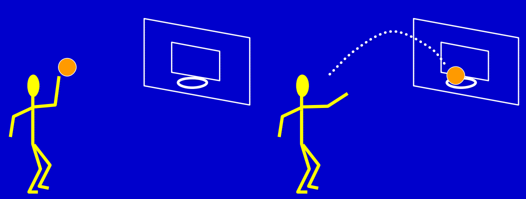
Spacetime Optimization

Simulation based methods

- Forward simulation [e.g. Baraff, Mirtich]
  - Highly realistic
  - Simulated character very hard to control
- Controllers [Raibert, Hodgins, Ngo, van de Pane]
  - Fast motion generation once controllers are computed
  - No set rules on controller generation

Spacetime constraints

- Animation is an optimal motion that achieves a given set of tasks
- Provides both realism and control



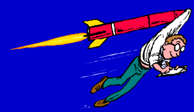
Simulation vs. Spacetime

Forward simulation
- initial value problem

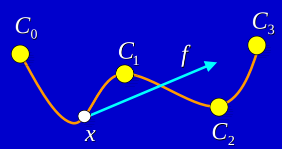
Spacetime constraints
- two-point boundary problem
- muscle forces vary as functions through time

Spacetime particle

A particle with a jet engine

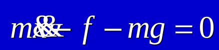


- Interpolate points at specific times
- Be fuel efficient



Equations of motion

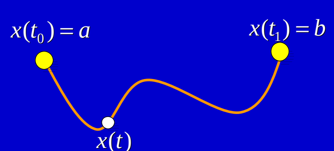
- Particle's position as a function of time *x(t)*
- Particle's mass *m*
- Time-varying jet force *f(t)*
- Constant gravitational force *mg*



$$m\ddot{x} - f - mg = 0$$

Constraints

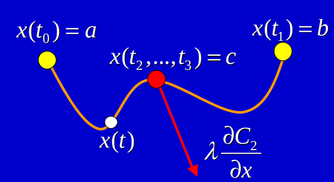
Fly from point *a* to point *b* in a fixed time period t_1-t_0



Mechanical constraints

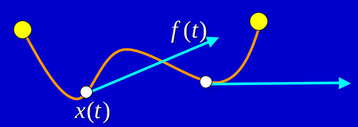
Constraints imposed by the environment

- – Forces which can act to satisfy the constraint



Jet engine “Muscle”

Force applied in arbitrary direction



Objective function

Minimize the rate of fuel consumption

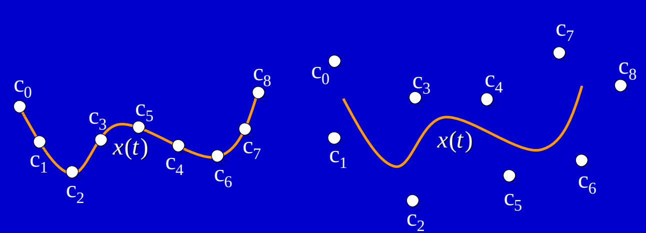
Proportional to the force magnitude integral

$$E = \int_{t_0}^{t_1} \| f(t) \|^2 \, dt$$

DOF representation

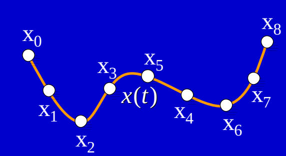
$$x_i(c_0^i, \ldots, c_n^i; t)$$
$$f_j(c_0^j, \ldots, c_n^j; t)$$

Defined in arbitrary basis:



Computing derivatives

Discretized samples use finite differences



$$\dot{x}_i = \frac{x_i - x_{i-1}}{h}$$

$$\ddot{x}_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2}$$

Constraints formulation

- Newtonian constraint

$$n_i = m\frac{x_{i+1} - 2x_i + x_{i-1}}{h^2} - f_i - mg = 0 \quad 1 < i < n$$

- Boundary constraints

$$c_a = x_1 - a = 0$$
$$c_b = x_n - b = 0$$

- Objective function

$$E = h\sum_i \| f_i \|^2$$

$$\underset{x_i, f_i}{\text{minimize}} \quad E \qquad \text{subject to} \quad \begin{Bmatrix} \begin{bmatrix} n_i & 1 < i < n \\ c_a & \\ c_b & \end{bmatrix} \end{Bmatrix}$$

Constraint derivatives

$$\frac{\partial n_i}{\partial x_j} = \begin{cases} 2m/h^2, & i = j \\ -m/h^2, & i \pm 1 \\ 0, & otherwise \end{cases}$$

$$\frac{\partial n_i}{\partial f_j} = \begin{cases} 1, & i = j \\ 0, & otherwise. \end{cases}$$

Objective function derivatives

$$\frac{\partial E}{\partial f_j} = 2 f_i$$

$$\frac{\partial^2 E}{\partial f_i \partial f_j} = \begin{cases} 2, & i = j \\ 0, & otherwise. \end{cases}$$

Spacetime optimization of complex structures

When optimizing a complex mechanical structure defined by its degrees of freedom

$$[q_0, q_1, \ldots, q_n]$$

things get a lot more complicated

- Newtonian constraints become significantly more complex
- Need to convert forces into generalized forces

Deriving Newtonian constraints

Start with Lagrange's equations of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} - Q = 0$$

Derive kinetic energy T and generalized forces Q

Newtonian transformation hierarchies

$$n_j = \sum_i \left[tr\left(\frac{\partial W_i}{\partial q_j} M_i \ddot{W}_i^T \right) + m_i g \frac{\partial W_i}{\partial q_j} c_i \right] +$$
$$\sum_k \left[\frac{\partial F_k p_k}{\partial q_j} \right] +$$
$$\sum_l \left[q_l^{\lambda} \frac{\partial C_{m_l}}{\partial q_j} \right]$$

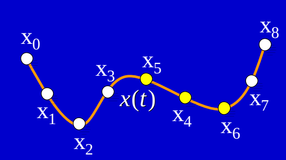
Muscles

Muscle force proportional to the difference between the current and desired parameter value

$$f_i = k_i \left(q_j^m - q_j \right)$$

Wavelet representation

- Fewer coefficients in flat regions
- Coefficients affects larger time intervals which leads to faster convergence



Advantages

- Intuitive constraint specification
- Change the feel of motion by modifying the objective function
- Produces natural looking not just physically correct motion

Importance of a good initial position

- Does not converge if the starting point is too far from the solution
- Hard to find the constraint hyper-surface
- Explosion of the number of unknowns

Parameter and constraint explosion

- Parameter space is proportional to
  - Number of DOFs
  - Length of the optimized time period
- Constraint count is proportional to the time period
- Constraint complexity is proportional to the number of DOFs