Subdivision surfaces

We can extend the idea of subdivision from curves to surfaces...

Building complex models

Subdivision surfaces

Chaikin’s use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a control polyhedron (or control mesh) to produce the limit surface

\[ \sigma = \lim_{j \to \infty} M^j \]

using splitting and averaging steps.

Reading

Recommended:

Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:

Loop averaging step

Once again we can use masks for the averaging step:

\[ Q_{\text{avg}} = \frac{\alpha(n)Q_1 + \cdots + Q_n}{\alpha(n) + n} \]

where

\[ \alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = 1 - \frac{3 + 2\cos(\pi/n))^2}{32} \]

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as \( G^1 \) continuity for surfaces.

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.

\[ Q^{(n)} = \frac{\varepsilon(n)Q + \cdots + Q_n}{\varepsilon(n) + n} \]

\[ T_{1}^{(n)} = \tau_1(n)Q_1 + \cdots + \tau_n(n)Q_n \]

\[ T_{2}^{(n)} = \tau_1(n)Q_1 + \cdots + \tau_{n-1}(n)Q_{n-1} \]

where

\[ \varepsilon(n) = \frac{3n}{\beta(n)} \quad \tau_i(n) = \cos(2\pi i/n) \]

Note that the eigenvalues of the related subdivision matrix have the form: \( \lambda_1 > \lambda_2 > \lambda_3 > \cdots \geq \lambda_n \geq 0 \).

How do we compute the normal?

Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!
Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask:

This gives rise to $G^0$ continuous surfaces (i.e., having positional but not tangent plane continuity)

Face schemes

4:1 subdivision of triangles is sometimes called a face scheme for subdivision, as each face begets more faces.

An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:

Catmull-Clark subdivision:

Note: after the first subdivision, all polygons are quadrilaterals in this scheme.

Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

This particular example uses the hybrid technique of DeRose, et al., which applies sharp subdivision rules at some creases for a finite number of steps, and then switches to smooth subdivision, giving more gentle creases. This technique was used in Geri's Game.

Vertex schemes

In a vertex scheme, each vertex begets more vertices. In particular, a vertex surrounded by $n$ faces is split into $n$ subvertices, one for each face:

Doo-Sabin subdivision:

The number edges (faces) incident to a vertex is called its valence. Edges with only once incident face are on the boundary. After splitting in this subdivision scheme, all non-boundary vertices are of valence 4.
Interpolating subdivision surfaces

Interpolating schemes are defined by

- splitting
- averaging only new vertices

The following averaging mask is used in butterfly subdivision:

Setting $t=0$ gives the original polyhedron, and increasing small values of $t$ makes the surface smoother, until $t=1/8$ when the surface is provably $C^1$. 