Knowledge Representation II
CSE 573

Logistics
• Reading for Wednesday
  Ch 11 “Planning”
• Projects
  Did we get everyone?
• Office Hour
  Monday 3-4pm
  Except... Today only 3-3:20

573 Topics

573 Topics

Review of “Last Time”
• Propositional Logic
  Resolution
  DPLL
  WalkSAT
• Expressiveness vs. Tractability
• Randomly Generating SAT

Resolution
If the unicorn is mythical, then it is immortal, but if it is not mythical, it is a mammal. If the unicorn is either immortal or a mammal, then it is horned.
Prove: the unicorn is horned.

\[ \begin{align*}
M = \text{mythical} & \quad & (-A \lor H) \\
I = \text{immortal} & \quad & (-I) \\
A = \text{mammal} & \quad & (M \lor A) \\
H = \text{horned} & \quad & (-M \lor I) \\
\end{align*} \]

DPLL (for real!)
Davis - Putnam - Loveland - Logemann

dpll(F, literal)
remove clauses containing literal
if (F contains no clauses) return true;
shorten clauses containing \(-\text{literal}\)
if (F contains empty clause) return false;
if (F contains a unit or pure L) return dpll(F, L);
choose V in F;
if (dpll(F, ¬V)) return true;
return dpll(F, V);
WalkSat

- Local search over space of complete truth assignments
  - With probability P: flip any variable in any unsatisfied clause
  - With probability (1-P): flip best variable in any unsat clause
    - Like fixed-temperature simulated annealing
- SAT encodings of N-Queens, scheduling
- Best algorithm for random K-SAT
  - Best DPLL: 700 variables
  - Walksat: 100,000 variables

Horn Theories

- Recall the special case of Horn clauses:
  - At most one positive literal / clause
  - E.g. from "If (fever) AND (vomiting) then FLU"
  - Unit propagation is refutation complete for Horn theories
    - Good implementation - linear time inference!
- Binary clauses
  - Linear-time inference

Random 3-SAT

- Random 3-SAT
  - Sample uniformly from space of all possible 3-clauses
    - n variables, l clauses
  - Which are the hard instances?
    - around l/n = 4.3

Random 3-SAT

- Varying problem size, n
  - Complexity peak appears to be largely invariant of algorithm
    - backtracking algorithms like Davis-Putnam
    - local search procedures like GSAT
  - What’s so special about 4.3?

Real-World Phase Transition Phenomena

- Many NP-hard problem distributions show phase transitions -
  - job shop scheduling problems
  - TSP instances from TSPLib
  - exam timetables @ Edinburgh
  - Boolean circuit synthesis
  - Latin squares (alias sports scheduling)
- Hot research topic: predicting hardness of a given instance, & using hardness to control search strategy (Horvitz, Kautz, Ruan 2001-3)
Themes

- **Expressiveness**
  Expressive but awkward
  No notion of objects, properties, or relations
  Number of propositions is fixed

- **Tractability**
  NPC in general
  Completeness / speed tradeoff
  Horn clauses, binary clauses

Logic-Based KR

- **Propositional logic**
  Syntax (CNF, Horn clauses, ...)
  Semantics (Truth Tables)
  Inference (FC, Resolution, DPLL, WalkSAT)
  Restricted Subsets

- **First-order logic**
  Syntax (quantifiers, skolem functions, ...)
  Semantics (Interpretations)
  Inference (FC, Resolution, Compilation)
  Restricted Subsets (e.g. Frame Systems)

Representing events, action & change

Propositional Logic vs. First Order

<table>
<thead>
<tr>
<th>Ontology</th>
<th>Facts (P, Q)</th>
<th>Objects, Properties, Relations</th>
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<tbody>
<tr>
<td>Syntax</td>
<td>Atomic sentences</td>
<td>Variables &amp; quantification</td>
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<tr>
<td></td>
<td>Connectives</td>
<td>Sentences have structure: terms</td>
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<tr>
<td></td>
<td></td>
<td>father-of(mother-of(X))</td>
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<tr>
<td>Semantics</td>
<td>Truth Tables</td>
<td>Interpreations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Much more complicated)</td>
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<tr>
<td>Inference</td>
<td>DPLL, GSAT</td>
<td>Unification</td>
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<td>Fast in practice</td>
<td>Forward, Backward chaining</td>
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<td>Prolog, theorem proving</td>
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<td>Complexity</td>
<td>NP-Completeness</td>
<td>Semi-decidable</td>
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FOL Definitions

- **Constants**: a, b, dog33.
  Name a specific object.

- **Variables**: X, Y.
  Refer to an object without naming it.

- **Functions**: dad-of
  Mapping from objects to objects.

- **Terms**: dad-of(dog33)
  Refer to objects

- **Atomic Sentences**: in(dad-of(dog33), food6)
  Can be true or false
  Correspond to propositional symbols P, Q

More Definitions

- **Logical connectives**: and, or, not, =>

- **Quantifiers**:
  ∀ Forall
  ∃ There exists

- **Examples**
  Dumbo is grey
  Elephants are grey
  There is a grey elephant

Quantifier / Connective Interaction

E(x) == “x is an elephant”
G(x) == “x has the color grey”

1. \( \forall x \, E(x) \land G(x) \)
2. \( \forall x \, E(x) \implies G(x) \)
3. \( \exists x \, E(x) \land G(x) \)
4. \( \exists x \, E(x) \implies G(x) \)
Nested Quantifiers:
Order matters!
\( \forall x \exists y P(x,y) \neq \exists y \forall x P(x,y) \)

• Examples
  - Every dog has a tail
  - Every dog shares a tail!
\( \forall d \exists t \text{ has}(d,t) \neq \exists t \forall d \text{ has}(d,t) \)

Someone is loved by everyone
\( \exists x \forall y \text{ loves}(y,x) \)

Propositional Logic: SEMANTICS

• “Interpretation” (or “possible world”)
• Specifically, TRUTH TABLES
  - Assignment to each variable either T or F
  - Assignment of T or F to each connective

\[
\begin{array}{c|c|c}
P & Q & P \wedge Q \\ 
T & T & T \\ 
T & F & F \\ 
F & T & F \\ 
F & F & F \\ 
\end{array}
\]

Models

• Depiction of one possible “real-world” model

Interpretations=Mappings
syntactic tokens \(\rightarrow\) model elements

Depiction of one possible interpretation, assuming
- Constants: Richard, John
- Functions: Leg(p,l), On(x,y)
- Relations: King(p)

Another interpretation, same assumptions
- Constants: Richard, John
- Functions: Leg(p,l)
- Relations: On(x,y)
- Relations: King(p)
Satisfiability, Validity, & Entailment

• S is valid if it is true in all interpretations
• S is satisfiable if it is true in some interpretation
• S is unsatisfiable if it is false in all interpretations

S1 entails S2 if for all interpretations where S1 is true, S2 is also true.

Skolemization

• Existential quantifiers aren’t necessary!
  Existential variables can be replaced by
  • Skolem functions (or constants)
  • Args to functions are all surrounding \( \forall \) variables

\[ \forall d \exists t \text{ has}(d, t) \]
\[ \forall d \text{ has}(d, f(d)) \]

\[ \exists x \forall y \text{ loves}(y, x) \]
\[ \forall y \text{ loves}(y, f()) \]
\[ \forall y \text{ loves}(y, f_{97}) \]

FOL Reasoning

• FO Forward & Backward Chaining
• FO Resolution
• Many other types of theorem proving
• Restricted representations
  • Description logics
  • Horn Clauses
• Compilation to SAT

Forward Chaining

• Given
  \[ \forall x \text{ lifeform}(x) \Rightarrow \text{mortal}(x) \]
  \[ \forall x \text{ mammal}(x) \Rightarrow \text{lifeform}(x) \]
  \[ \forall x \text{ dog}(x) \Rightarrow \text{mammal}(x) \]
  \[ \text{dog}(\text{fido}) \]

• Prove
  \[ \text{mortal}(\text{fido}) \]

\[ \forall x \text{ dog}(x) \Rightarrow \text{mammal}(x) \]
\[ \text{dog}(\text{fido}) \]
\[ \text{mammal}(\text{fido}) \]

Unification

• Emphasize variables with ?
• Useful for FO inference (modus ponens, ...)
  Also for compilation of FOPC \( \rightarrow \) propositional

• Unify(\( \Phi \), \( \Psi \)) returns “mgu”
  \[ \text{Unify(city}(a), \text{city}(kent)) \text{ returns } a/kent \]

• Substitute(expr, mapping) returns new expr
  \[ \text{Substitute(connected}(a, b), \{a/kent\}) \text{ returns } \text{connected}(kent, b) \]

Unification Examples

• Unify(road(\( ?a \), kent), road(seattle, \( ?b \))
• Unify(road(\( ?a \), \( ?a \)), road(seattle, kent))
• Unify(f(g(\( ?x \), dog), \( ?y \)), f(g(cat, \( ?y \), dog)
• Unify(f(g(\( ?x \)), f(\( ?x \)))
Resolution
[Robinson 1965]

\{ (p \lor \alpha), (\neg p \lor \beta \lor \gamma) \} \vdash (\alpha \lor \beta \lor \gamma)

Recall Propositional Case:
- Literal in one clause
- Its negation in the other
- Result is disjunction of other literals

First-Order Resolution
[Robinson 1965]

\{ (p(\beta) \lor a(a), (\neg p(q) \lor b(\gamma) \lor c(\beta))) \}

\vdash (a(a) \lor b(q) \lor c(\beta))

- Literal in one clause
- The negation of something which unifies in the other
- Result is disjunction of other literals / mgu

First-Order Resolution

• Is it the case that \( \Sigma \models \Phi \)?
• Method
  Let \( \vartheta = \Sigma \land \neg \Phi \)
  Convert \( \vartheta \) to clausal form
  - Standardize variables
  - Move quantifiers to front, skolemize to remove \( \exists \)
  - Replace \( \Rightarrow \) with \( \lor \) and \( \neg \)
  - Demorgan's laws...
  Resolve until get empty clause

Example

• Given

\forall x \text{ man}(x) \Rightarrow \text{mortal}(x)
\forall x \text{ woman}(x) \Rightarrow \text{mortal}(x)
\forall x \text{ person}(x) \Rightarrow \text{man}(x) \lor \text{woman}(x)
\text{person(kelly)}

• Prove

\text{mortal(kelly)}

KR with Description Logics

Abox

mother(jane)
child-of(jane, bob)
...

Tbox

person
father
mother
grandmother

Example Continued

\{ \neg\text{m}(\beta), d(\beta) \} \quad \{ \neg \text{w}(\gamma), d(\gamma) \} \quad \{ \neg \text{p}(\beta), \text{m}(\beta), \text{w}(\beta) \} \quad \{ \text{p}(k) \} \quad \{ \neg \text{d}(k) \}

\{ \text{m}(k), \text{w}(k) \} \quad \{ \text{w}(k), \text{d}(k) \} \quad \{ \}
Tbox
• Term definitions
• FO Language + inference organized into a taxonomy, e.g:
  father(x) = person(x) \land male(x) \land \exists y \text{ childof}(y,x)
  parent(x) = person(x) \land \exists y \text{ childof}(y,x)
• Complexity of classifying new terms
  subsumption

Subsumption hierarchy →

Abox
• Assertions
• Abox – separate language + inference for "propositional" assertions using Tbox terms
  e.g. person(kelley)

Debate
• Restricted language thesis
  Disjunction, negation, particularization, order…
  Natural kinds
• Restricted classification thesis
  Concepts using contingent information:
  Treatable disease, democratic country, illegal act
• Counterargument
• Constructs: Omit vs limit
  Completeness
  Efficiency

Compilation to Prop. Logic I
• Typed Logic
  ∀_city a,b connected(a,b)
• Universe
  Cities: seattle, tacoma, enumclaw
• Equivalent propositional formula:

Compilation to Prop. Logic II
• Universe
  • Cities: seattle, tacoma, enumclaw
  • Firms: IBM, Microsoft, Boeing
• First-Order formula
  ∀_city c \exists_{firm} f \text{ hasHQ}(c, f)
• Equivalent propositional formula

Hey!
• You said FO Inference is semi-decidable
• But you compiled it to SAT
  Which is NP Complete
• So now we can always do the inference?!?
  Tho it might take exponential time…
• Something seems wrong here…..????
Restricted Forms of FO Logic

- **Known, Finite Universes**
  Compile to SAT
- **Frame Systems**
  Ban certain types of expressions
- **Horn Clauses**
  Aka Prolog
- **Function-Free Horn Clauses**
  Aka Datalog