CSE567: Digital Systems Design

Homework 2 Solution

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1 Problem 1

The identity is false. This can be seen by writing out the truth table for each side of the equality.

2 Problem 2a

\[
\begin{align*}
 f(A, B, C, D, E) &= (A + BC + \bar{D})(\bar{B}C + \bar{D} + E)(A + \bar{E})(AD + E) \\
 f(A, B, C, D, E) &= (ABC + AD + AE + BC + BCD + BCE + D + \bar{DE})(AD + AE + \bar{AD}E) \\
 f(A, B, C, D, E) &= (AE + \bar{BC} + \bar{D})(AD + AE) \\
 f(A, B, C, D, E) &= (AE + \bar{BC} + \bar{D})(AD + AE) \\
 f(A, B, C, D, E) &= (AE + ABCD)
\end{align*}
\]

3 Problem 2b

\[
\begin{align*}
 f(A, B, C, D, E) &= (\bar{A} + B\bar{E})(B\bar{E} + C + D)(\bar{C} + E) \\
 f(A, B, C, D, E) &= (\bar{A}C + \bar{A}D + B\bar{E})(\bar{C} + E) \\
 f(A, B, C, D, E) &= (\bar{A}CE + \bar{A}CD + \bar{A}DE + B\bar{C}\bar{E})
\end{align*}
\]

4 Problem 3a

\[
\begin{align*}
 f(A, B, C, D, E) &= (AB) + (CD) + (A\bar{C}) + (\bar{C}E) \\
 f(A, B, C, D, E) &= (A(B + \bar{C})) + (D(C + \bar{E})) \\
 f(A, B, C, D, E) &= (A + D)(A + C + \bar{E}) + (B + \bar{C} + D)
\end{align*}
\]

5 Problem 3b

\[
\begin{align*}
 f(A, B, C, D, E, F) &= AB(\bar{C} + D)\bar{E} + F \\
 f(A, B, C, D, E, F) &= (AB + F)(\bar{C} + D + F)(E + F) \\
 f(A, B, C, D, E, F) &= (A + F)(B + F)(\bar{C} + D + F)(E + F)
\end{align*}
\]
6 Problem 4a

\[ f(A, B, C, D) = D(A + BC) + \bar{D}(\bar{A}(\bar{B} + \bar{C})) \]

7 Problem 4b

The best way to do this problem is to notice that an or gate is identical to a nand gate with both its inputs negated. Now, convert all of the AND gates to NAND gates. Look at the left most OR gate. If we convert it to a NAND gate it will already have one of its inputs negated (the NAND gate does this to BC). We just have to negate the other input and we have an OR gate, so add an inverter to the A signal. Now look at the rightmost OR gate and change it to a NAND gate. Because both of its inputs come from NAND gates, its inputs are already negated, and it still acts as an OR gate.

8 Problem 5

If a function and its dual are equivalent, then the complement of the function and the complement of the dual are also equivalent. The complement of the dual is the same as the function with its inputs inverted (the only difference between duality and deMorgan’s law is that deMorgan’s law also flips the signs of the variables). Thus, we need a function that, when its inputs are complemented, produces the same result as if its output was complemented. In a truth table this means that whenever a sequence is true, its complement must be false (so, for example, if 001 is true, 110 needs to be false). This yields 16 possible functions that are the dual of themselves in the three-variable case.

9 Problem 6

Anything that needs an inverter cannot be implemented with only AND and OR gates.

10 Problem 7

You cannot implement either AND or OR using just XOR gates.
11 Problem 8a
Problem 8b

\[ f(w, x, y, z) = w'xy'z' + w'y'z + xy'z + w'xz + w'x'yz + wz \]
13 Problem 8c