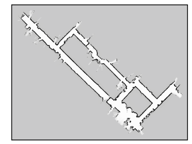
CSE-571
Robotics

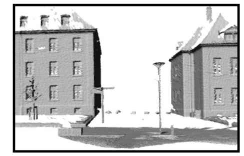
Mapping

1

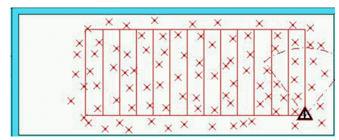
Types of SLAM-Problems

Grid maps or scans





Sparse landmarks



RGB / Depth Maps

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Problems in Mapping

- Sensor interpretation
  - How do we extract relevant information from raw sensor data?
  - How do we represent and integrate this information over time?
- Robot locations have to be known
  - How can we estimate them during mapping?

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Occupancy Grid Maps

- Introduced by Moravec and Elfes in 1985
- Represent environment by a grid.
- Estimate the probability that a location is occupied by an obstacle.
- Key assumptions
  - Occupancy of individual cells is independent

$$Bel(m_t) = P(m_t \mid u_1, z_2 \ldots, u_{t-1}, z_t) = \prod_{x,y} Bel(m_t^{[xy]})$$

  - Robot positions are known!

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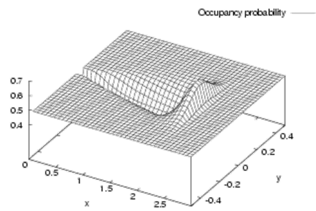
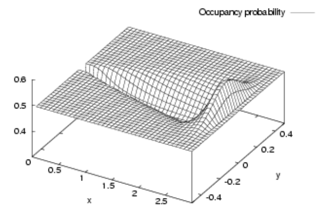
Inverse Sensor Model for Occupancy Grid Maps

Combination of linear function and Gaussian:





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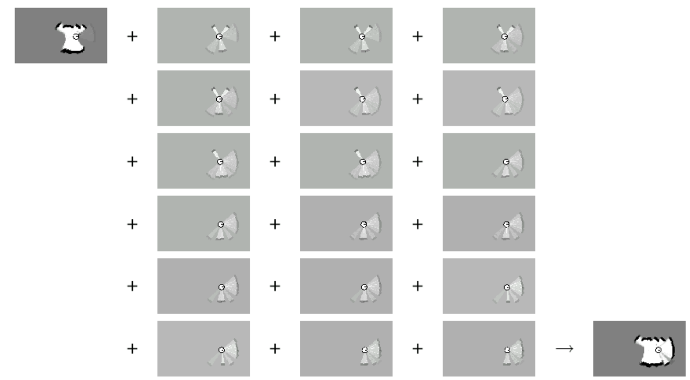
Incremental Updating of Occupancy Grids (Example)



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Alternative for Lidar: Counting

- For every cell count
  - hits(*x*,*y*): number of cases where a beam ended at <*x*,*y*>
  - misses(*x*,*y*): number of cases where a beam passed through <*x*,*y*>

$$Bel(m^{[xy]}) = \frac{\mathrm{hits}(x,y)}{\mathrm{hits}(x,y) + \mathrm{misses}(x,y)}$$

- Assumption: *P*(*occupied*(*x*,*y*)) = *P*(*reflects*(*x*,*y*))

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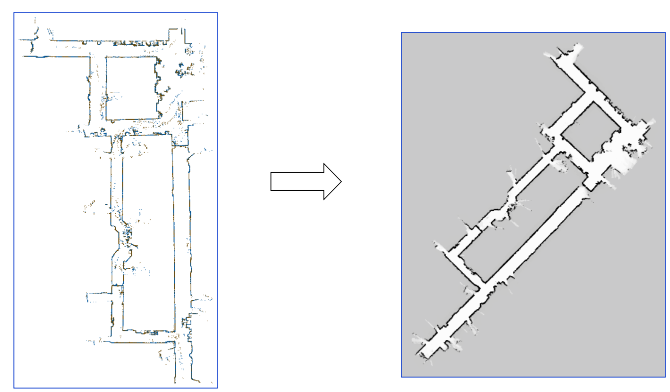
Occupancy Grids: From scans to maps



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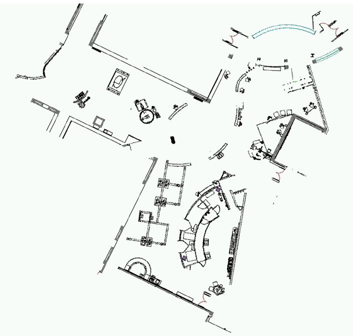
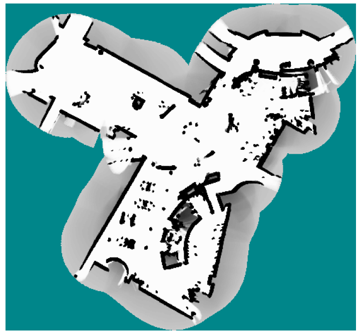
Tech Museum, San Jose



CAD map



occupancy grid map

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OctoMap

A Probabilistic, Flexible, and Compact 3D Map Representation for Robotic Systems

K.M. Wurm, *A. Hornung*,
M. Bennewitz, C. Stachniss, W. Burgard

University of Freiburg, Germany

http://octomap.sf.net

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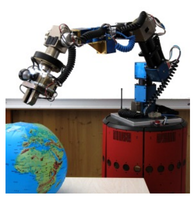
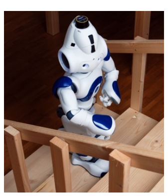
Robots in 3D Environments

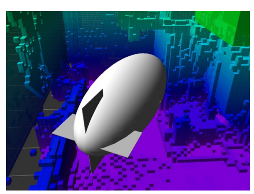


Mobile manipulation

Outdoor navigation



Humanoid robots



Flying robots

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3D Map Requirements

- Full 3D Model
  - Volumetric representation
  - Free-space
  - Unknown areas (e.g. for exploration)
- Can be updated
  - Probabilistic model
    (sensor noise, changes in the environment)
  - Update of previously recorded maps
- Flexible
  - Map is dynamically expanded
  - Multi-resolution map queries
- Compact
  - Memory efficient
  - Map files for storage and exchange

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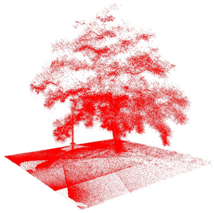
Map Representations

Pointclouds



- Pro:
  - No discretization of data
  - Mapped area not limited
- Contra:
  - Unbounded memory usage
  - No direct representation of free or unknown space

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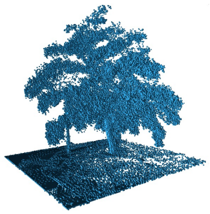
Map Representations

3D voxel grids



- Pro:
  - Probabilistic update
  - Constant access time
- Contra:
  - Memory requirement
    - Extent of map has to be known
    - Complete map is allocated in memory

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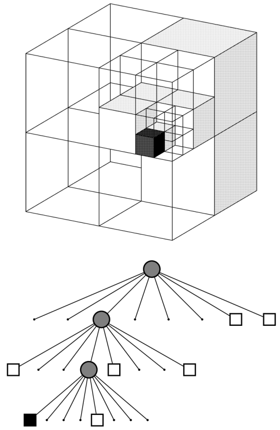
Map Representations

Octrees



- Tree-based data structure
- Recursive subdivision of space into octants
- Volumes allocated as needed
- Multi-resolution

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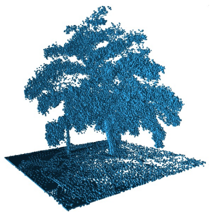
Map Representations

Octrees



- Pro:
  - Full 3D model
  - Probabilistic
  - Flexible, multi-resolution
  - Memory efficient
- Contra:
  - Implementation can be tricky (memory, update, map files, …)

- Open source implementation as C++ library available at http://octomap.sf.net

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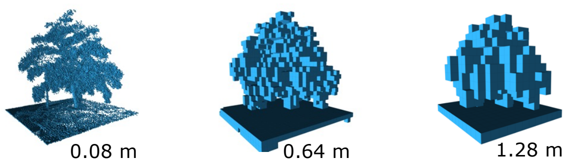
Probabilistic Map Update

- Clamping policy ensures updatability [Yguel '07]

$$L(n) \in [l_{\min}, l_{\max}]$$

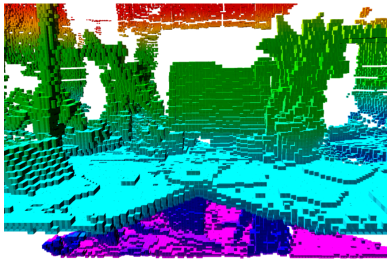
- Update of inner nodes enables multi-resolution queries

$$L(n) = \max_{i=1..8} L(n_i)$$



Examples

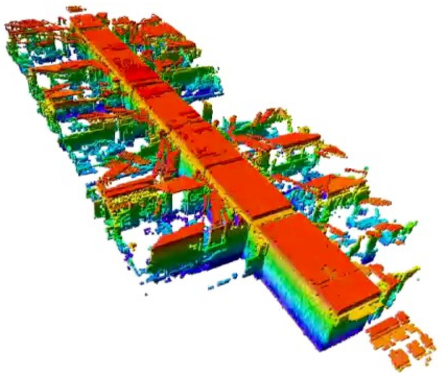
- Cluttered office environment



Map resolution: 2 cm

Examples: Office Building

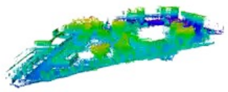
- Freiburg, building 079



Examples: Large Outdoor Areas

- Freiburg computer science campus

(292 x 167 x 28 m³, 20 cm resolution)



Examples: Tabletop

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Memory Usage

Map dataset	Mapped area [m^3]	Resolution [m]	Memory consumption [MB] Full grid	No compr.	Lossless compr.	File size [MB] All data	Binary
FR-079 corridor	43.8 × 18.2 × 3.3	0.05	80.54	73.64	41.70	15.80	0.67
		0.1	10.42	10.90	7.25	2.71	0.14
Freiburg outdoor	292 × 167 × 28	0.20	654.42	188.09	130.39	49.75	2.00
		0.80	10.96	4.56	4.13	1.53	0.08
New College (Epoch C)	250 × 161 × 33	0.20	637.48	91.43	50.70	18.71	0.99
		0.80	10.21	2.35	1.81	0.64	0.05

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CSE-571
Robotics

SLAM: Simultaneous Localization and Mapping

Many slides courtesy of Ryan Eustice, Cyrill Stachniss, John Leonard

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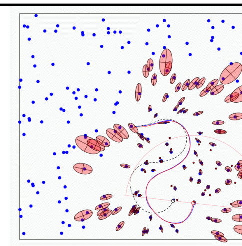
The SLAM Problem



A robot is exploring an unknown, static environment.

Given:

- The robot's controls
- Observations of nearby features

Estimate:

- Map of features
- Path of the robot

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SLAM Applications

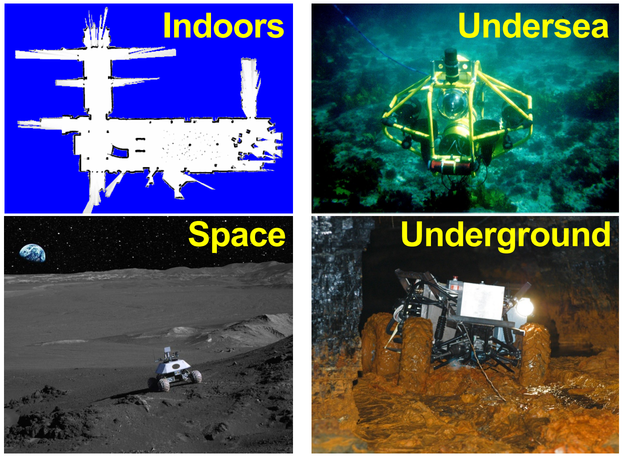

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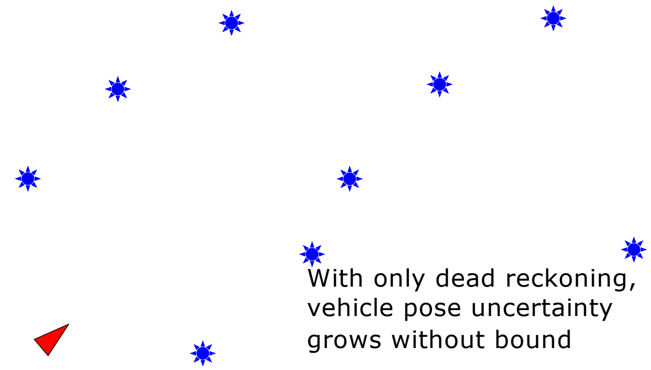
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leon

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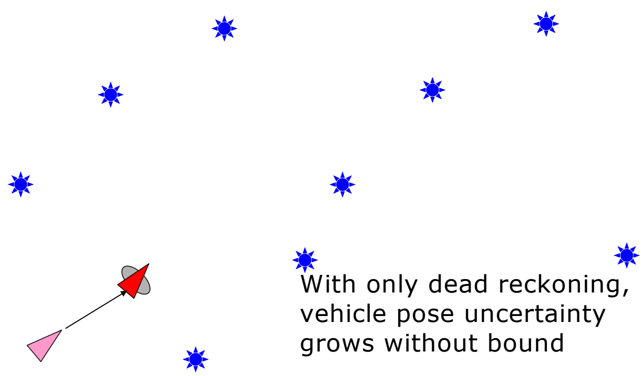
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leon

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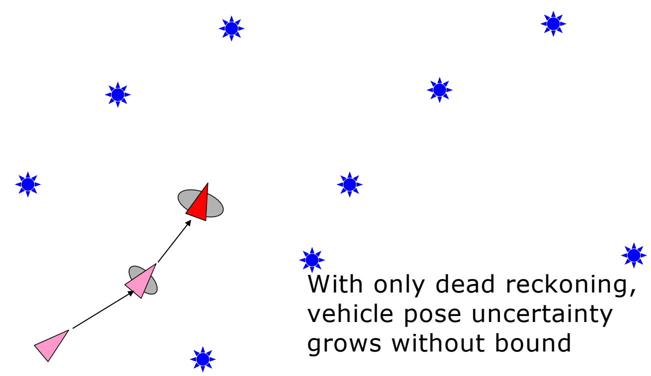
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leon

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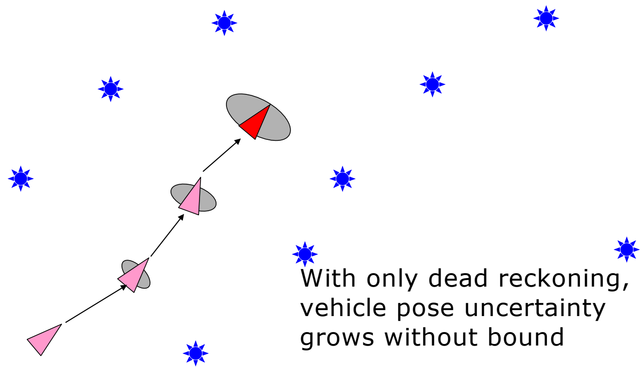
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leon

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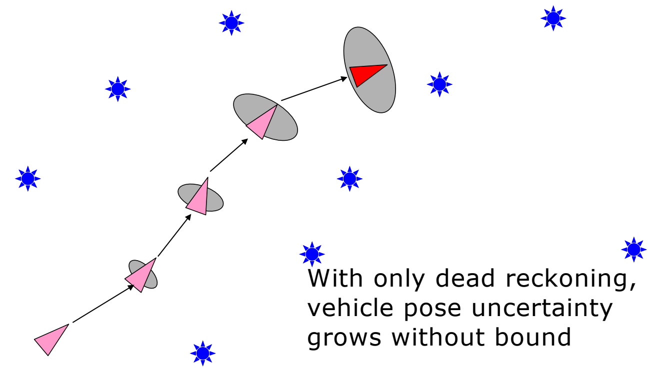
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leon

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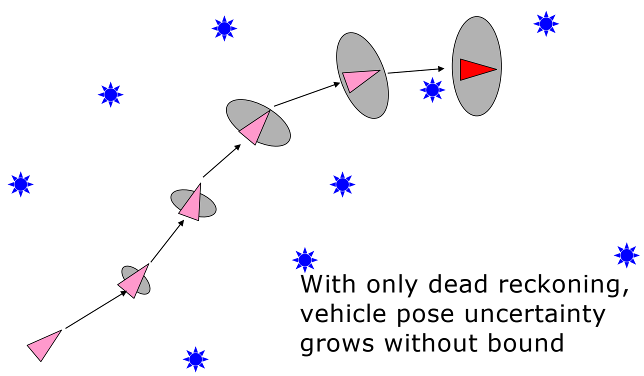
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leon

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Mapping with Raw Odometry

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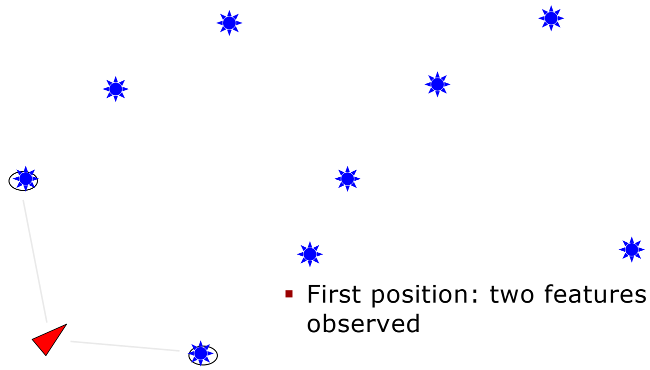
Repeat, with Measurements of Landmarks



- First position: two features observed

Courtesy J. Leon

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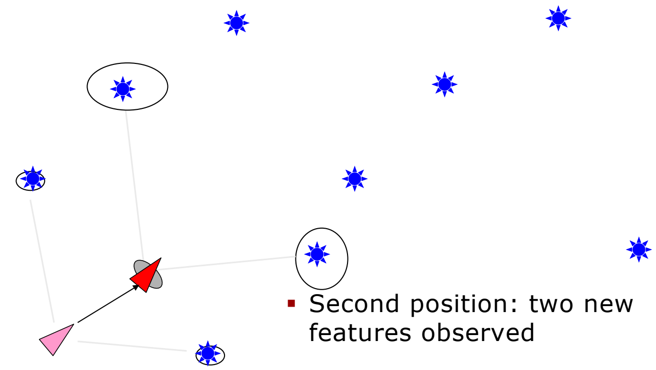
Illustration of SLAM with Landmarks



- Second position: two new features observed

Courtesy J. Leon

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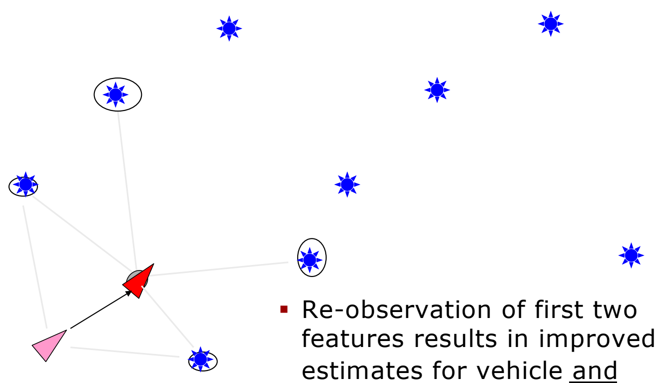
Illustration of SLAM with Landmarks



- Re-observation of first two features results in improved estimates for vehicle and feature

Courtesy J. Leon

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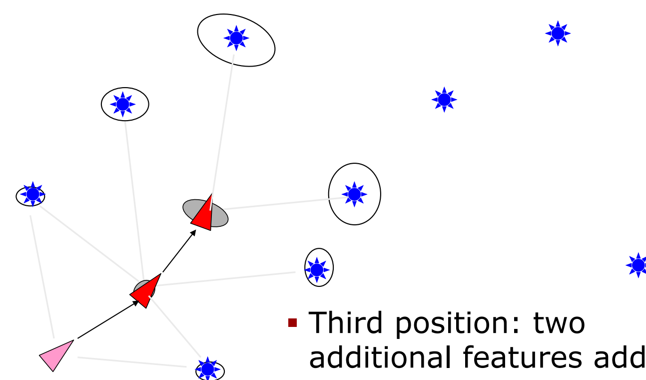
Illustration of SLAM with Landmarks



- Third position: two additional features added to map

Courtesy J. Leon

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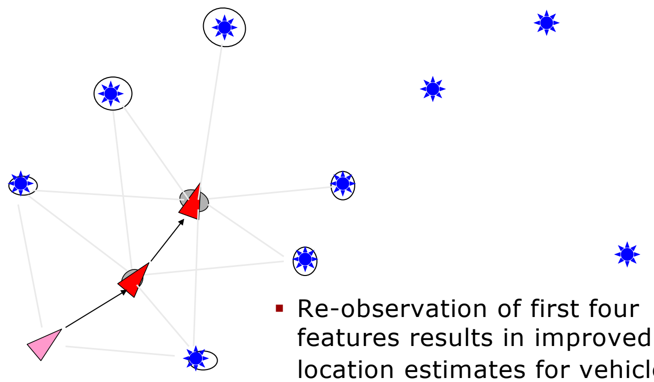
Illustration of SLAM with Landmarks



- Re-observation of first four features results in improved location estimates for vehicle and all features

Courtesy J. Leon

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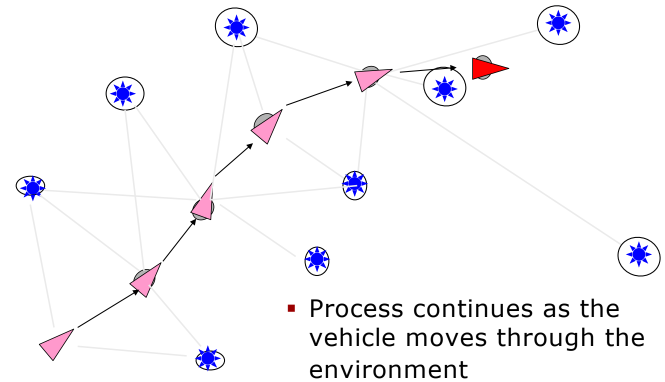
Illustration of SLAM with Landmarks



- Process continues as the vehicle moves through the environment

Courtesy J. Leon

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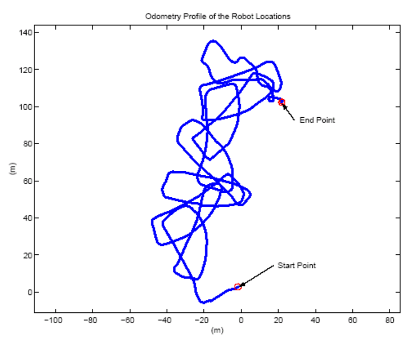
SLAM Using Landmarks



Courtesy J. Leon

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Test Environment (Point Landmarks)

Courtesy J. Leon

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View from Vehicle

Courtesy J. Leon

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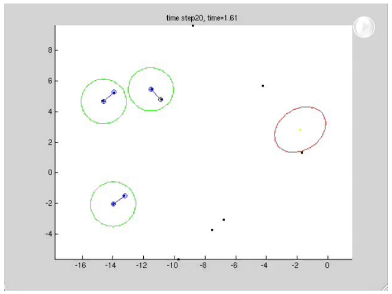
SLAM Using Landmarks

1. Move
2. Sense
3. Associate measurements with known features
4. Update state estimates for robot and previously mapped features
5. Find new features from unassociated measurements
6. Initialize new features
7. Repeat

MIT Indoor Track



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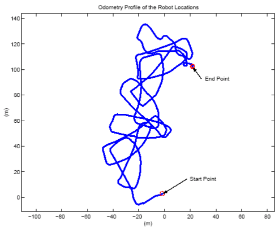
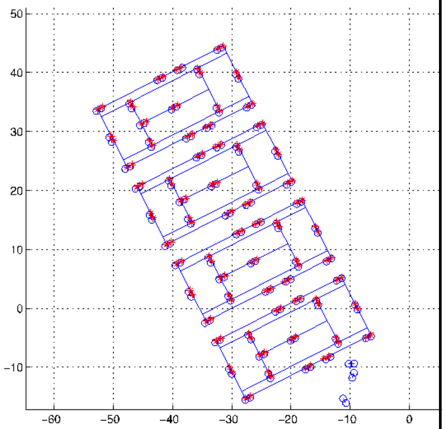
Comparison with Ground Truth



odometry



SLAM result

Courtesy J. Leon

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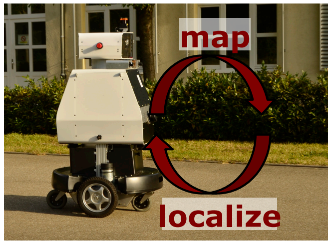
Simultaneous Localization and Mapping (SLAM)

- Building a map and locating the robot in the map at the same time
- Chicken-and-egg problem



Courtesy: Cyrill Stachni

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Definition of the SLAM Problem

Given

- The robot's controls

$$u_{1:T} = \{u_1, u_2, u_3, \ldots, u_T\}$$

- Observations

$$z_{1:T} = \{z_1, z_2, z_3, \ldots, z_T\}$$

Wanted

- Map of the environment

$$m$$

- Path of the robot

$$x_{0:T} = \{x_0, x_1, x_2, \ldots, x_T\}$$

Courtesy: Cyrill Stachni

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Two Main Paradigms

Kalman filter

Graph-based

Courtesy: Cyrill Stachni

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EKF SLAM

- Application of the EKF to SLAM
- Estimate robot's pose and locations of landmarks in the environment
- Assumption: known correspondences
- State space (for the 2D plane) is

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \ldots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}})^T$$

Courtesy: Cyrill Stachni

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EKF SLAM: State Representation

- Map with n landmarks: (3+2n)-dimensional Gaussian
- Belief is represented by

$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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EKF SLAM: State Representation

- More compactly

$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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EKF SLAM: State Representation

- Even more compactly (note: $x_R \to x$)

$$\underbrace{\begin{pmatrix} x \\ m \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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EKF SLAM: Filter Cycle

1. State prediction
2. Measurement prediction
3. Measurement
4. Data association
5. Update

Courtesy: Cyrill Stachni

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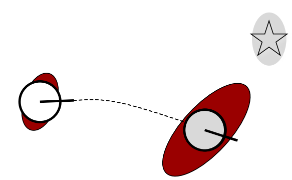
EKF SLAM: State Prediction



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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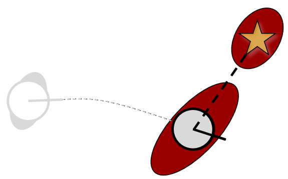
EKF SLAM: Measurement Prediction



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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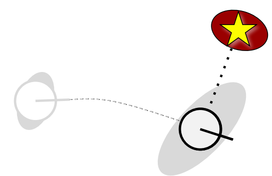
EKF SLAM: Obtained Measurement



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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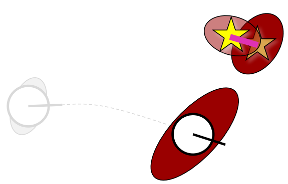
EKF SLAM: Data Association and Difference Between h(x) and z



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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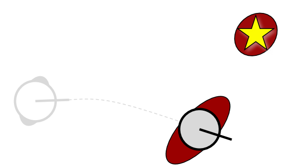
EKF SLAM: Update Step



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachni

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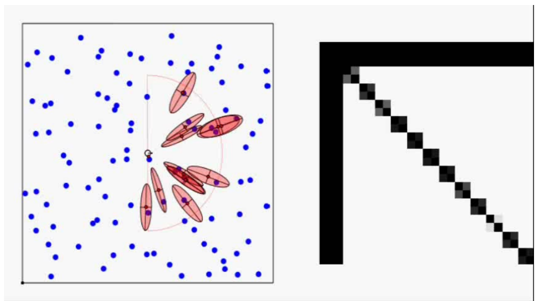
EKF SLAM Correlations



Blue path = true path **Red path** = estimated path **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] …
- Single hypothesis data association

Courtesy: M. Montemer

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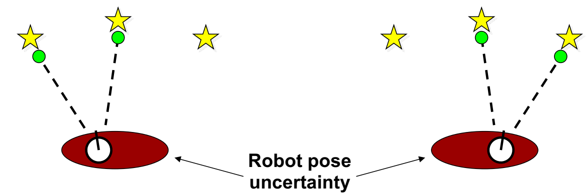
Data Association in SLAM



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
  - EKF SLAM is brittle in this regard
- Pose error correlates data associations

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Loop-Closing

- Loop-closing means recognizing an already mapped area
- Data association under
  - high ambiguity
  - possible environment symmetries
- Uncertainties **collapse** after a loop-closure (whether the closure was correct or not)

Courtesy: Cyrill Stachni

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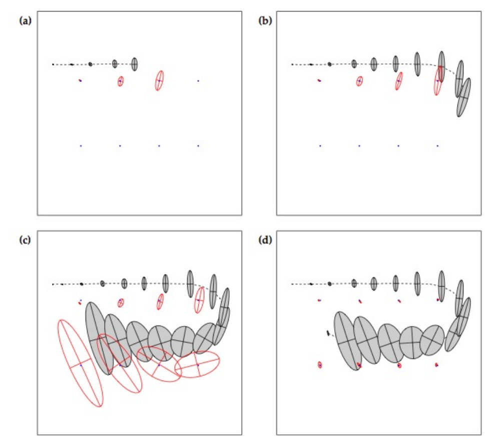
Online SLAM Example



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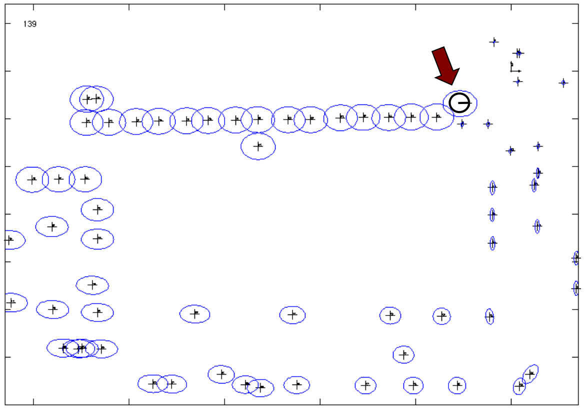
Before the Loop-Closure



Courtesy: K. Arras

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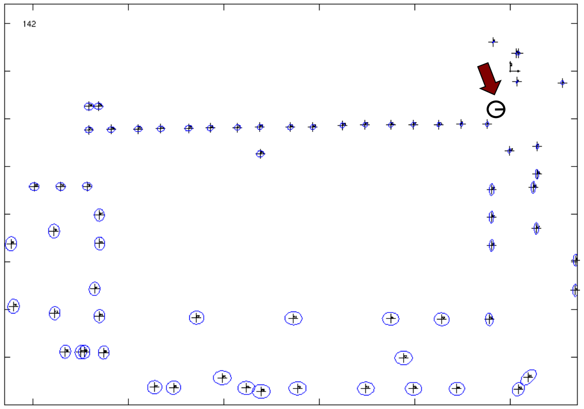
After the Loop-Closure



Courtesy: K. Arras

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Example: Victoria Park Dataset

Courtesy: E. Neb

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Victoria Park: Data Acquisition

Courtesy: E. Neb

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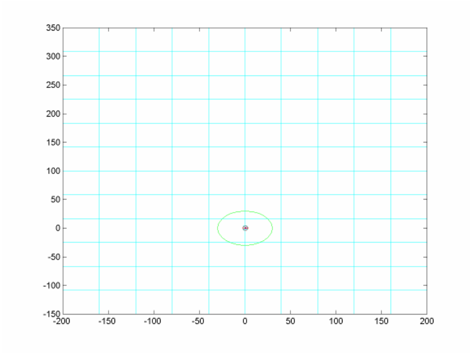
Victoria Park: EKF Estimate



Courtesy: E. Nebot

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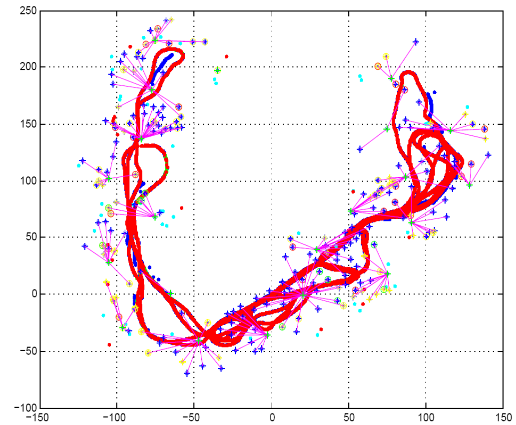
Victoria Park: EKF Estimate



Courtesy: E. Nebot

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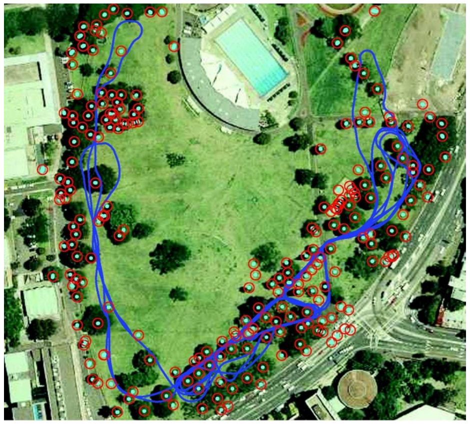
Victoria Park: Landmarks



Courtesy: E. Nebot

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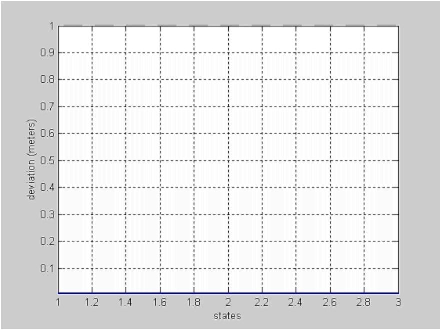
Victoria Park: Landmark Covariance



Courtesy: E. Nebot

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Andrew Davison: MonoSLAM

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EKF SLAM Summary

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

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EKF Algorithm

1. **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:
2. Prediction:
3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ $\longleftarrow$ $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$ $\longleftarrow$ $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$
5. Correction:
6. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ $\longleftarrow$ $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ $\longleftarrow$ $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ $\longleftarrow$ $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$
9. Return μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \qquad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

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Literature

EKF SLAM

- "Probabilistic Robotics", Chapter 10
- Smith, Self, & Cheeseman: "Estimating Uncertain Spatial Relationships in Robotics"
- Dissanayake et al.: "A Solution to the Simultaneous Localization and Map Building (SLAM) Problem"
- Durrant-Whyte & Bailey: "SLAM Part 1" and "SLAM Part 2" tutorials

Courtesy: Cyrill Stachni

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Graph-SLAM

- Full SLAM technique
- Generates probabilistic links
- Computes map only occasionally
- Based on Information Filter form

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Information Form

- Represent posterior in canonical form

$$\Omega = \Sigma^{-1} \quad \text{Information matrix}$$

$$\xi = \Sigma^{-1}\mu \quad \text{Information vector}$$

- One-to-one transform between canonical and moment representation

$$\Sigma = \Omega^{-1}$$

$$\mu = \Omega^{-1}\xi$$

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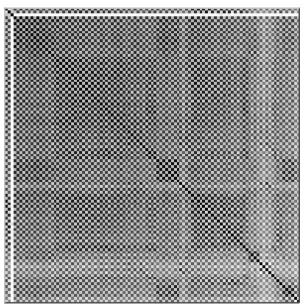
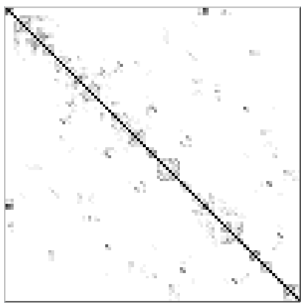
Information vs. Moment Form



Correlation matrix



Information matrix

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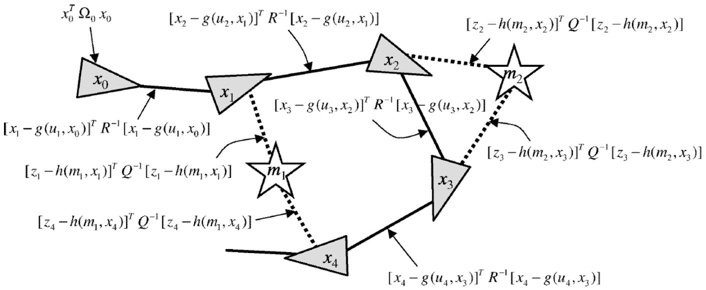
Graph-SLAM Idea



Sum of all constraints:

$$J_{\text{GraphSLAM}} = x_0^T \Omega_0 x_0 + \sum_t [x_t - g(u_t, x_{t-1})]^T R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)]^T Q^{-1} [z_t - h(m_{c_t}, x_t)]$$

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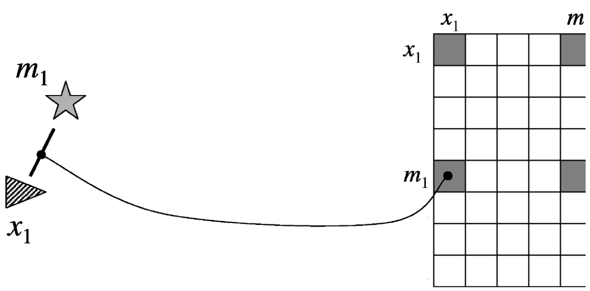
Graph-SLAM Idea (1)



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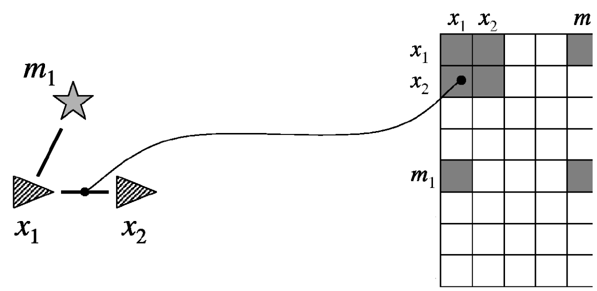
Graph-SLAM Idea (2)



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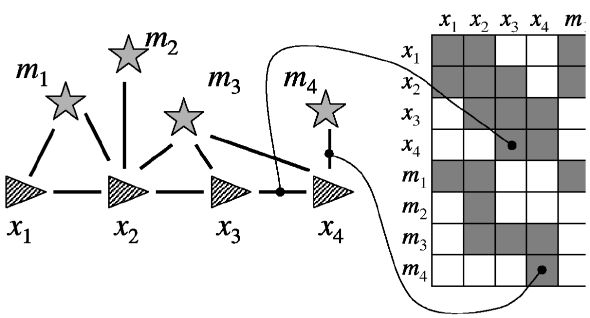
Graph-SLAM Idea (3)



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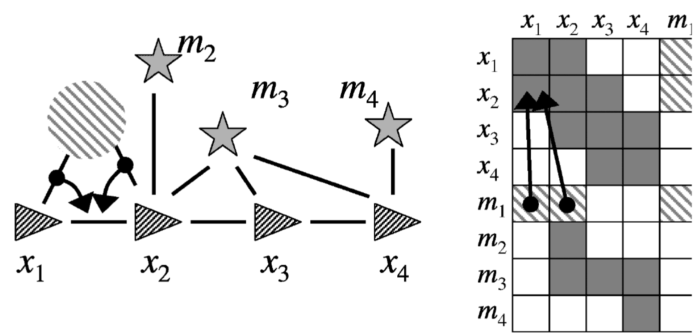
Graph-SLAM Inference (1)



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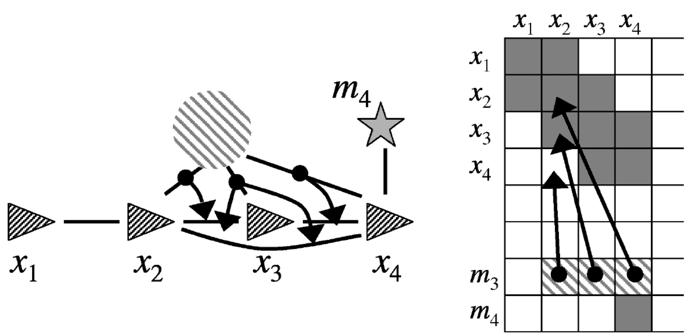
Graph-SLAM Inference (2)



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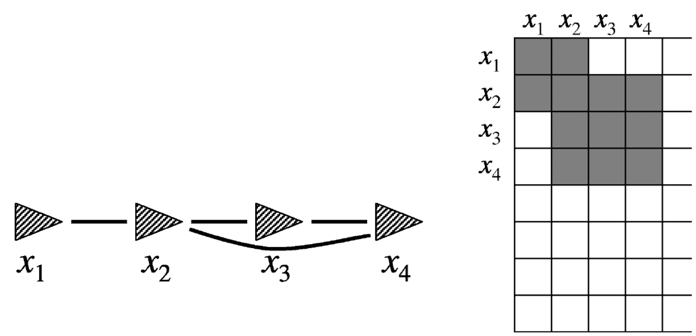
Graph-SLAM Inference (3)



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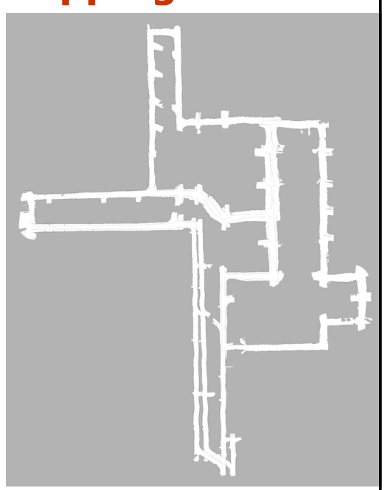
Mine Mapping



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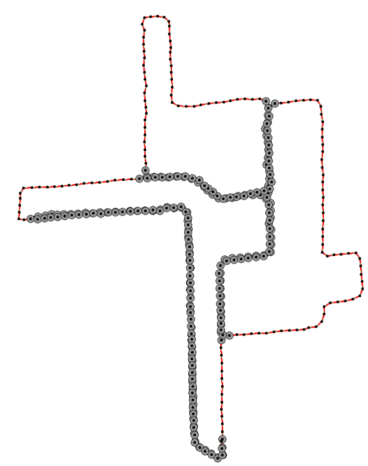
Mine Mapping: Data Associations



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Efficient Map Recovery

- Information matrix inversion can be avoided if only best map estimate is required
- Minimize constraint function $J_{GraphSLAM}$ using standard optimization techniques (gradient descent, Levenberg Marquardt, conjugate gradient)

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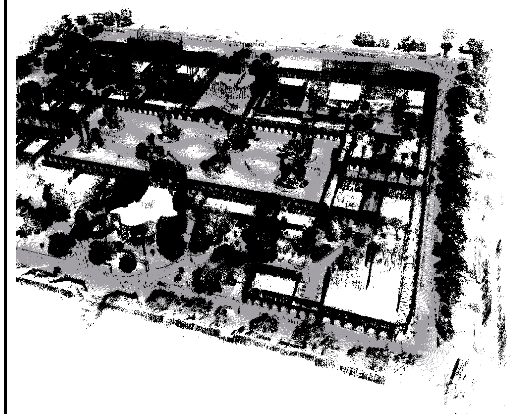
3D Outdoor Mapping



10^8 features, 10^5 poses, only few secs using cg.

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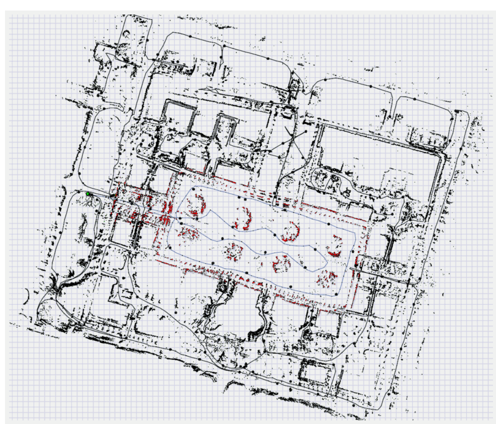
Map Before Optimization



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Map After Optimization

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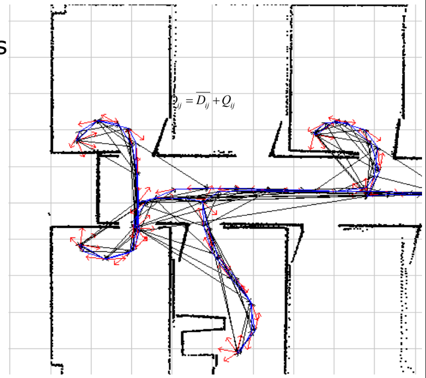
Robot Poses and Scans [Lu and Milios 1997]

- Successive robot poses connected by odometry
- Laser scan matching yields constraints between poses
- Loop closure based on map patches created from multiple scans



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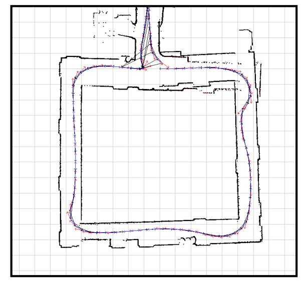
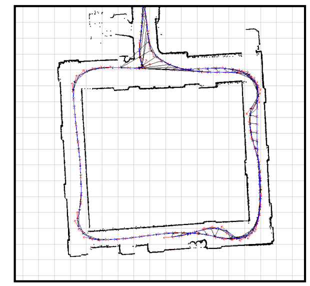
Loop Closure

- Use scan patches to detect loop closure
- Add new position constraints
- Deform the network based on covariances of matches



Before loop closure



After loop closure

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Mapping the Allen Center

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Graph-SLAM Summary

- Adresses full SLAM problem
- Constructs link graph between poses and poses/landmarks
- Graph is sparse: number of edges linear in number of nodes
- Inference performed by building information matrix and vector (linearized form)
- Map recovered by reduction to robot poses, followed by conversion to moment representation, followed by estimation of landmark positions
- ML estimate by minimization of $J_{GraphSLAM}$
- Data association by iterative greedy search

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