CSE-571
Robotics

Probabilistic Motion and Sensor Models

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$$Bel(x_t) = \eta \ P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1}$$

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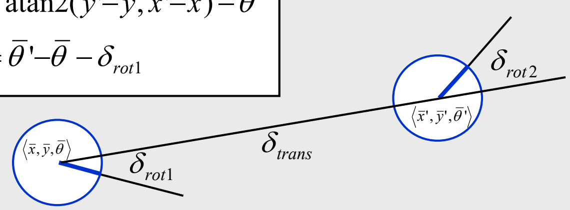
Probabilistic Kinematics

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$
$$\delta_{rot1} = \mathrm{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$
$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



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Noise Model for Motion

- The measured motion is given by the true motion corrupted with noise.

$$\hat{\delta}_{rot1} = \delta_{rot1} + \varepsilon_{\alpha_1 |\delta_{rot1}| + \alpha_2 |\delta_{trans}|}$$
$$\hat{\delta}_{trans} = \delta_{trans} + \varepsilon_{\alpha_3 |\delta_{trans}| + \alpha_4 |\delta_{rot1} + \delta_{rot2}|}$$
$$\hat{\delta}_{rot2} = \delta_{rot2} + \varepsilon_{\alpha_1 |\delta_{rot2}| + \alpha_2 |\delta_{trans}|}$$

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Sample Odometry Motion Model

1. Algorithm **sample_motion_model**(u, x):

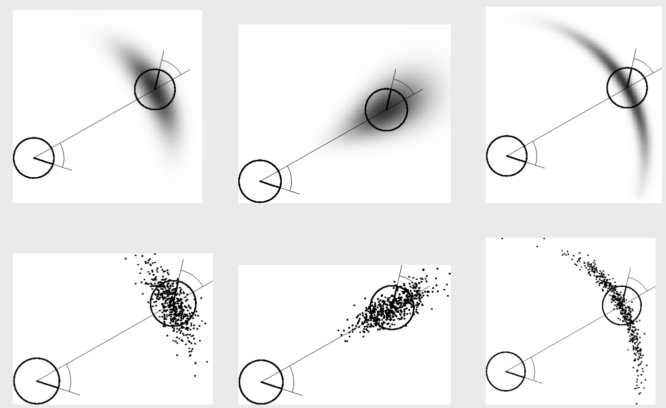
   $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle, x = \langle x, y, \theta \rangle$

1. $\hat{\delta}_{rot1} = \delta_{rot1} + \text{sample}(\alpha_1 |\delta_{rot1}| + \alpha_2\, \delta_{trans})$
2. $\hat{\delta}_{trans} = \delta_{trans} + \text{sample}(\alpha_3\, \delta_{trans} + \alpha_4\, (|\delta_{rot1}| + |\delta_{rot2}|))$
3. $\hat{\delta}_{rot2} = \delta_{rot2} + \text{sample}(\alpha_1 |\delta_{rot2}| + \alpha_2\, \delta_{trans})$

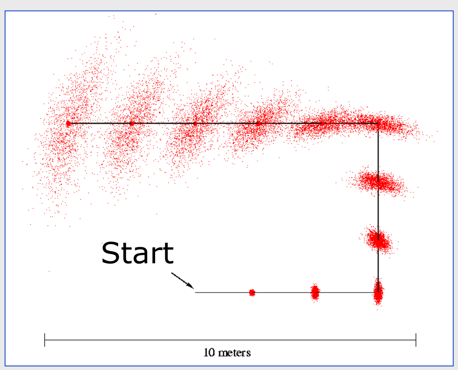
4. $x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$
5. $y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$
6. $\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

7. Return $\langle x', y', \theta' \rangle$

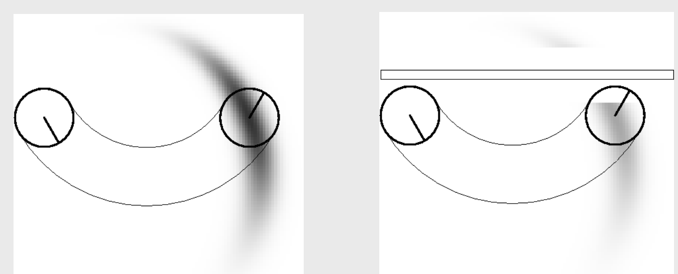
Examples (odometry based)



Sample-based Motion



Motion Model with Map



$P(x \mid u, x')$ $\quad$ $P(x \mid u, x', m) \approx P(x \mid m)\, P(x \mid u, x')$

- When does this approximation fail?

Probabilistic Sensor Models

Beam-based
Scan-based
Landmarks

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$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

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Sensors for Mobile Robots

- Contact sensors: Bumpers, touch sensors
- Internal sensors
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
  - Encoders, torque
- Proximity sensors
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors: Cameras, depth cameras
- Satellite-style sensors: GPS, MoCap

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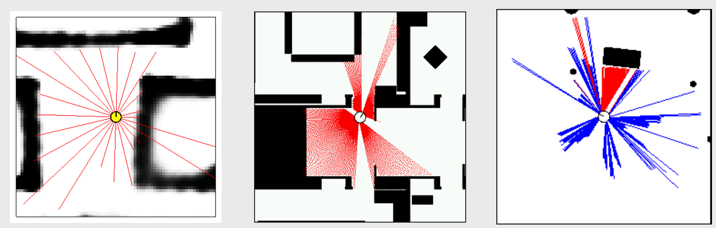
Proximity Sensors



- The central task is to determine *P(z|x)*, i.e. the probability of a measurement *z* given that the robot is at position *x* in a known map.
- **Question**: Where do the probabilities come from?
- **Approach**: Let's try to explain a measurement.

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Beam-based Sensor Model

- Scan z consists of K measurements.

$$z = \{z_1, z_2, \ldots, z_K\}$$

Beam-based Sensor Model

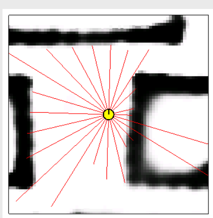
- Scan z consists of K measurements.

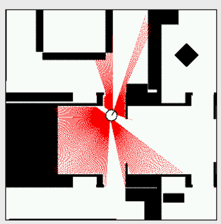
$$z = \{z_1, z_2, \ldots, z_K\}$$

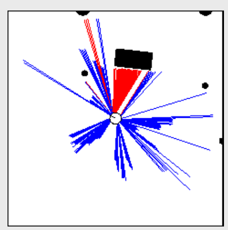
- Individual measurements are independent given the robot position and a map.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

Beam-based Sensor Model







$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

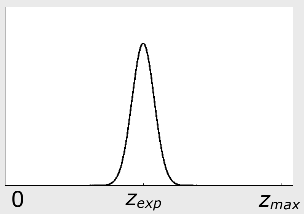
See book Section 6.3.4 on exponential smoothing of model.

Proximity Measurement

- Measurement can be caused by …
  - a known obstacle.
  - cross-talk.
  - an unexpected obstacle (people, furniture, …).
  - missing all obstacles (total reflection, glass, …).
- Noise is due to uncertainty …
  - in measuring distance to known obstacle.
  - in position of known obstacles.
  - in position of additional obstacles.
  - whether obstacle is missed.

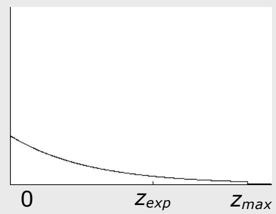
Beam-based Proximity Model

Measurement noise



$$P_{hit}(z \mid x,m)=\eta \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(z-z_{exp})^2}{\sigma^2}}$$

Unexpected obstacles



$$P_{unexp}(z \mid x,m)=\eta\, \lambda\, e^{-\lambda z}$$

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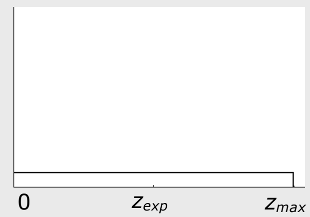
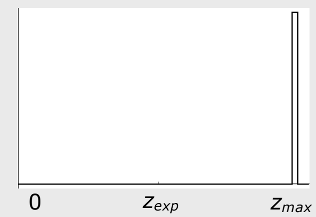
Beam-based Proximity Model

Random measurement



$$P_{rand}(z \mid x,m)=\eta \frac{1}{z_{max}}$$

Max range



$$P_{max}(z \mid x,m)=\eta \frac{1}{z_{small}}$$

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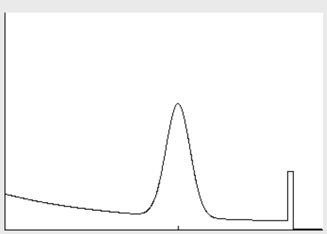
Mixture Density



$$P(z \mid x,m)=\begin{pmatrix}\alpha_{\text{hit}}\\ \alpha_{\text{unexp}}\\ \alpha_{\text{max}}\\ \alpha_{\text{rand}}\end{pmatrix}^T \cdot \begin{pmatrix}P_{\text{hit}}(z \mid x,m)\\ P_{\text{unexp}}(z \mid x,m)\\ P_{\text{max}}(z \mid x,m)\\ P_{\text{rand}}(z \mid x,m)\end{pmatrix}$$

How can we determine the model parameters?

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Approximation

- Maximize log likelihood of the data z:

$$P(z \mid z_{\text{exp}})$$

- Search parameter space.
- EM to find mixture parameters
  - Assign measurements to densities.
  - Estimate densities using assignments.
  - Reassign measurements.

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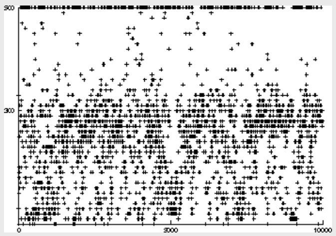
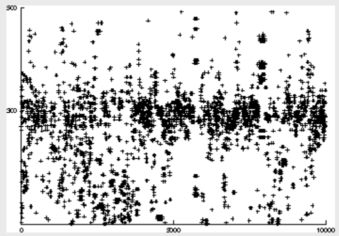
Raw Sensor Data

Measured distances for expected distance of 300 cm.



Sonar



Laser

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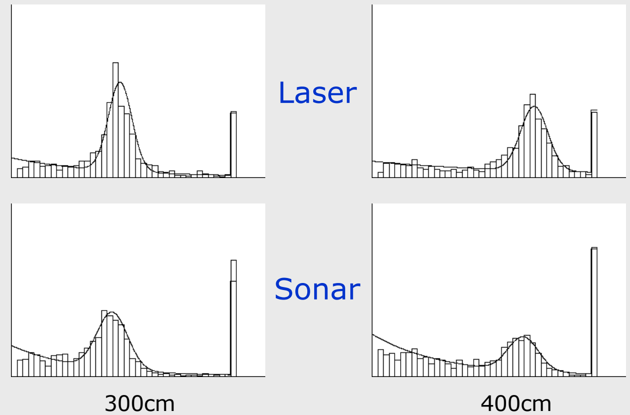
Approximation Results



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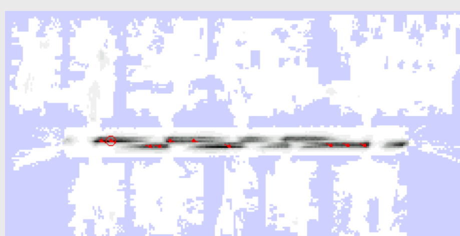
Example

z



$P(z|x,m)$

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Summary Beam-based Model

- Assumes independence between beams.
  - Justification?
  - Overconfident!
- Models physical causes for measurements.
  - Mixture of densities for these causes.
- Implementation
  - Learn parameters based on real data.
  - Different models can be learned for different angles at which the sensor beam hits the obstacle.
  - Determine expected distances by ray-tracing.
  - Expected distances can be pre-processed.

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Scan-based Model

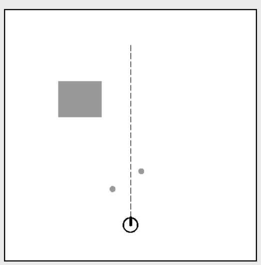
- Beam-based model is …
  - not smooth for small obstacles and at edges
  - not very efficient.

- Idea: Instead of following along the beam, just check the end point.

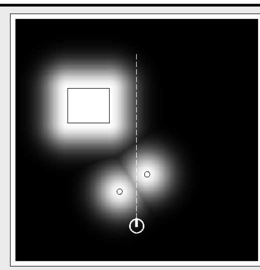
Scan-based Model

- Probability is a mixture of …
  - a Gaussian distribution with mean at distance to closest obstacle,
  - a uniform distribution for random measurements, and
  - a small uniform distribution for max range measurements.
- Again, independence between different components is assumed.

Example

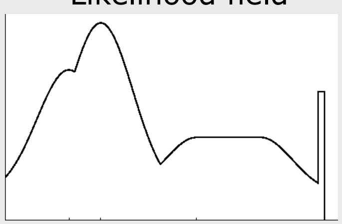


Map *m*



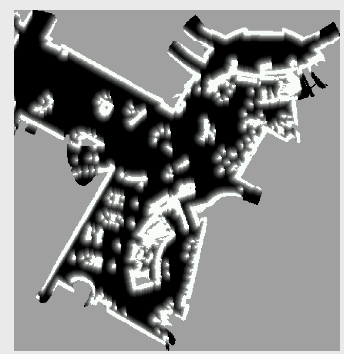
Likelihood field

$P(z|x,m)$



San Jose Tech Museum

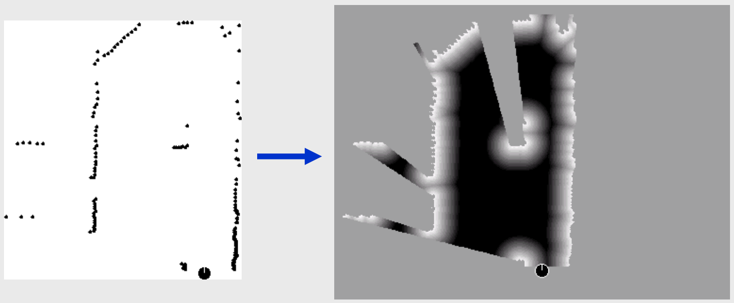
Occupancy grid map



Likelihood field

Scan Matching

- Extract likelihood field from scan and use it to match different scan.



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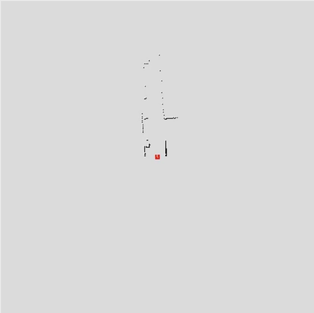
Scan Matching

- Extract likelihood field from first scan and use it to match second scan.



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Properties of Scan-based Model

- Highly efficient, uses 2D tables only.
- Smooth w.r.t. to small changes in robot position.
- Allows gradient descent, scan matching.
- Ignores physical properties of beams.

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Additional Models of Proximity Sensors

- Map matching (sonar,laser): generate small, local maps from sensor data and match local maps against global model.
- Scan matching (laser): map is represented by scan endpoints, match scan into this map using ICP, correlation.
- Features (sonar, laser, vision): Extract features such as doors, hallways from sensor data.

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Landmarks

- Active beacons (*e.g.* radio, GPS)
- Passive (*e.g.* visual, retro-reflective)
- Standard approach is triangulation

- Sensor provides
  - distance, or
  - bearing, or
  - distance and bearing.

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Distance and Bearing

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Probabilistic Model

1. Algorithm **landmark_detection_model**(z,x,m):
   $z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$
2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$
3. $\hat{\alpha} = \operatorname{atan2}(m_y(i) - y, m_x(i) - x) - \theta$
4. $p_{\det} = \operatorname{prob}(\hat{d} - d, \varepsilon_d) \cdot \operatorname{prob}(\hat{\alpha} - \alpha, \varepsilon_\alpha)$
5. Return $z_{\det} p_{\det} + z_{\mathrm{fp}} P_{\mathrm{uniform}}(z \mid x, m)$

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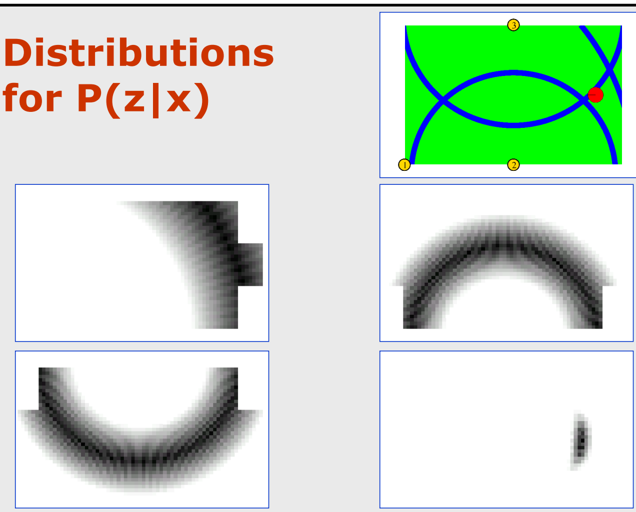
Distributions for P(z|x)



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Summary of Parametric Motion and Sensor Models

- Explicitly modeling uncertainty in motion and sensing is key to robustness.
- In many cases, good models can be found by the following approach:
  1. Determine parametric model of noise free motion or measurement.
  2. Analyze sources of noise.
  3. Add adequate noise to parameters (eventually mix densities for noise).
  4. Learn (and verify) parameters by fitting model to data.
  5. Likelihood of measurement is given by "probabilistically comparing" the actual with the expected measurement.
- It is important to be aware of the underlying assumptions!