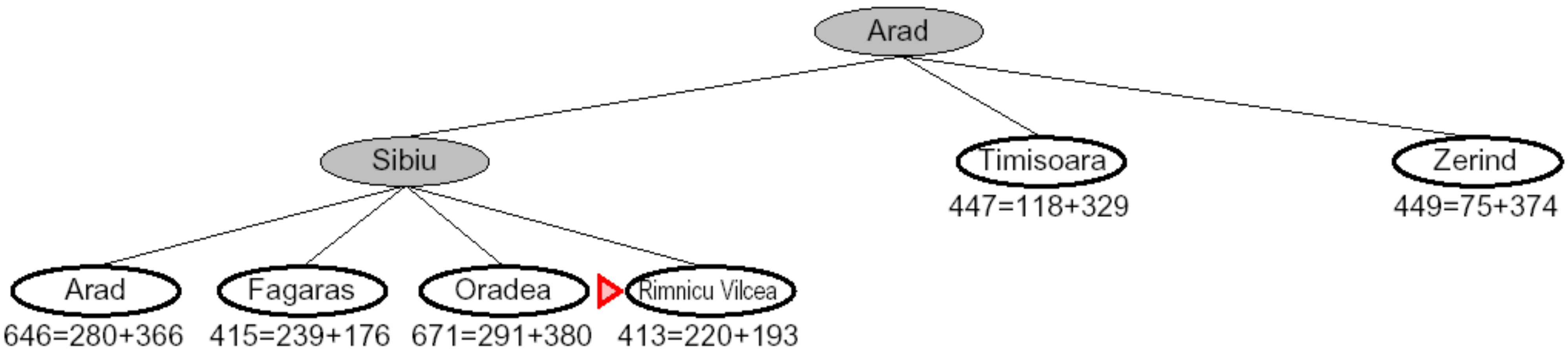


Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374

A* Example



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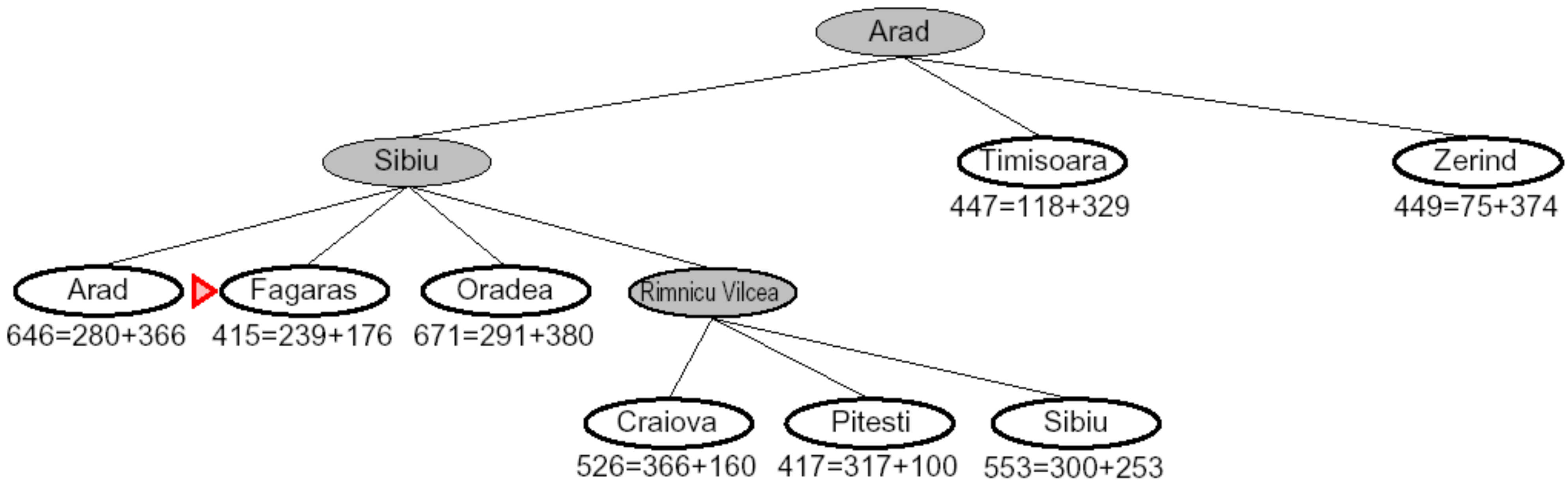
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193

A* Example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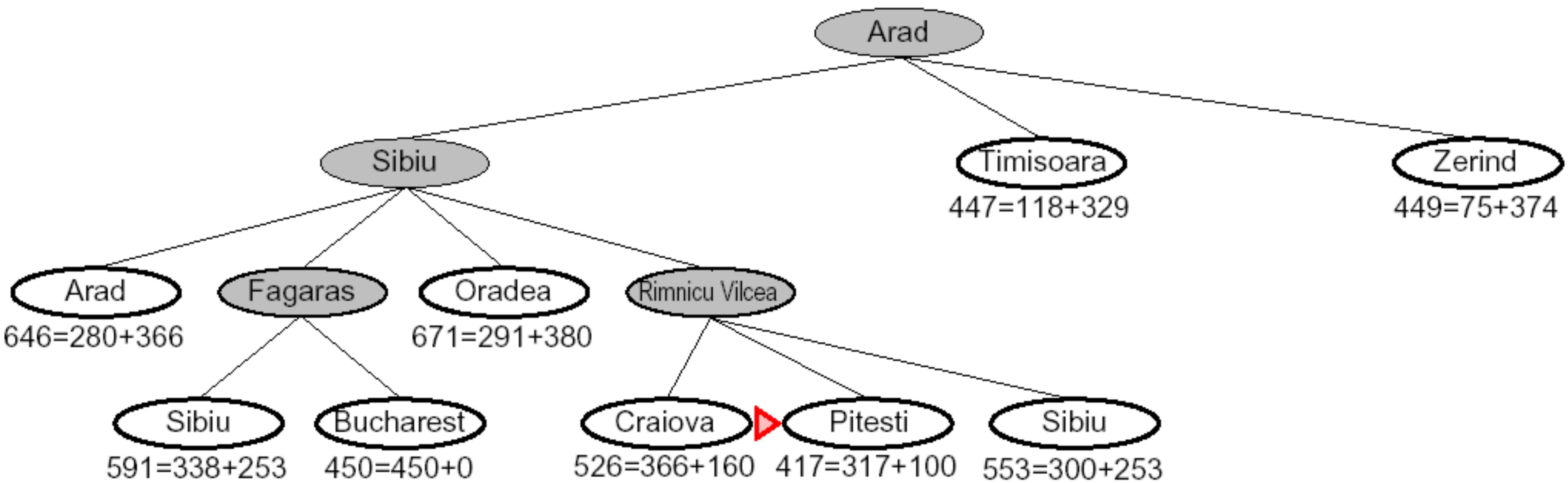
A* Example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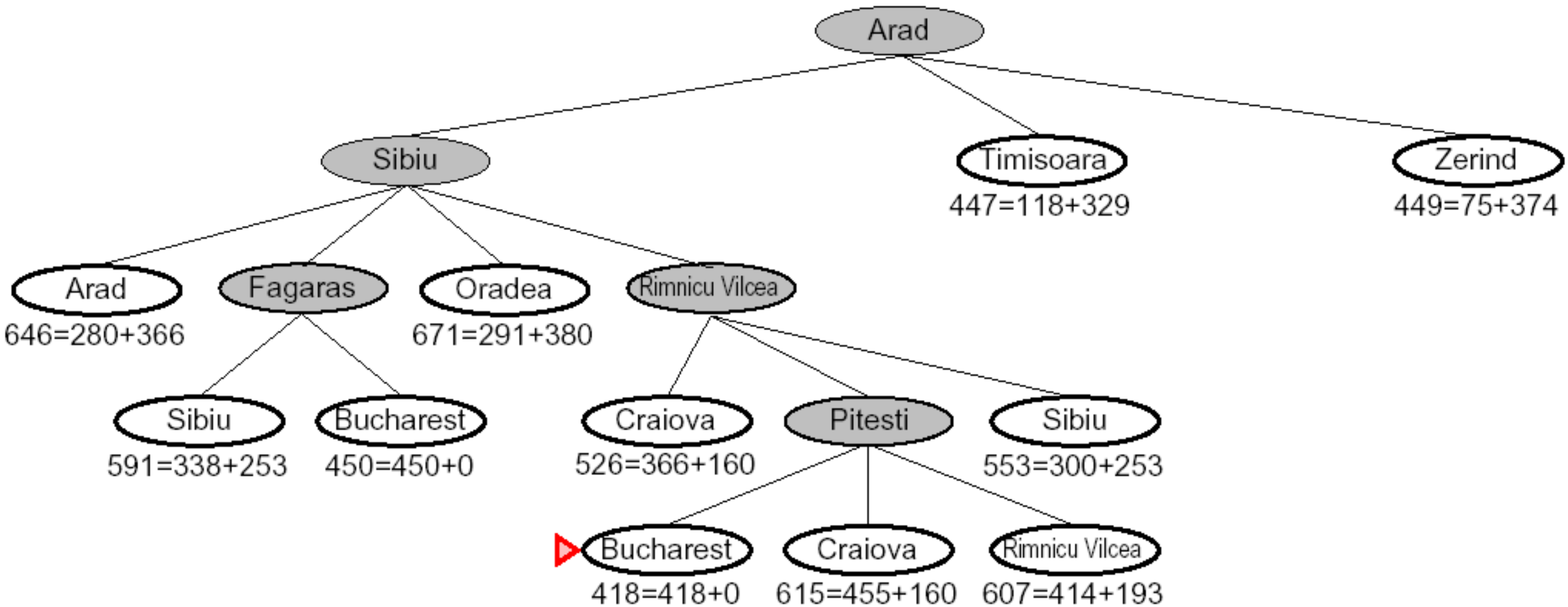
A* Example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Craiova
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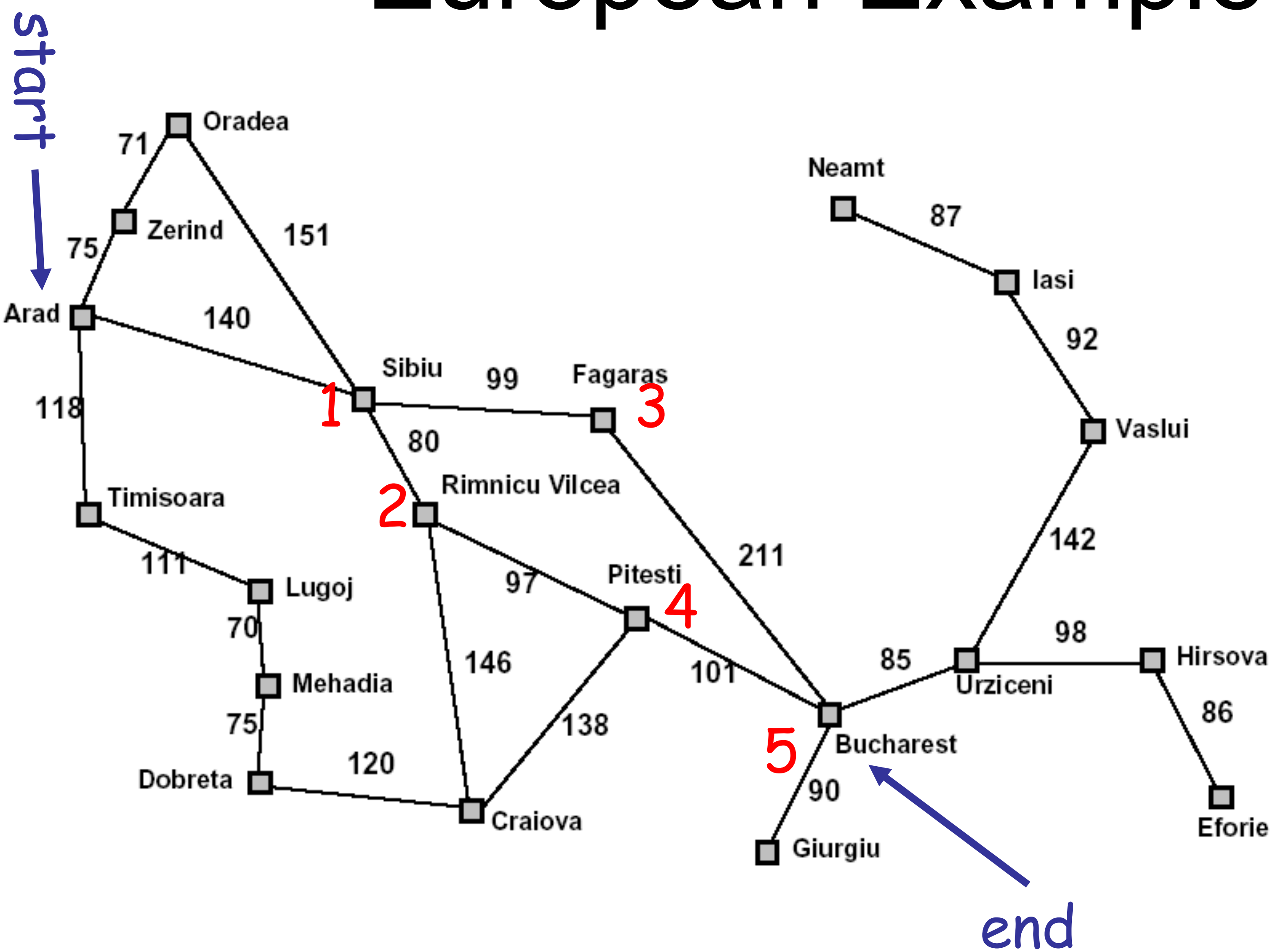
Rimnicu Vilcea
607=414+193

European Example



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* Summary

- Pros

  Produces optimal cost solution!

  Does so quite quickly (focused)

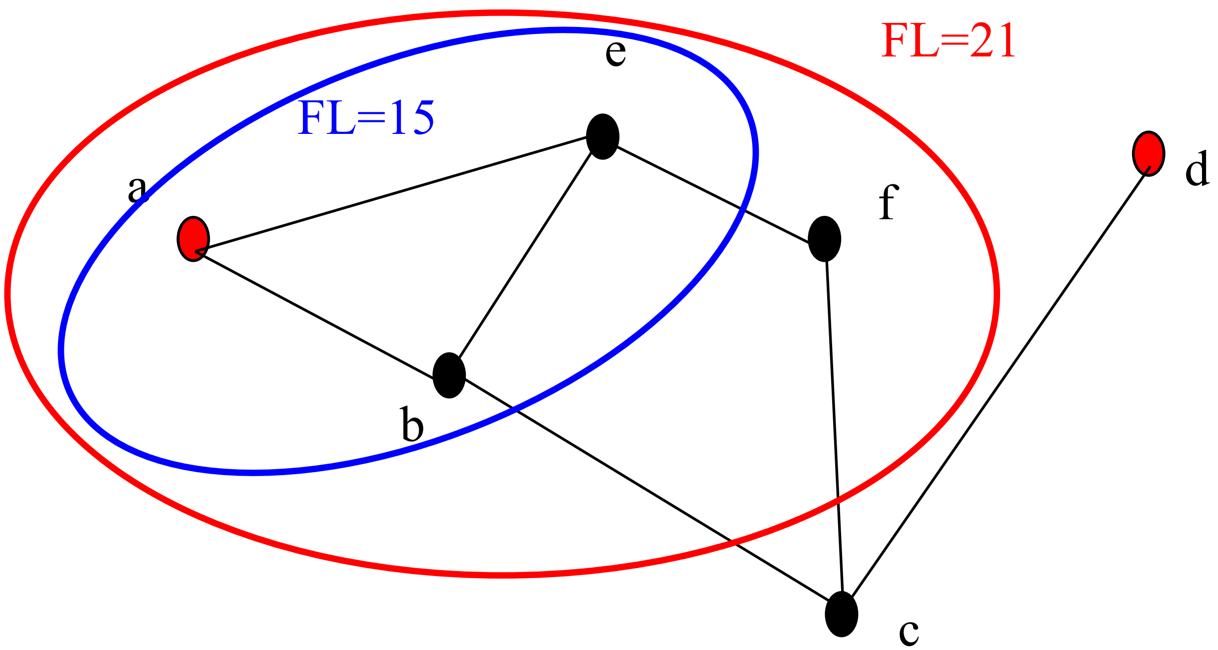
- Cons

  Maintains priority queue…

  Which can get exponentially big ☹

Iterative-Deepening A*

- Like iterative-deepening depth-first, but...
- Depth bound modified to be an f-limit
  - Start with f-limit = h(start)
  - Perform depth-first search (using stack, no queue)
    - Prune any node if f(node) > f-limit
    - Next f-limit = min-cost of any node pruned



IDA* Analysis

- Complete & Optimal (ala A*)
- Space usage ∝ depth of solution
- Each iteration is DFS - no priority queue!
- # nodes expanded relative to A* ??
  - Depends on # unique values of heuristic function
  - In 8 puzzle: few values ⇒ close to # A* expands
  - In traveling salesman: each f value is unique

    ⇒ $1+2+\ldots+n = O(n^2)$ where n=nodes A* expands

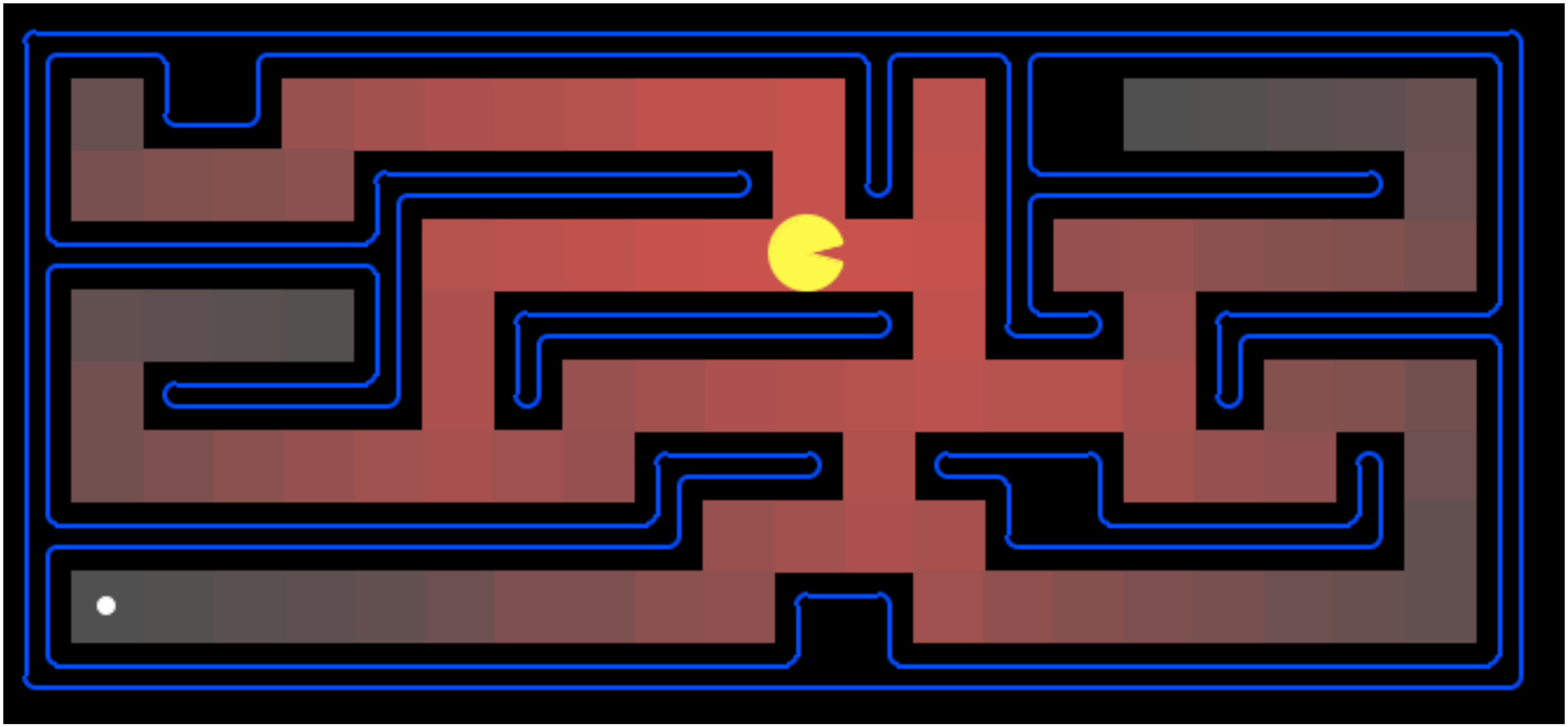
    if n is too big for main memory, n^2 is too long to wait!

Forgetfulness

- A* used exponential memory
- How much does IDA* use?
  - During a run?
  - In between runs?
    - SMA*

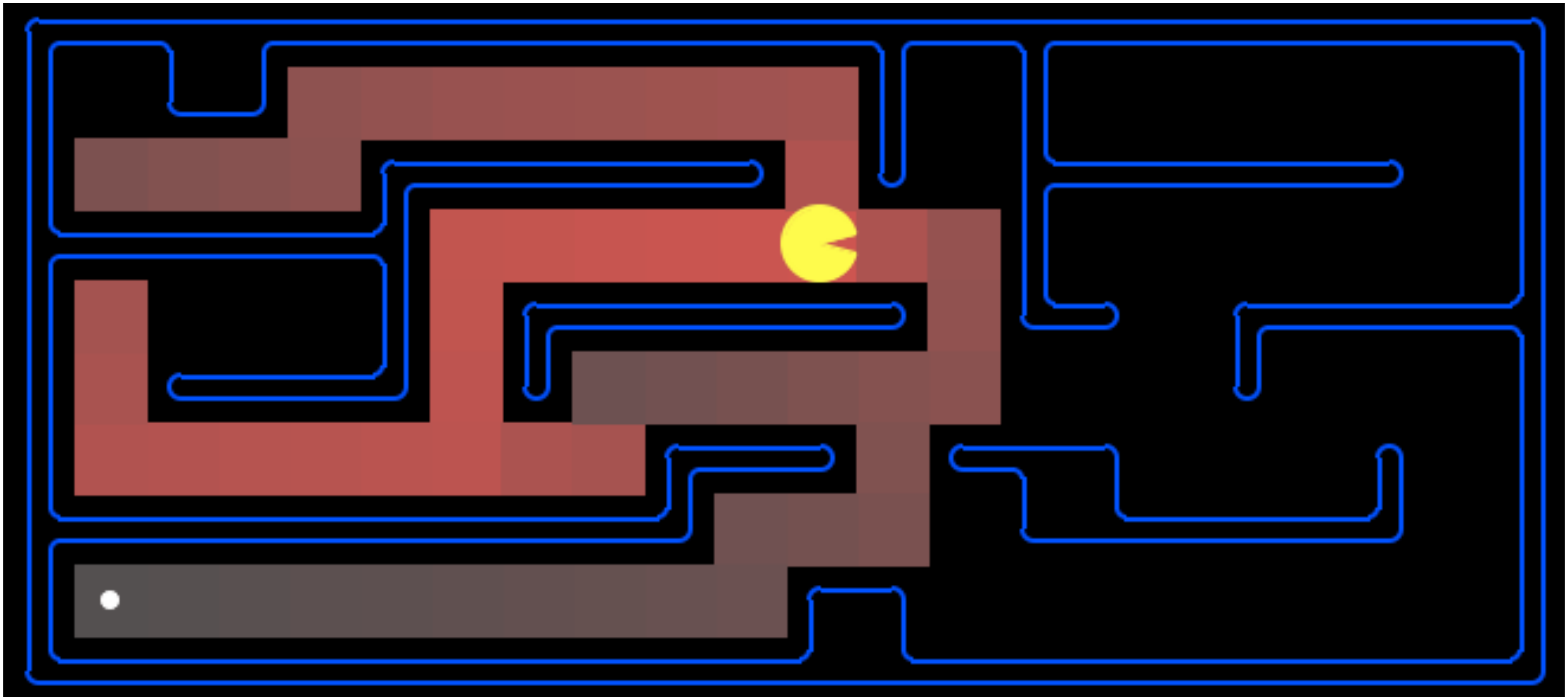
Which Algorithm?

- Uniform cost search (UCS):



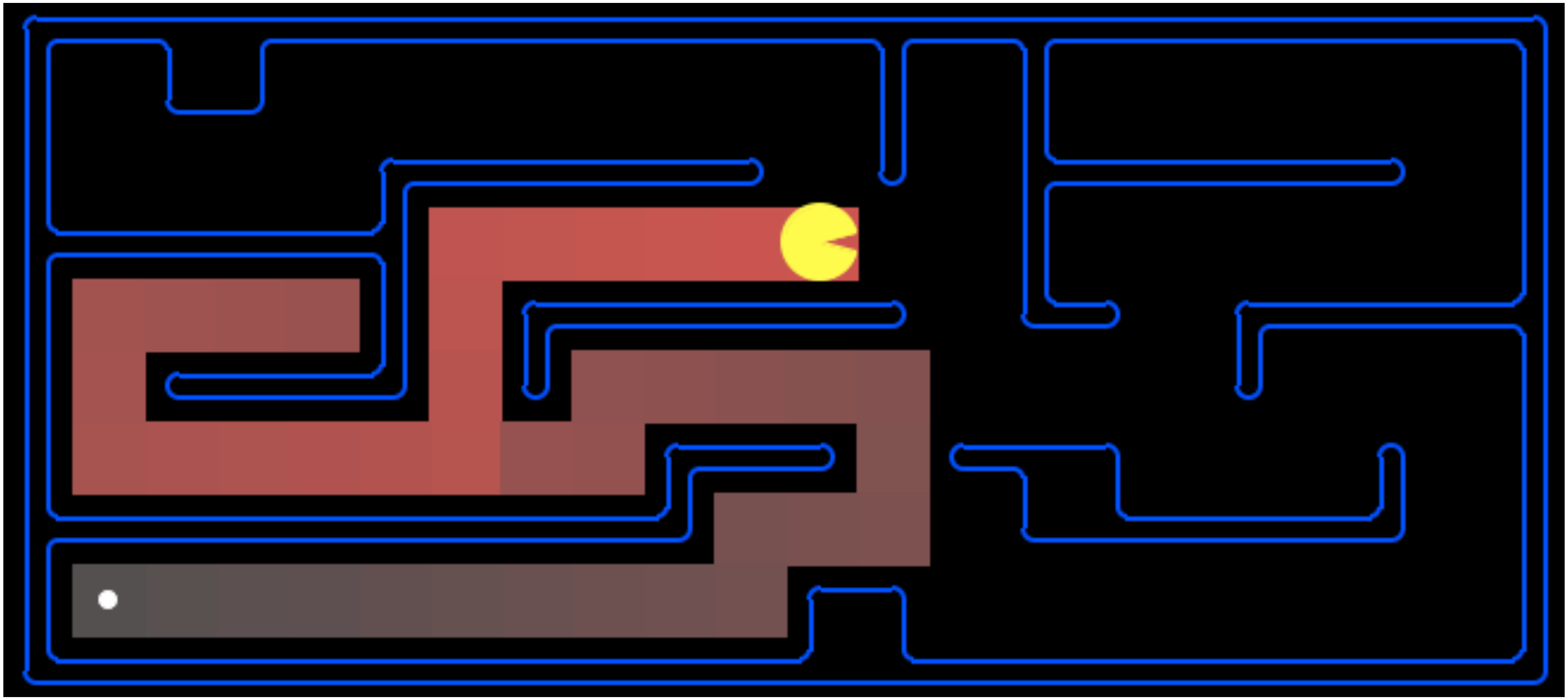
Which Algorithm?

- A*, Manhattan Heuristic:



Which Algorithm?

- Best First / Greedy, Manhattan Heuristic:



Demo

http://qiao.github.io/PathFinding.js/visual/

SUGGESTED BY **Fernando Centurion**