Interactive Control of Rigid Body Motion

Keyframing
- Interpolates motion from "key" positions
- Perfect control over the motion
- Realistic look difficult to achieve

Simulation
- Solves equations to compute motion
- Perfect realism of the motion
- The motion is difficult to control
Production

Story dictates the motion
Artists must control the behavior

Rigid Body Motion

Interactive Control
- Extended rigid body dynamics
- Differential control

Problem Statement

Compute a rigid body motion that achieves the desired goals

Problem Statement

Compute control parameters $u$ such that $q(t_i) = c_i$ at times $t_i = t_0, \ldots, t_n$
Main Challenges

The function is nonlinear
- Motion is a solution to nonlinear DEs
- The function domain is high dimensional
  - For a single rigid body $u \in \mathbb{R}^3 \times \text{SO}(3) \times \mathbb{R}^3 \times \mathbb{R}^3$
- The function is discontinuous
  Artist must control the behavior
Extended Dynamics

Plausible dynamics instead of correct

\[ q_0 = \begin{pmatrix} x_0 \\ \theta_0 \\ v_0 \\ \omega_0 \end{pmatrix} \quad \ddot{q} = \frac{F}{M} \quad u = \begin{pmatrix} q_0 \\ n \end{pmatrix} \quad q = \frac{F}{M} \]
Extended Dynamics

Plausible dynamics instead of correct

\[ u = \begin{pmatrix} q_0 \\ n \\ \vdots \end{pmatrix} \quad \dot{q} = \frac{F}{M} \]

Differential Control

\[ q' = S(t, u) \]

Interactive Control

1) Evaluate \( \delta c_i \) for current parameters \( u \)

2) Compute \( \delta u \) such that \( \delta c_i = \frac{\partial S(t, u)}{\partial u} \delta u \)

3) Update parameters \( u' = u + \varepsilon \delta u \)

4) Repeat with \( u = u' \)
Important Details

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Simulation must be fast

Polygonal bodies

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Derivative evaluation

- Finite differences are slow and inaccurate
- Specialized automatic differentiation technique

Important Details

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Convergence

- Interaction
- Local sampling

Discrete Domain

\( S(t, u) \)
\[ q_{1c} = F_1(u) \]

\[ q_{t_{c1}} = C(F_1(u), u) \]

\[ S(t, u) = F_2(C(F_1(u), u), u) \]

\[ S(t, u) = F_2 \circ C \circ F_1 \]
Main Challenge

*Differential Control assumes $S(t,u)$ is continuous with respect to $u* 

Discrete search/optimization

Following Derivatives

*Works well for “local” discrete change*

Convergence
Discrete Domain

Discrete Search/Optimization

Sampling

Summary

*Interactive Control*

- Continuous optimization
  - Differential Control
- Discrete optimization
  - Sampling
  - Randomized Path Planning