Useful equations

Phong Shading Model

$$I_{phong} = k_e + k_a I_a + \sum_j \left[I_{l_j} \left[k_d \left(\mathbf{N} \cdot \mathbf{L}_j \right)_+ + k_s \left(\mathbf{V} \cdot \mathbf{R}_j \right)_+^{n_s} \right] \min\left\{ 1, \frac{1}{a_0 + a_1 d_j + a_2 d_j^2} \right\} \right]$$

Raytracing Model

$$I_{pixel} = I_{phong} + k_r I_{reflect} + k_t I_{transmit}$$

Reflection Angle

$$\mathbf{r} = 2(\mathbf{l} \cdot \mathbf{n})\mathbf{n} - \mathbf{l}$$

Transmission Angle

$$\eta = \frac{\eta_i}{\eta_t}$$

$$\cos\theta_i = \mathbf{n} \cdot \mathbf{v}$$

$$\cos\theta_t = \sqrt{1 - \eta^2 (1 - \cos^2 \theta_i)}$$

Note that Total Internal Reflection occurs when the square root is imaginary.

$$\mathbf{t} = (\eta \cos\theta_i - \cos\theta_t)\mathbf{n} - \eta\mathbf{v}$$

Cross Product (implemented as ‘vector.cross(othervector)’ in vecmath.h)

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} bz - cy \\ cx - az \\ ay - bx \end{bmatrix}$$

Finding two perpindicular vectors both perpindicular to a given input vector

$$\mathbf{I} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\mathbf{O_1} = \left\| \begin{matrix} 1 \\ 1 \\ (a+b)/c \end{matrix} \right\|$$

$$\mathbf{O_2} = \left\| \mathbf{O_1} \times \mathbf{I} \right\|$$

Rectangular to Polar

$$r = \left| \begin{matrix} x \\ y \\ z \end{matrix} \right|$$

$$\phi = \cos^{-1} \frac{z}{r}$$

$$\theta = \sin^{-1} \frac{y}{r \sin \phi} = \cos^{-1} \frac{x}{r \sin \phi}$$

$$x = r \cos \theta \sin \phi$$
$$y = r \sin \theta \sin \phi$$
$$z = r \cos \phi$$